Dimensional Crossover in a Spin-imbalanced Fermi Gas

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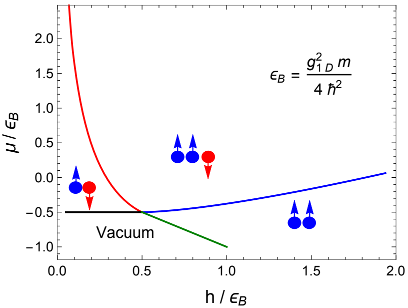
Phase diagram in 1D : Bethe Ansatz

$$\hat{H}_{1\text{D}} = \int dz \Big[\sum_{\sigma=\uparrow,\downarrow} \hat{\psi}_\sigma^\dagger(z) \big(\hat{H}^{\text{sp}} - \mu_\sigma \big) \hat{\psi}_\sigma(z) + g_{1\text{D}} \hat{\psi}_\uparrow^\dagger(z) \hat{\psi}_\downarrow^\dagger(z) \hat{\psi}_\downarrow(z) \hat{\psi}_\uparrow(z) \Big]$$



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- No long-range order

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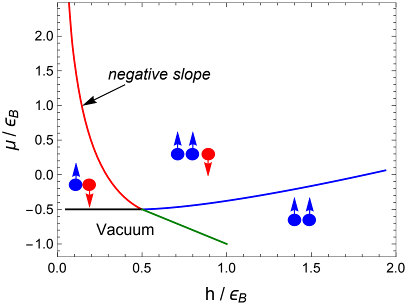

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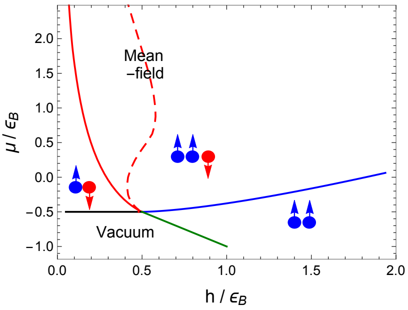

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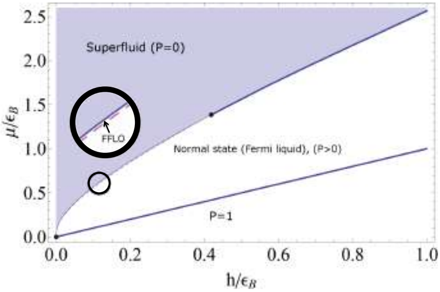

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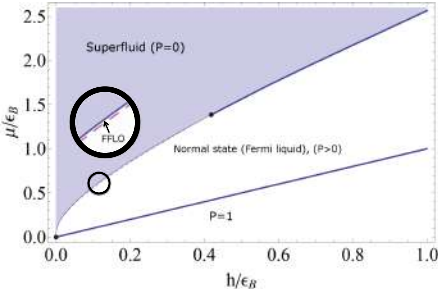

- Small FFLO region (weak nesting)
- Long-range order
- Positive slope
- Other phases proposed : Deformed Fermi surface, Mixed phase, etc.

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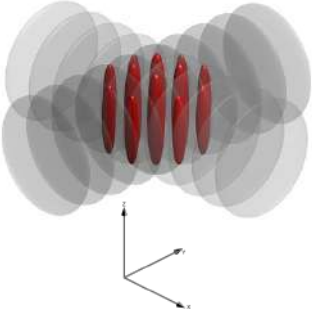

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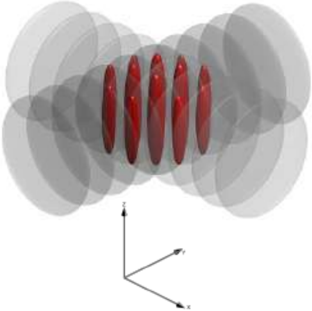

Controllable parameters : lattice depth, densities, interaction strength

Application : realizing the 1D model

Necessary conditions :

- $V_0/E_R \gg 1 \implies J \to 0$ (isolated tubes)
- Low density, and T $\ll$ band-gap

  $\implies$ transverse motion frozen to the lowest energy level

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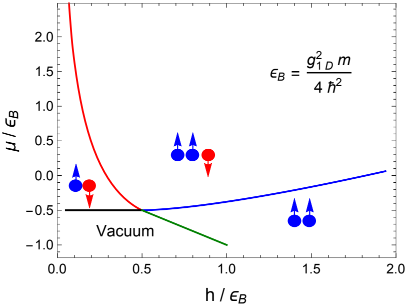
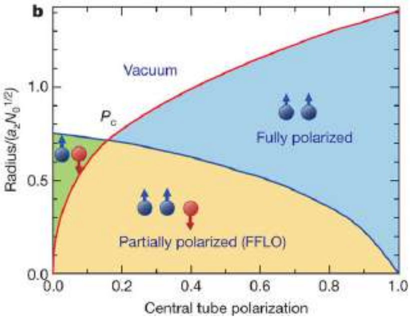
Olshanii's mapping to an effective 1D model :

$$\frac{d_\perp \hbar\omega_\perp}{g_{1\mathrm{D}}} = \frac{d_\perp}{2a_s} + \frac{\zeta(1/2)}{2\sqrt{2}}$$

Agrees with experiment! (near unitarity)





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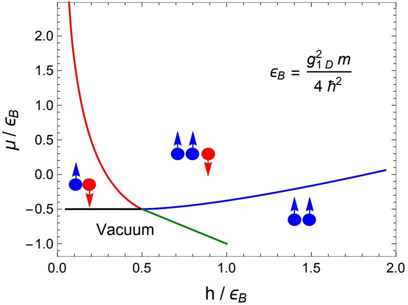

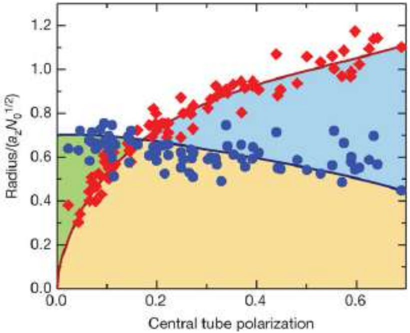

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Assumptions :

- J small $\rightarrow$ use tight-binding model
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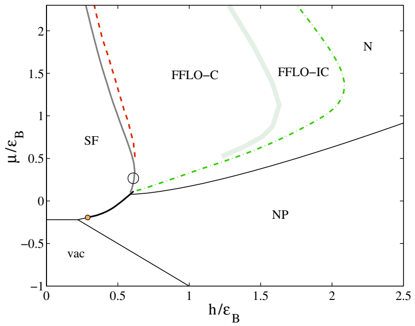

PRL **99**, 250403 (2007) : $J/\varepsilon_B = 0.08$

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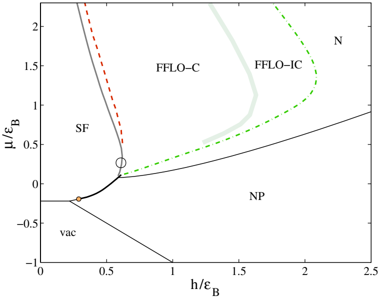

- 1D-like structure for all interactions for small J.

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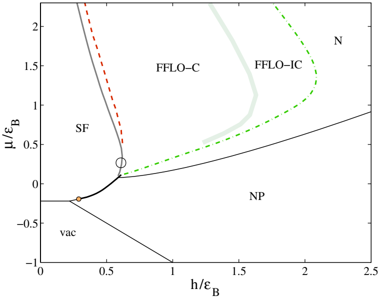


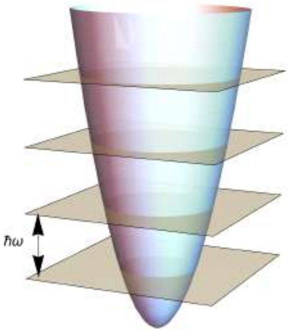
- 1D-like structure for all interactions for small J.
- predicts a turning point

  $\rightarrow$ not seen in experiments

  $\rightarrow$ need a more accurate model

What we did

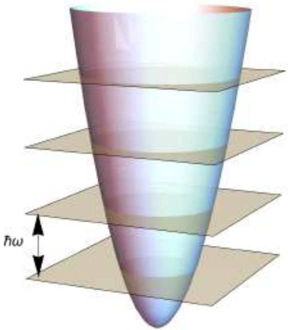
- Consider a single tube - model as a cylindrical harmonic trap
- 1D $\rightarrow$ 3D crossover happens as density (or μ) increases



What we did

- Consider a single tube - model as a cylindrical harmonic trap
- 1D $\rightarrow$ 3D crossover happens as density (or μ) increases
- Find mean-field phase diagram as a function of a_s and T
- Map to an effective 1D model for $\mu < 2\hbar\omega_\perp$

  $\rightarrow$ density corrections to Olshanii's mapping



Setting up the equations

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Diagonalize the BdG Hamiltonian :

$$\hat{H}^{\mathrm{MF}} = \sum_n [(E_n - h)\hat{\gamma}^\dagger_{n_\uparrow}\hat{\gamma}_{n_\uparrow} + (E_n + h)\hat{\gamma}^\dagger_{n_\downarrow}\hat{\gamma}_{n_\downarrow} + (\varepsilon_n - E_n)] - \frac{1}{g}\int d^3r |\Delta(\vec{r})|^2$$

where $\begin{pmatrix} \hat{H}^{\mathrm{sp}} - \mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & \mu - \hat{H}^{\mathrm{sp}} \end{pmatrix} \begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix} = E \begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix}, \quad E_n \geqslant 0$

Regularization

Ground state energy ($T = 0$) :

$$\mathcal{E} = \sum_n [\alpha(E_n - h) + \varepsilon_n - E_n] - \frac{1}{g} \int d^3r \, |\Delta(\vec{r})|^2 \, .$$

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For large n, $|\varepsilon_n - E_n| \ll \varepsilon_n \implies$ use perturbation theory

$$\implies \mathcal{E} = \mathcal{E}_{\text{exact}} \underbrace{- \sum_n{}' \langle n|\hat{\Delta}\hat{\Delta}^\dagger|n\rangle/(2\varepsilon_n) - \frac{1}{g}\int d^3r |\Delta(\vec{r})|^2}_{\text{divergences cancel out}}$$

Ansatz for $\Delta(\vec{r})$

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- Variational parameters : Δ_0, ξ, q
- Allowed states : FF, BCS ($q = 0$), Normal ($\Delta_0 = 0$), and breached-pair ($q = 0$)
- LO ansatz yields very similar results

Breached-pair state

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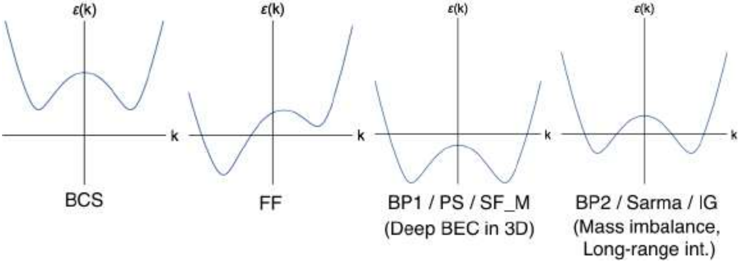
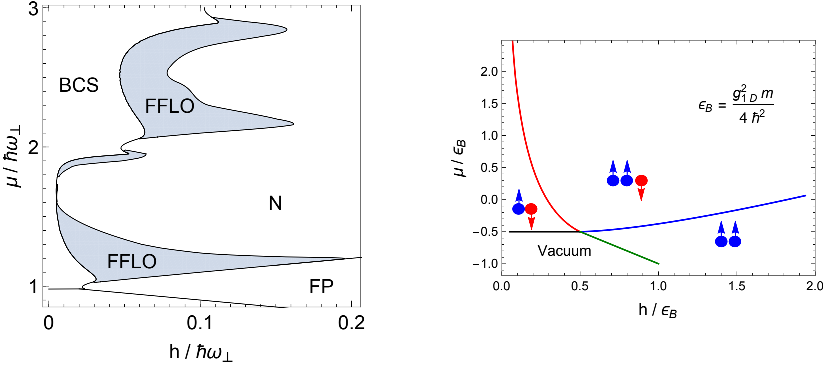
Example dispersions in 1D :

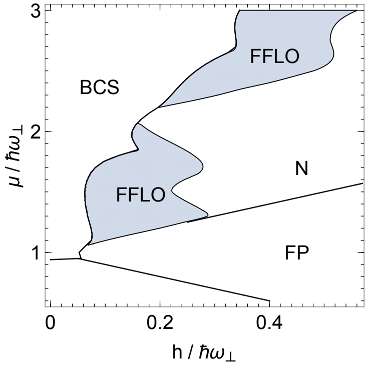


Phase diagram at weak interactions ($a_s = -d_\perp/3$)



1D-like structure that repeats as new channels open

Change with stronger interactions ($a_s = -2d_\perp/3$)



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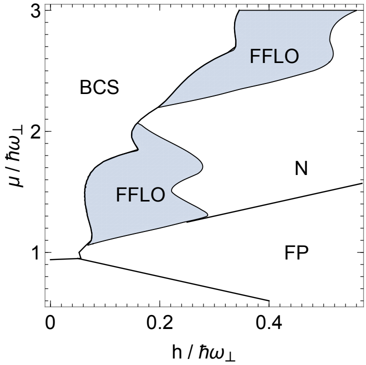
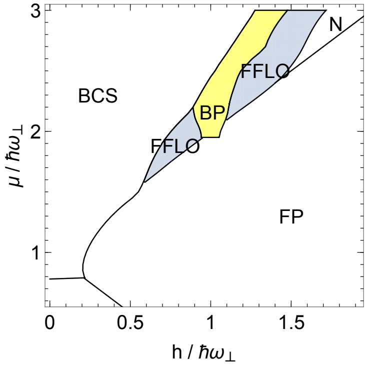


- Crossover to 3D happens at a lower density (μ)
- The SF region grows

At unitarity



- Stable BP phase emerges
- 3D-like for all densities

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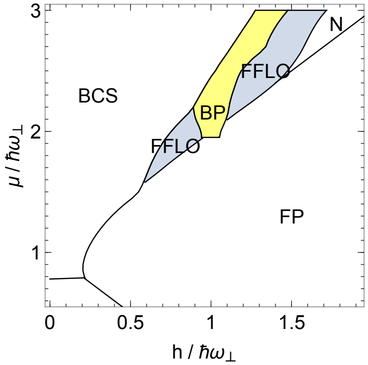

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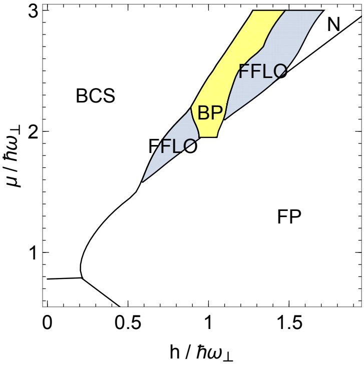

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- DFT produces 3D-like behavior (+BP)!

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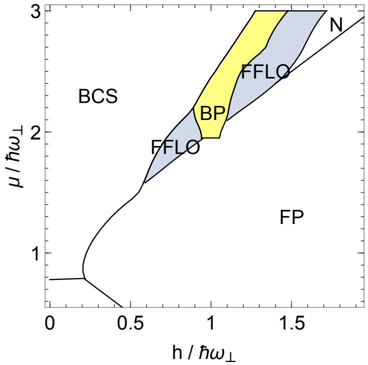
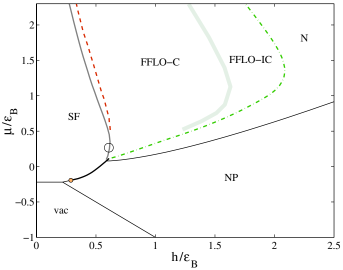
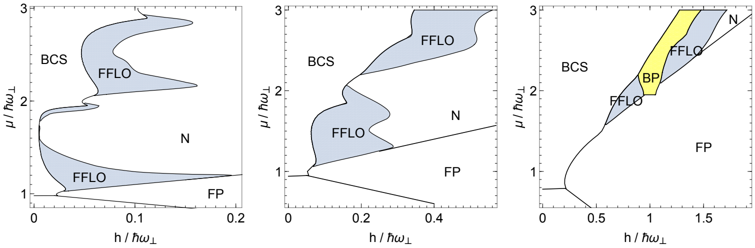


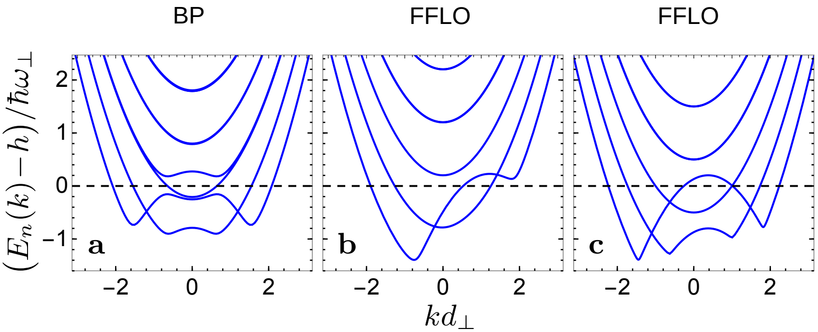


Does not agree with mean-field results with Olshanii's mapping

Phase diagrams



FF and BP dispersions



Different curves denote different transverse modes

Mapping to effective 1D model for $\mu < 2\hbar\omega_\perp$

Degenerate 2nd-order perturbation theory :

$$\frac{d_\perp \hbar\omega_\perp}{g_{1D}} = f\left(\frac{a_s}{d_\perp}, \frac{\mu}{\hbar\omega_\perp}, \frac{\Delta_0}{\hbar\omega_\perp}, \frac{\xi}{d_\perp}, q d_\perp\right)$$

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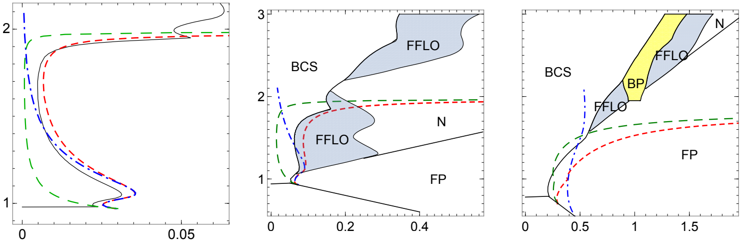
Consider the limit $\Delta_0, q \to 0$, $\xi/d_\perp \to 1$:

$$\frac{1}{\tilde{g}_{1D}} = \frac{1}{2\tilde{a}_s} + \frac{\zeta(\frac{1}{2}, 2-\tilde{\mu})}{2\sqrt{2}} - \frac{\sqrt{2}}{\pi}\Theta(\tilde{\mu}-1)\sum_{j=1}^{\infty}\frac{2^{-2j}}{\sqrt{j+1-\tilde{\mu}}}\tan^{-1}\sqrt{\frac{\tilde{\mu}-1}{j+1-\tilde{\mu}}},$$

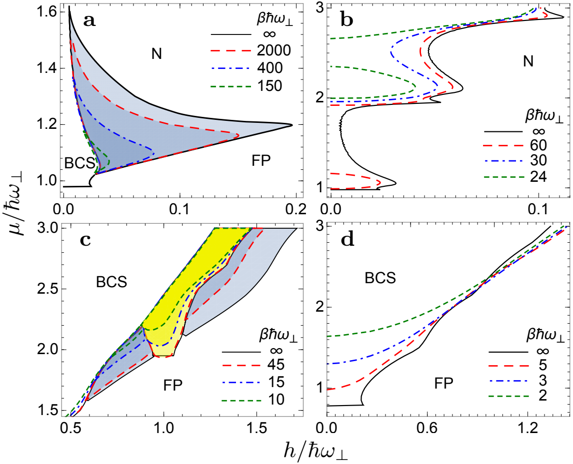
As $\mu \to \hbar\omega_\perp$, $1/\tilde{g}_{1D} = 1/(2\tilde{a}_s) + \zeta(1/2)/(2\sqrt{2})$ (Olshanii!)

Comparisons of effective models



Red → Mean-field with our mapping
Blue → Mean-field with Olshanii's mapping
Green → Bethe Ansatz with our mapping

Effect of temperature



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