Collective dynamics of solitons in superfluids

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Erich J. Mueller
PRL 118, 260402 (2017)

NSF
ARO - MURI
Animation: M. Reichl

Superfluids

- Not just dissipationless transport
- Cooper pairs can have rich dynamics

Wave function of Cooper pairs $\equiv$ order parameter

$$\Delta(x,t)$$

Superfluids

- Not just dissipationless transport
- Cooper pairs can have rich dynamics

Wave function of Cooper pairs $\equiv$ order parameter

$$\Delta(x,t)$$

spatial structure

- vortices
- solitons

has dynamics

- Josephson junctions
- superconducting Qbits

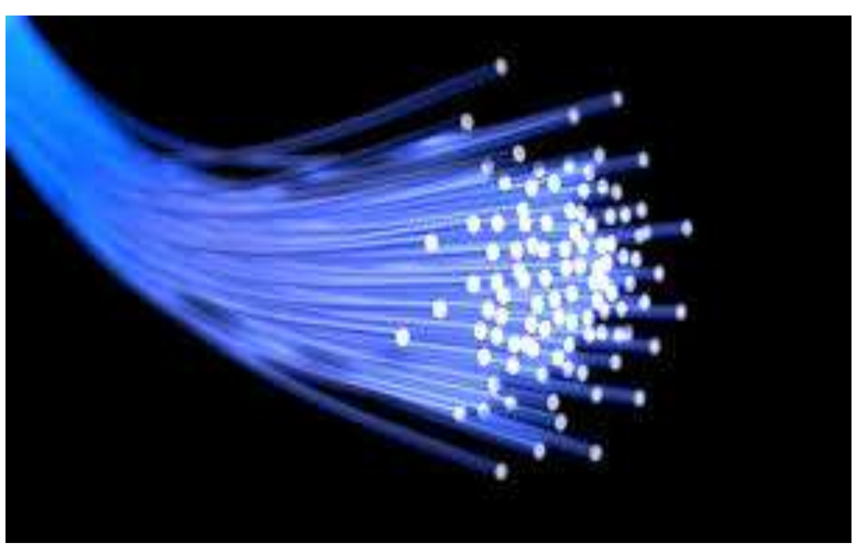
What are solitons

- Solitons are persistent nonlinear waves

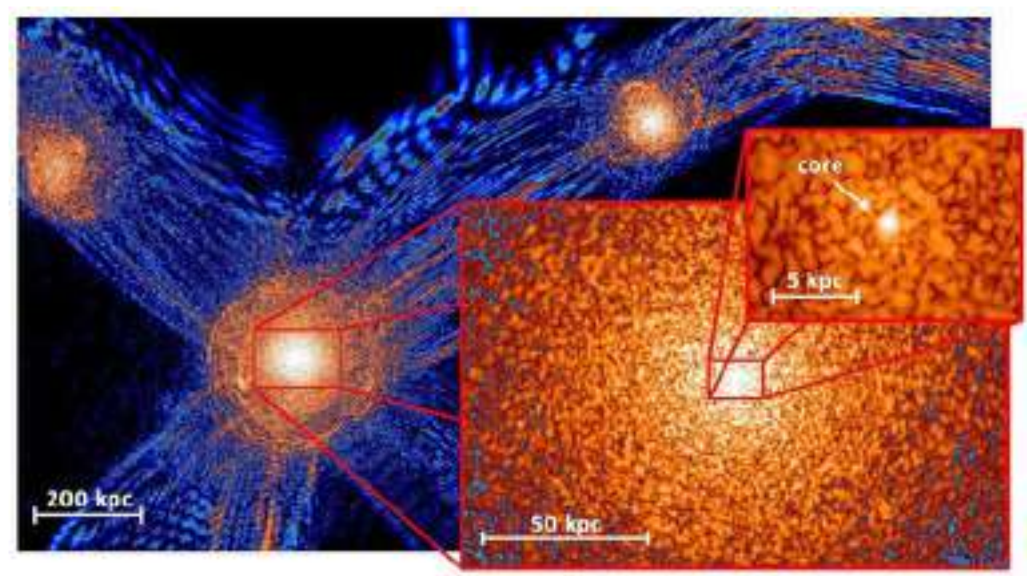
  "Pulses" which travel without changing shape

Hydrodynamics

Weather



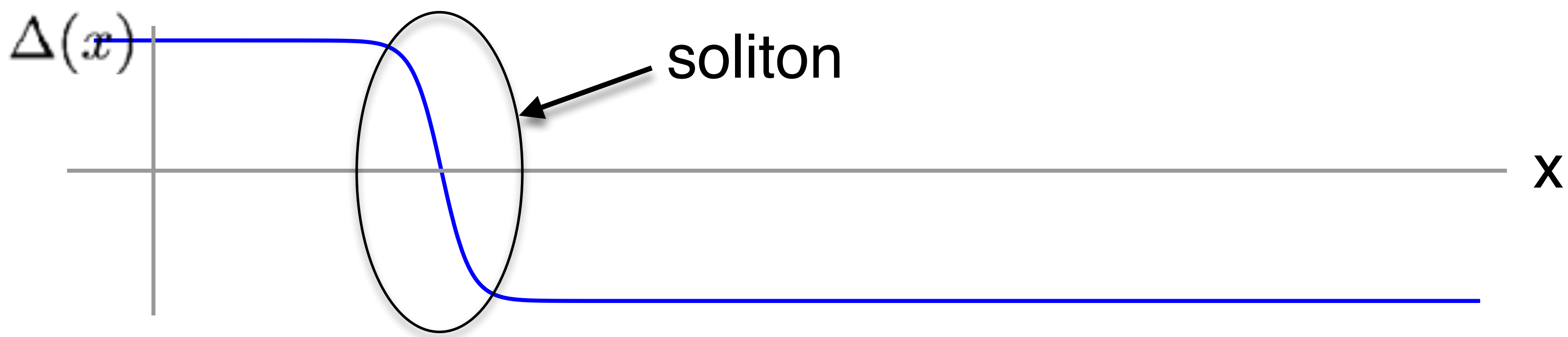
Telecommunications



Cosmology

Solitons in superfluids

- Sharp changes in the order parameter



Solitons in superfluids

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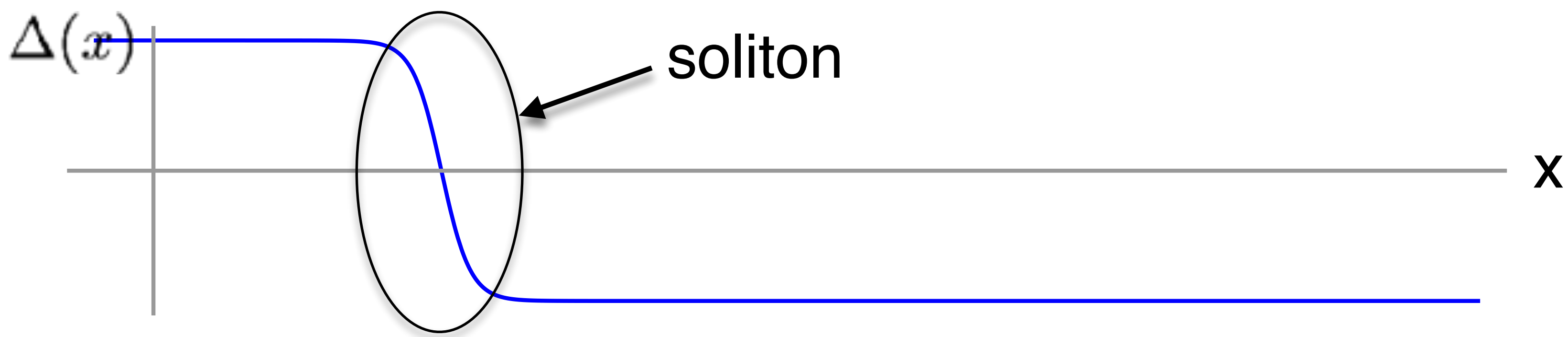

stationary soliton

$\mathrm{Im}\ \Delta(x) = \mathrm{const.} \neq 0$

traveling soliton

Solitons in superfluids

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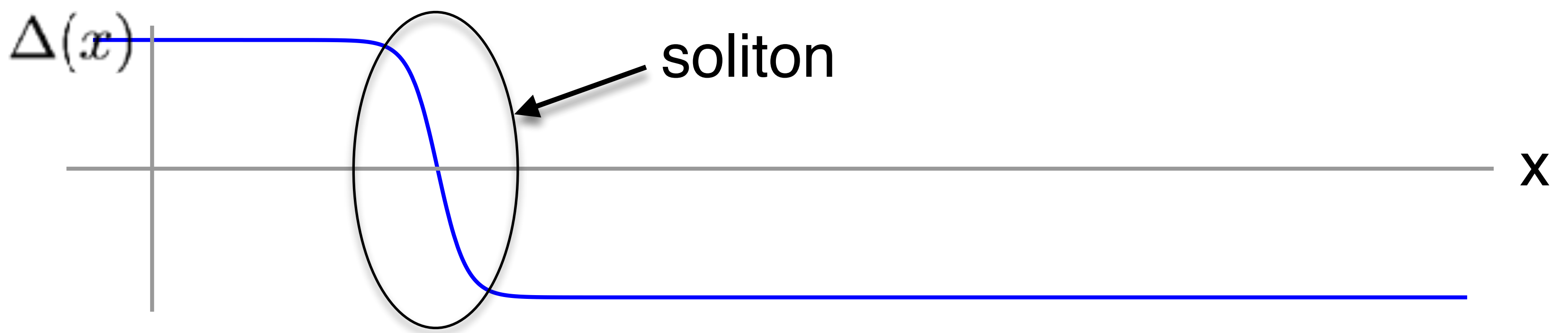

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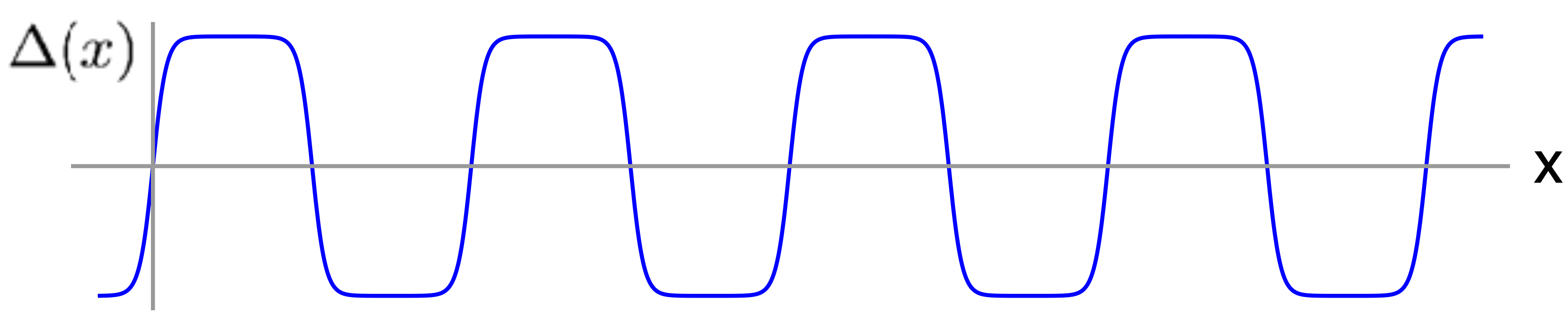
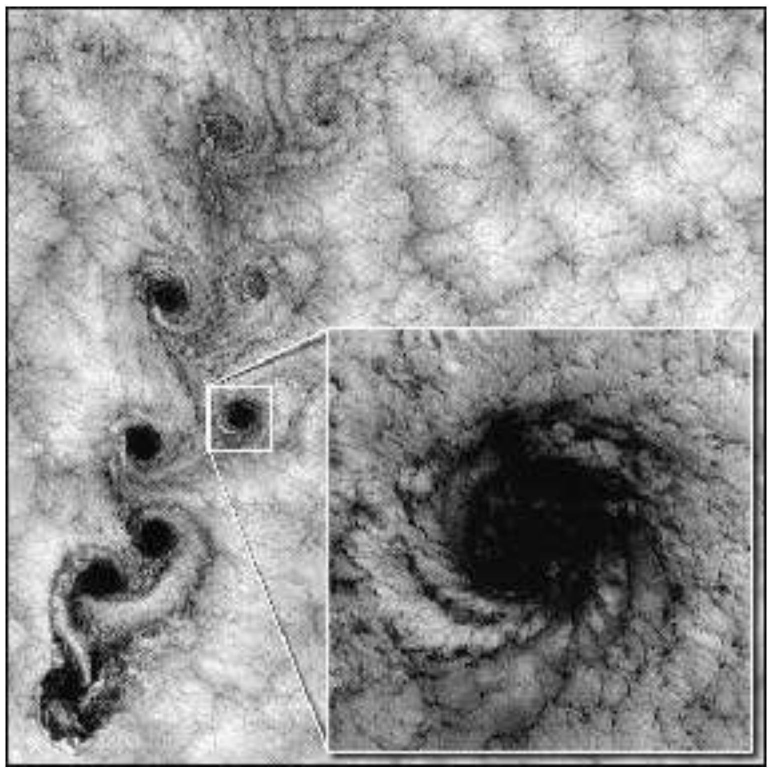


stationary
soliton train

Why study collective modes of solitons

- Understanding emergent nonlinear structures

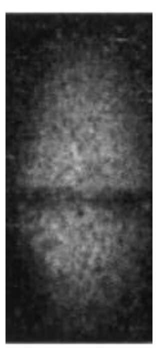


develop theoretical framework to identify collective degrees of freedom

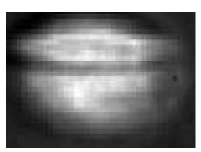
Why study collective modes of solitons

- Understanding emergent nonlinear structures
- Understanding far-from-equilibrium properties of superfluids
  (e.g., soliton trains are generated in phase transitions)

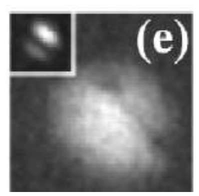
Solitons in experiments:



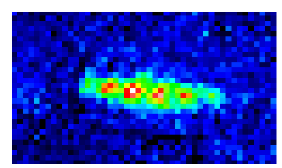
Sengstock 1999
PRL



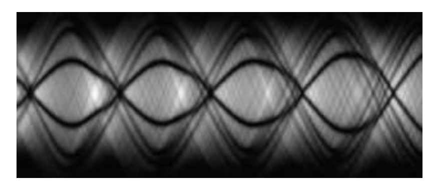
Philips 2000
Science



Cornell 2001
PRL

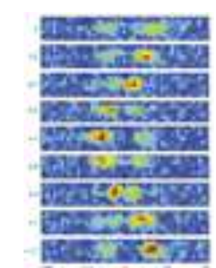


Wieman 2006
PRL

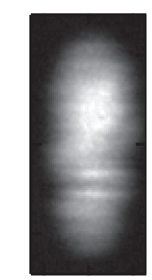


Oberthaler 2008 PRL

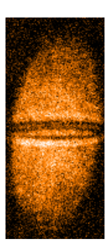
Ferrari 2014
PRL

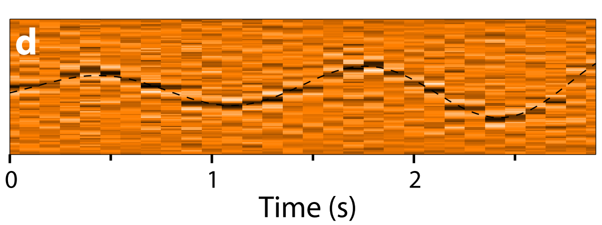


Hulet 2014
Nat. Phys.



Spielman
2016





Zwierlein 2013, 2014, 2016
PRL

Why study collective modes of solitons

- Understanding emergent nonlinear structures
- Understanding far-from-equilibrium properties of superfluids (e.g., soliton trains are generated in phase transitions)
- Collective modes are cool and characterize system dynamics

Collective excitations of a superfluid

- Fluctuations about a stationary solution $\Delta_0(x)$

$$\Delta(x,t) = \Delta_0(x) + \delta(x,t)$$

Decompose fluctuations into Fourier components

$$\delta(x,t) = \sum_k \delta_k e^{i(kx - \omega_k t)}$$

$\omega(k)$: collective-mode spectrum

Collective excitations of a superfluid

- Fluctuations about a stationary solution $\Delta_0(x)$

$$\Delta(x,t) = \Delta_0(x) + \delta(x,t)$$

Decompose fluctuations into Fourier components

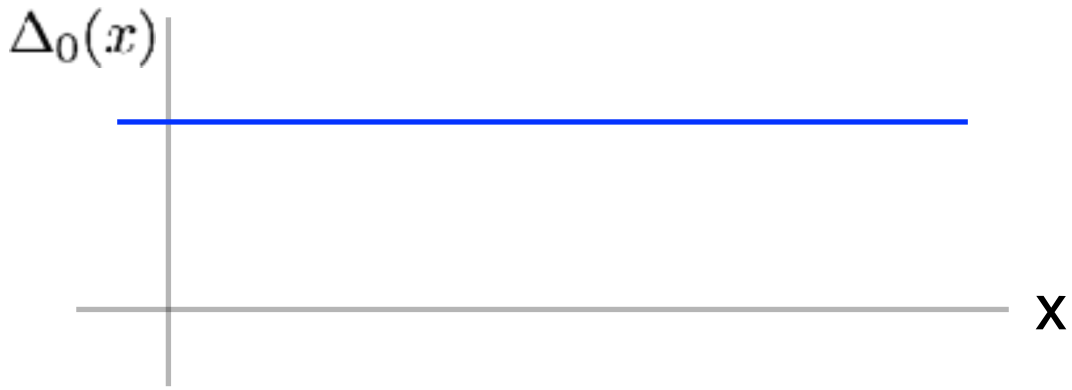
$$\left.\begin{array}{l} \delta(x,t) = \sum_k \delta_k e^{i(kx-\omega_k t)} \\ \omega(k) : \text{collective-mode spectrum} \end{array}\right\} \text{only one branch of collective modes (e.g., sound waves)}$$

In general,

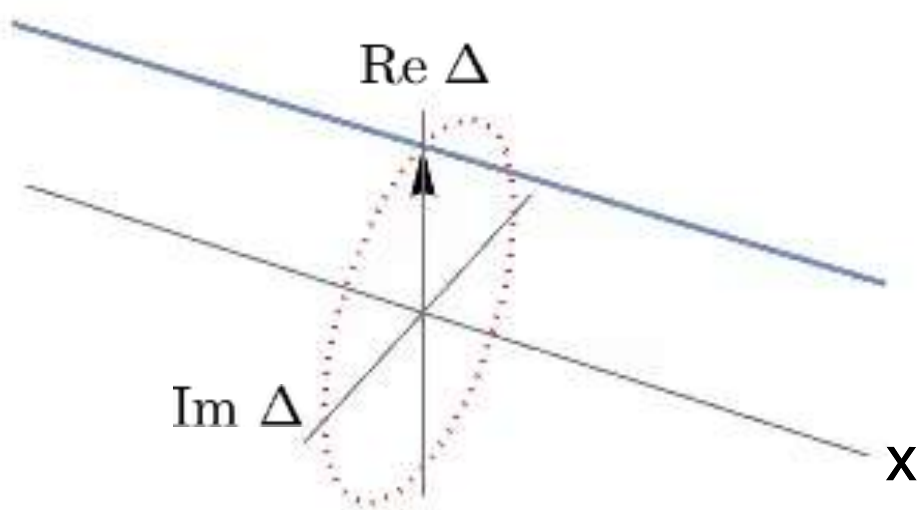
$$\delta(x,t) = \sum_j \sum_k \delta_k^{(j)} e^{i\left(kx-\omega_k^{(j)} t\right)}$$

$\omega^{(j)}(k)$: collective-mode spectrum with multiple branches

Collective modes of a uniform superfluid



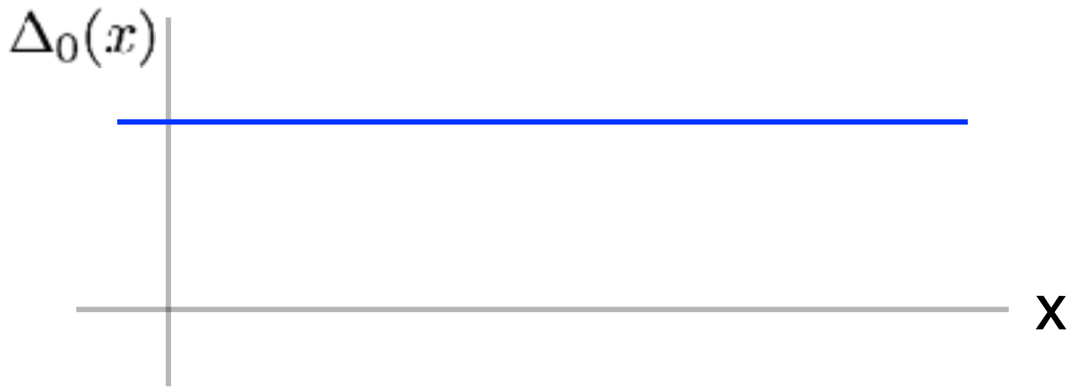
Symmetry under phase rotation



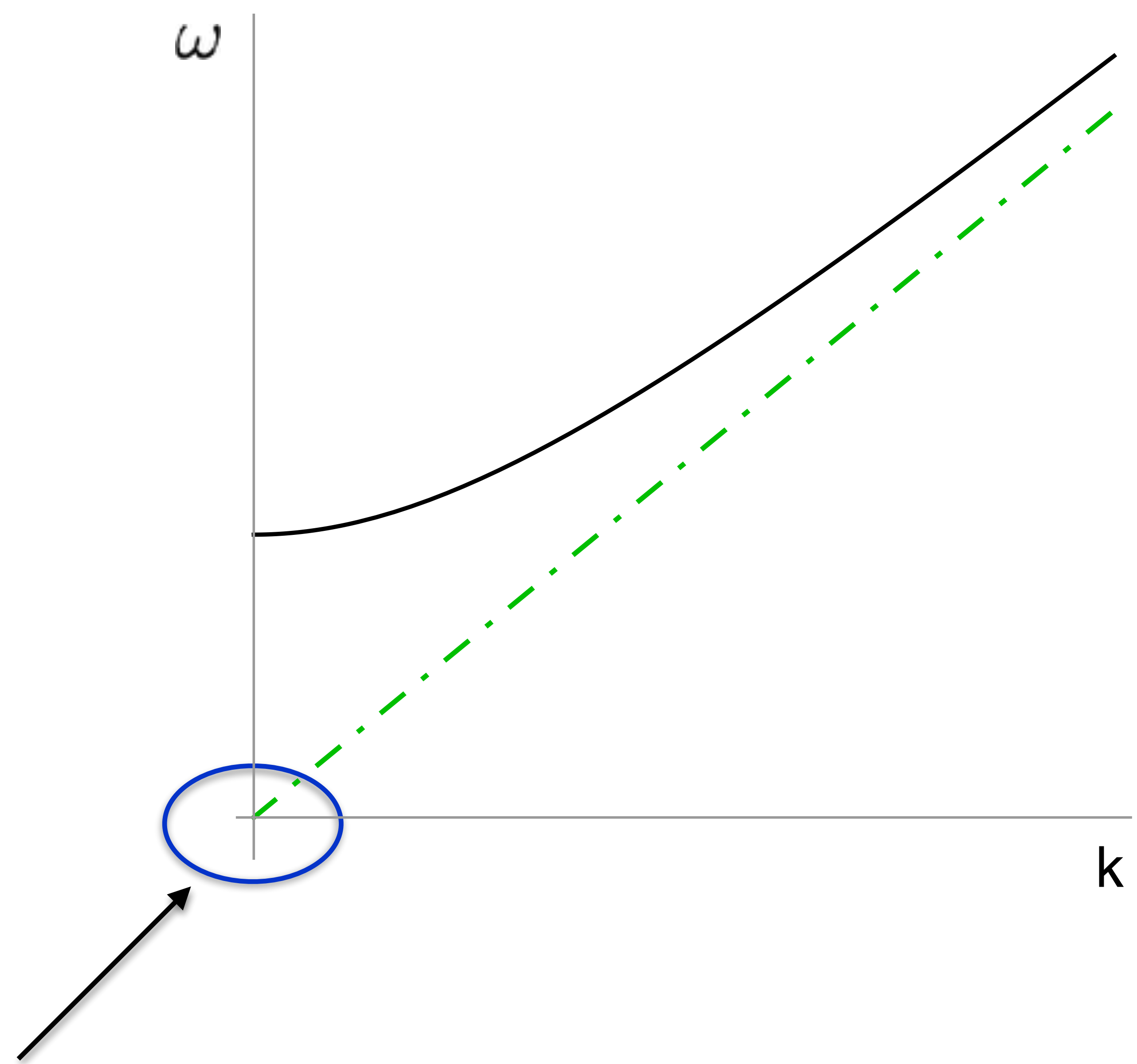
$\Longrightarrow$

One zero-energy collective mode
(Goldstone's theorem)

Collective modes of a uniform superfluid



Collective-mode spectrum



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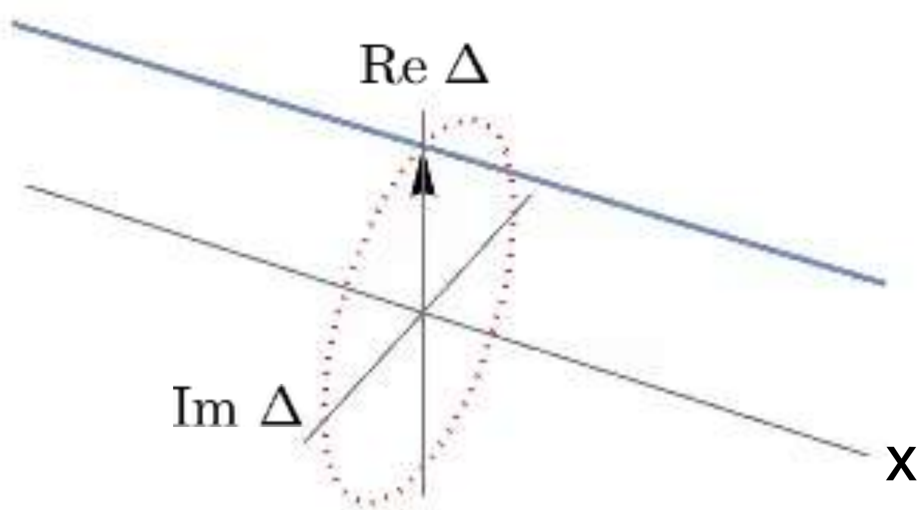

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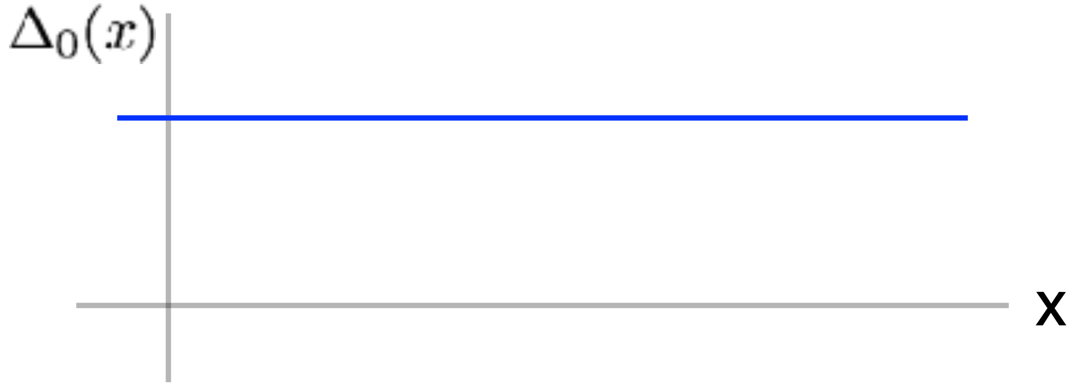

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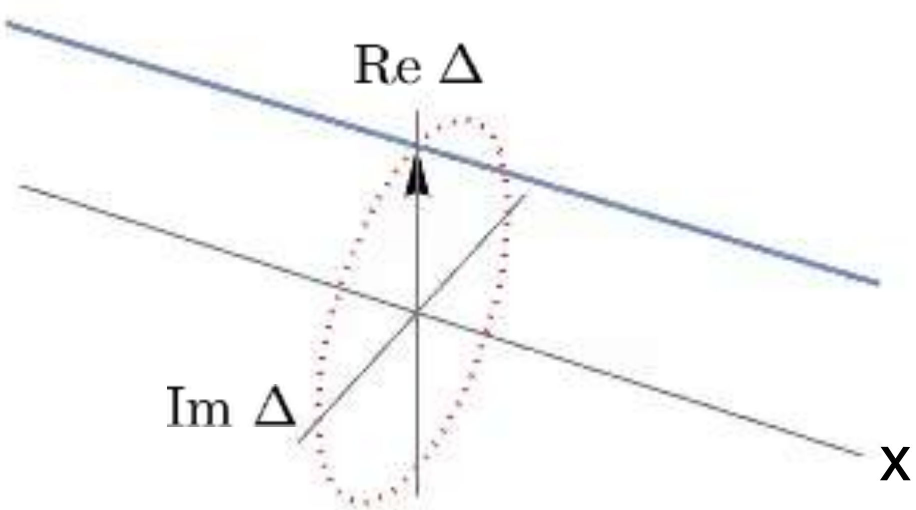

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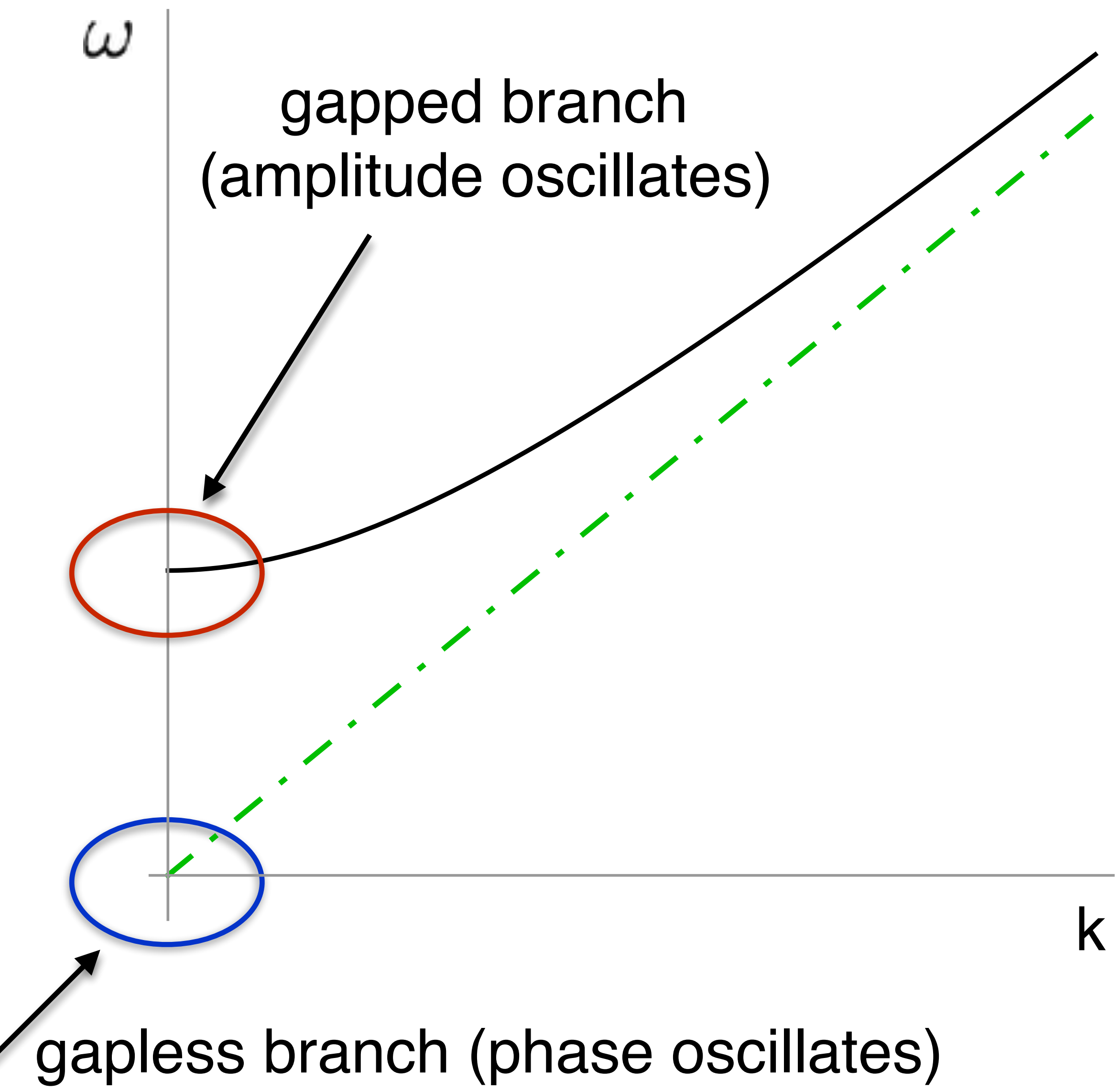
Collective-mode spectrum



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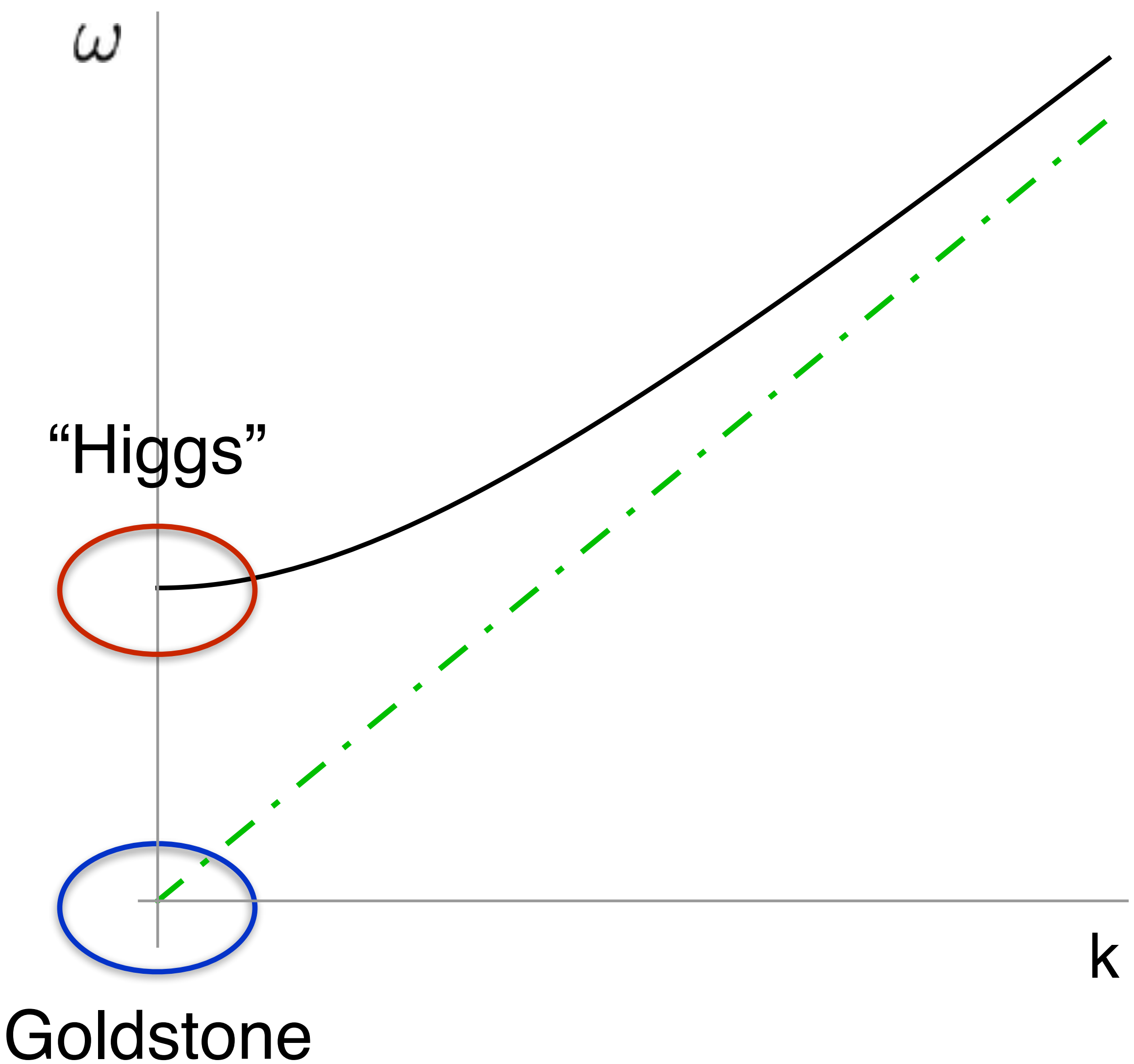

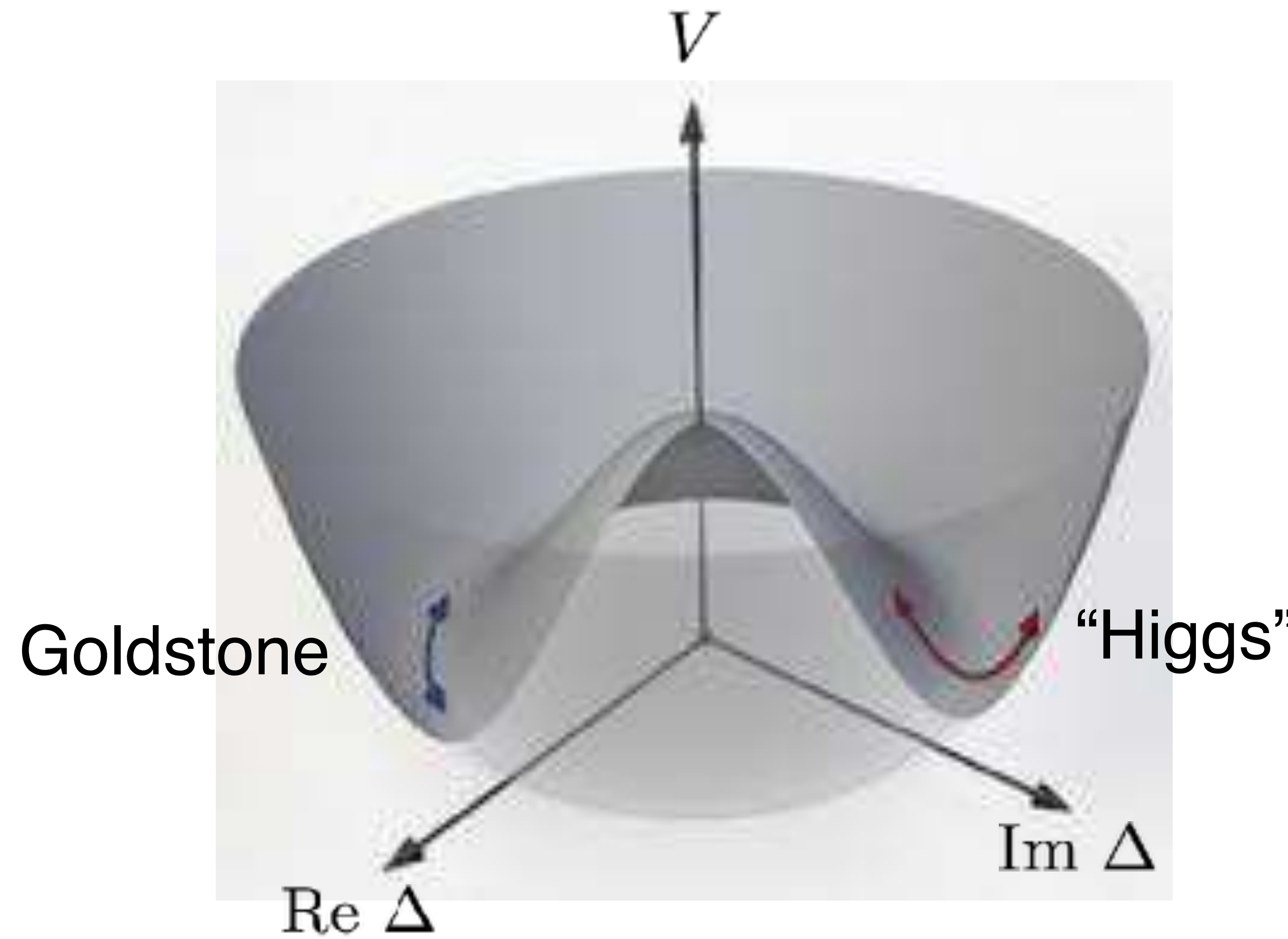

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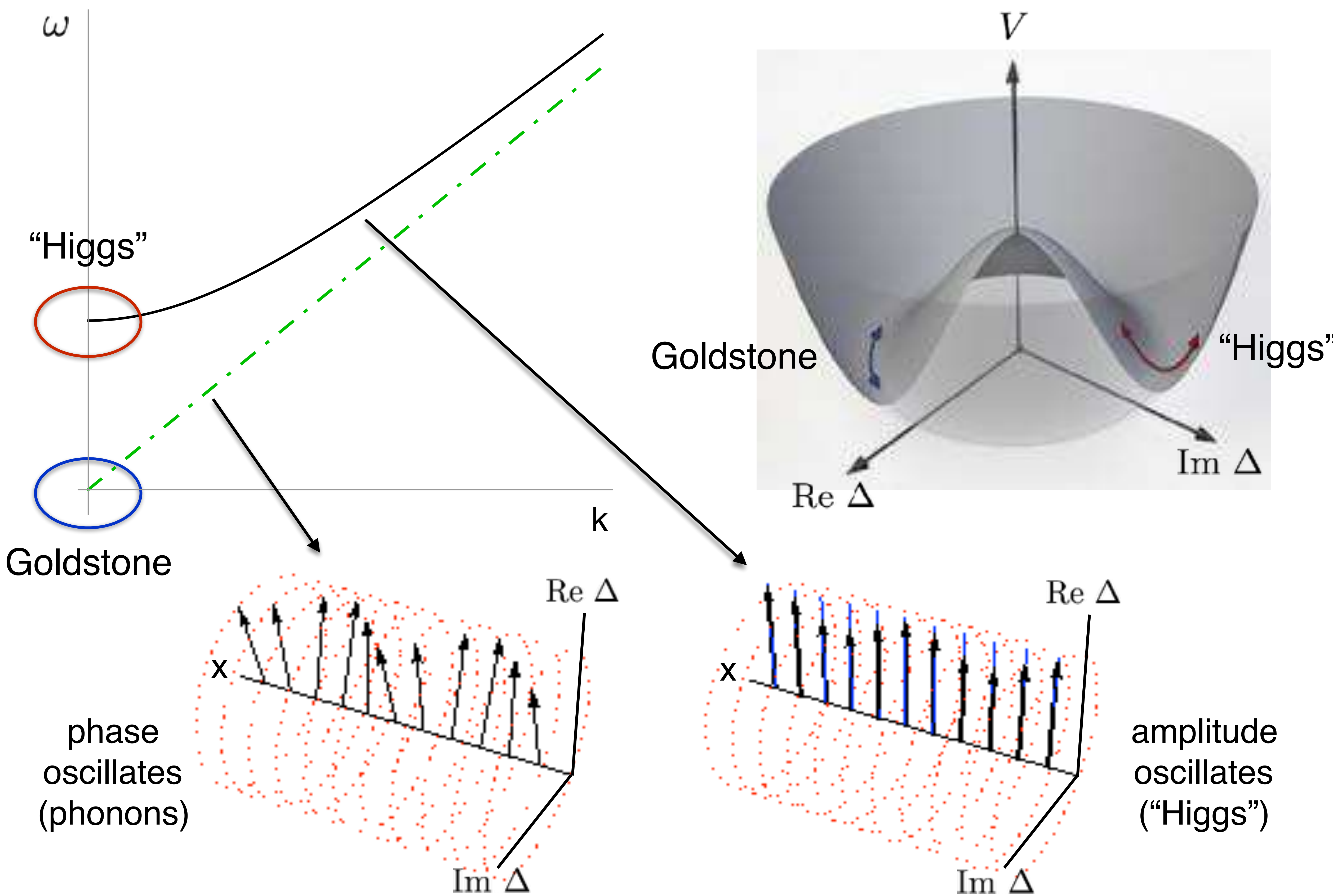
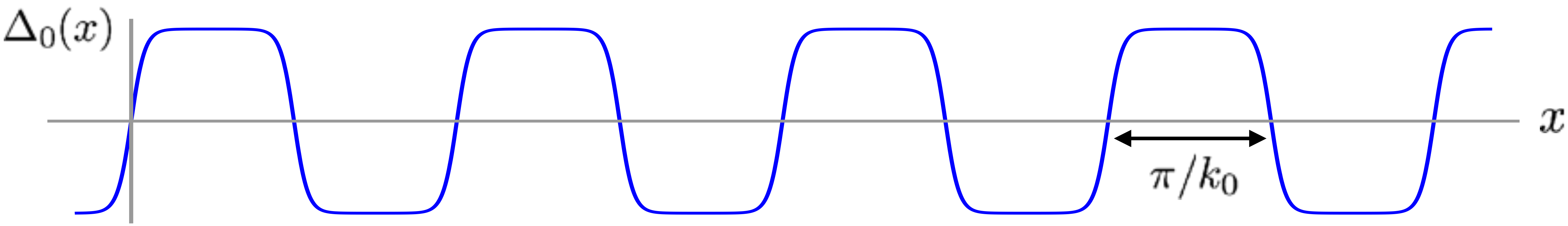
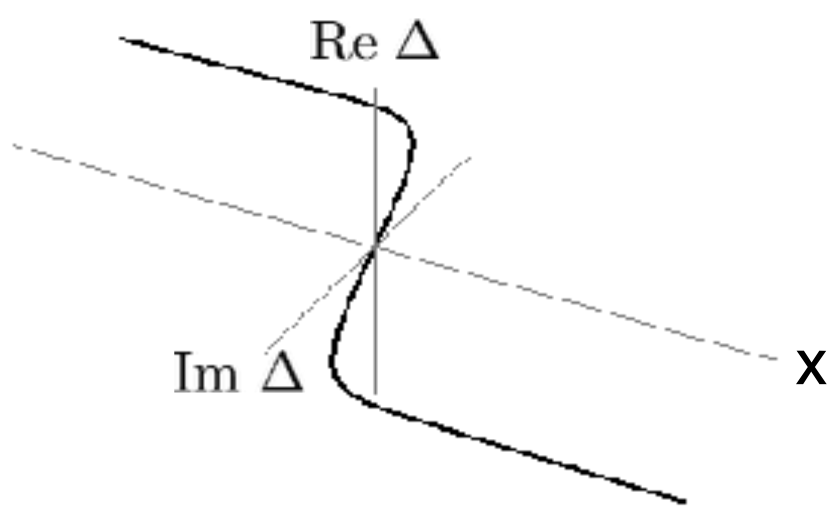


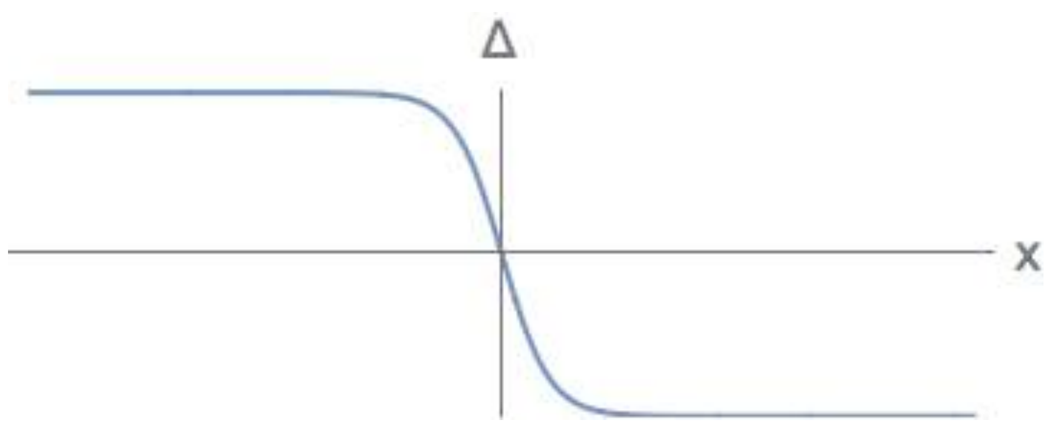
Collective modes of a soliton train



Gauge symmetry (phase rotation)



Translational symmetry



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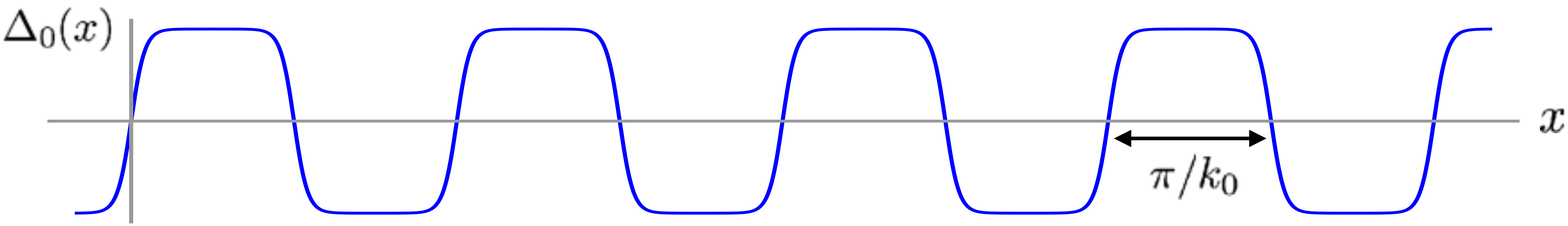

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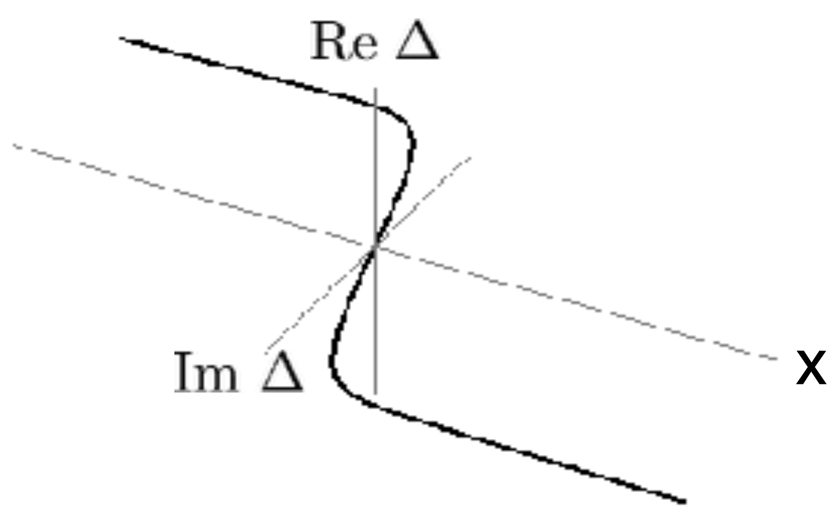

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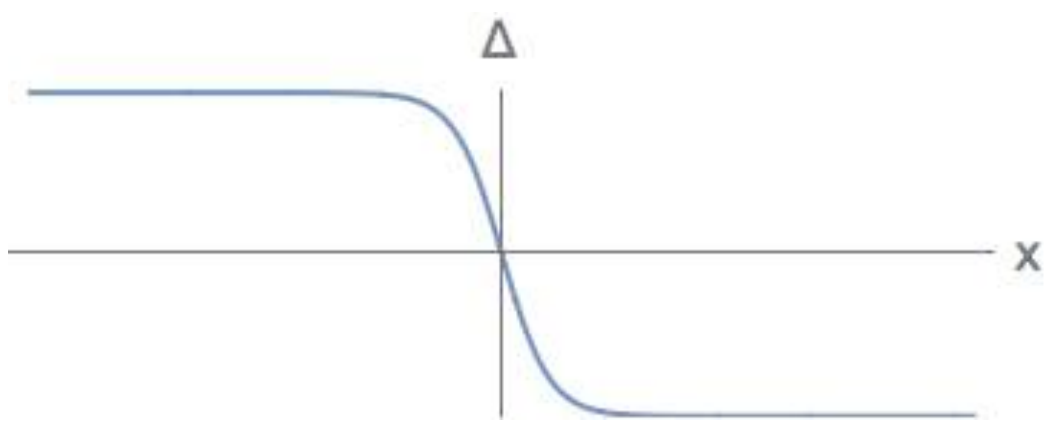
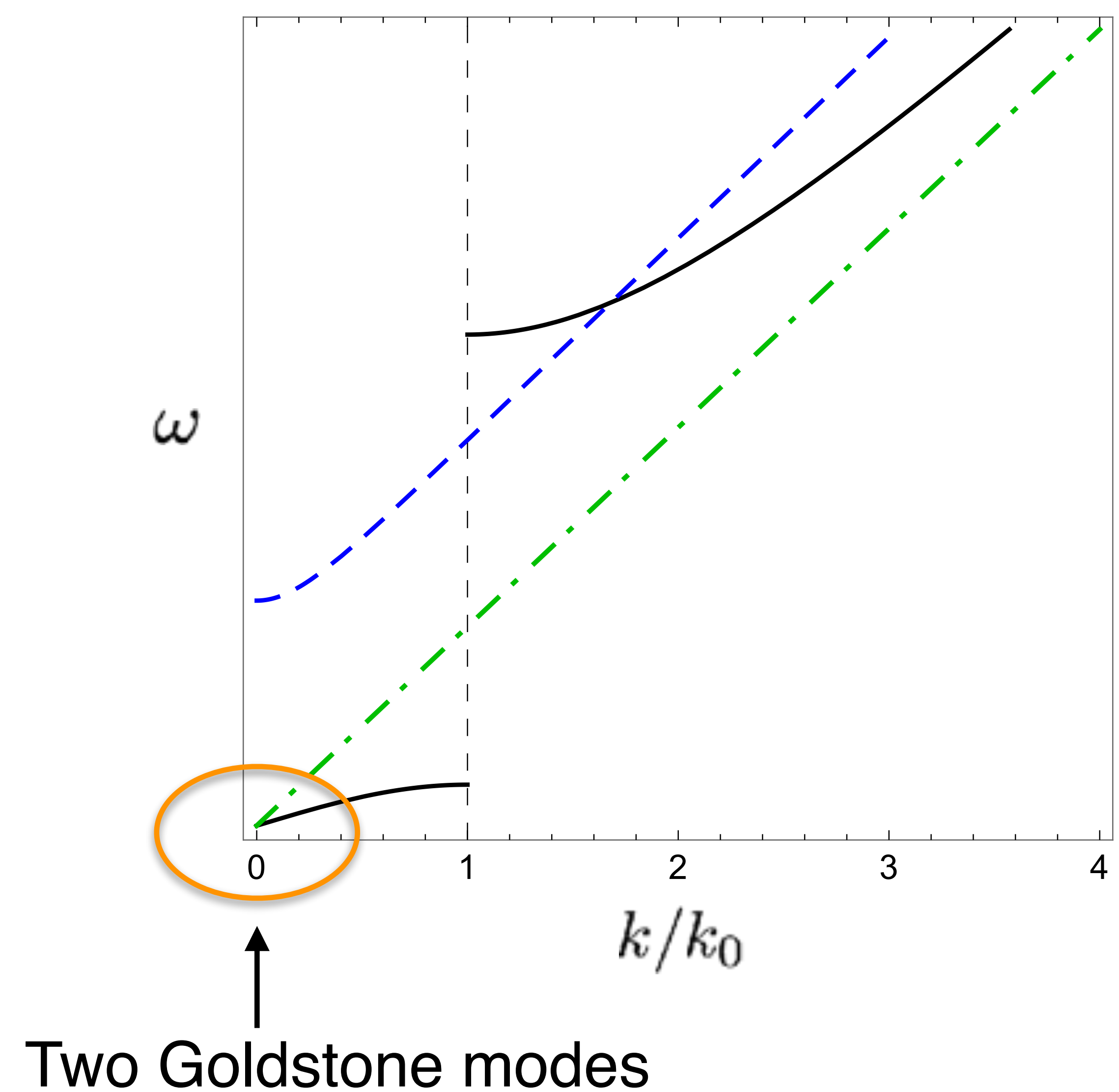
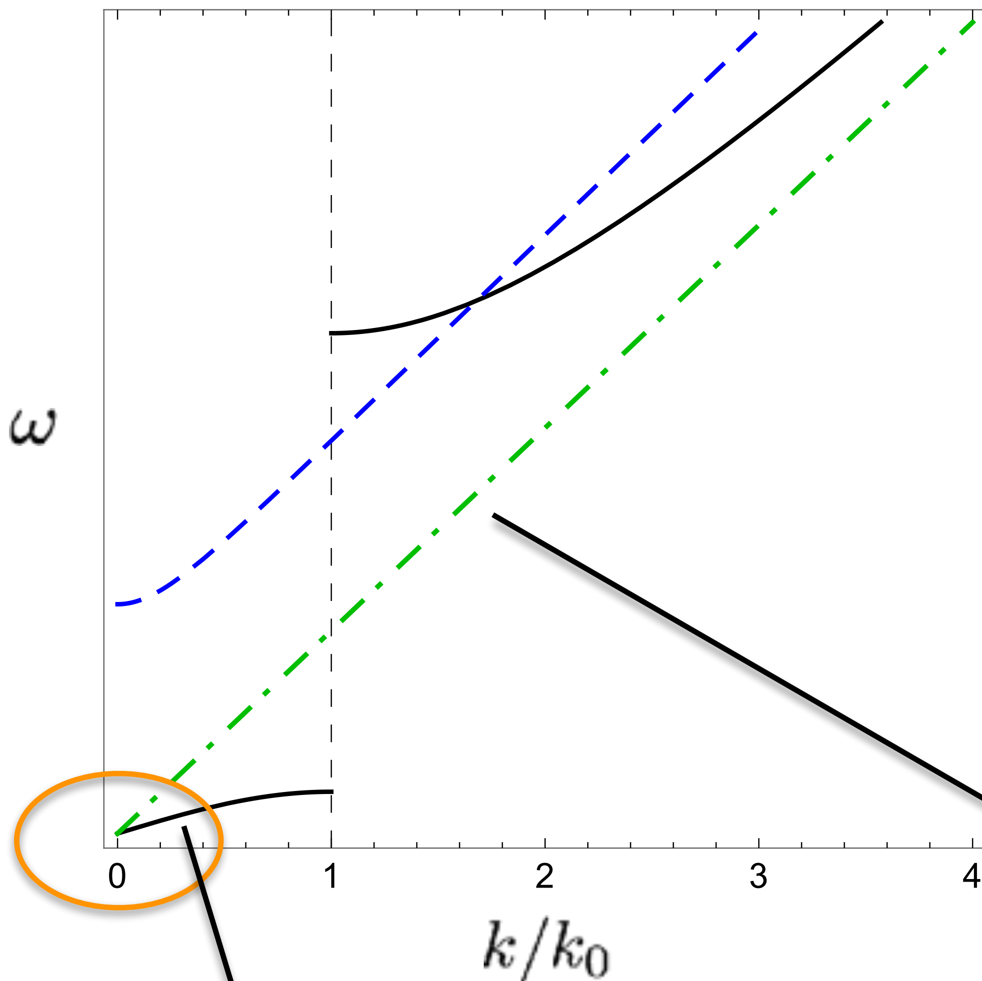


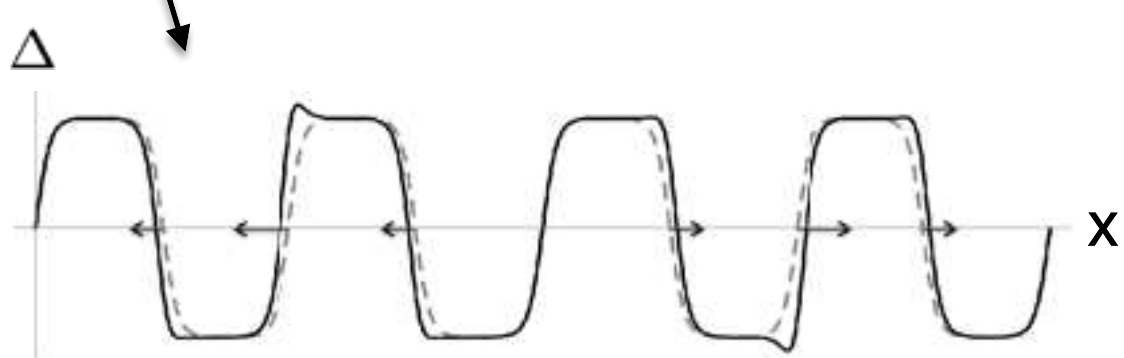
Collective-mode spectrum



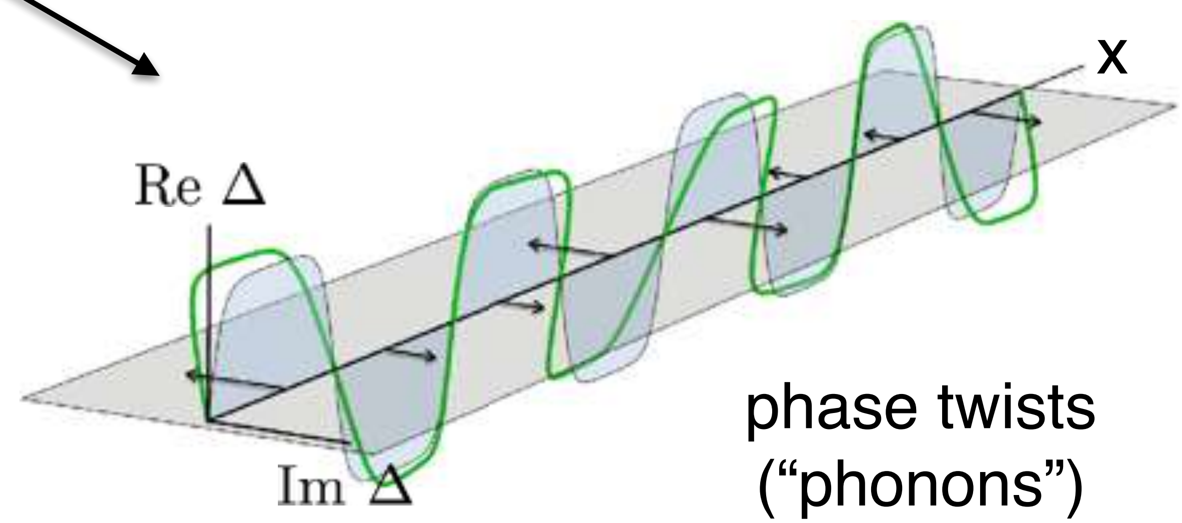
Collective modes of a soliton train

Collective-mode spectrum





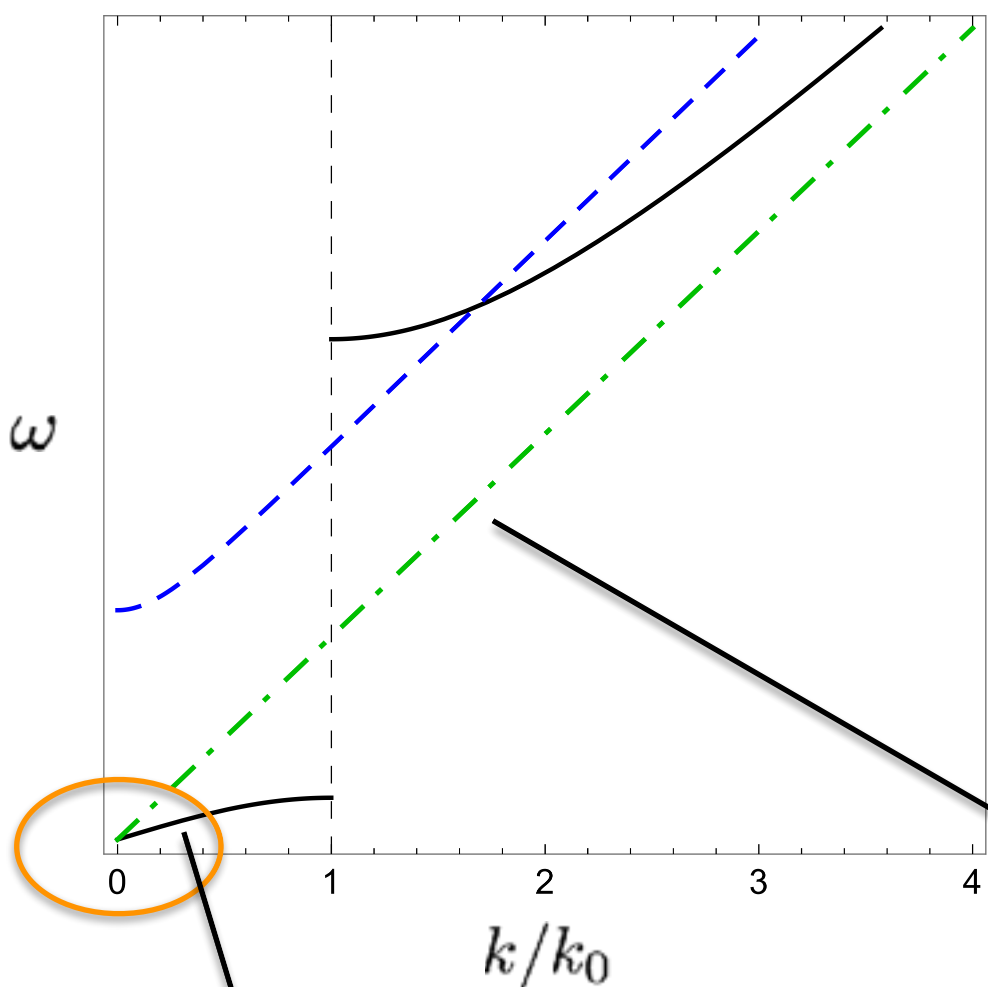
separation oscillates ("elastic mode")

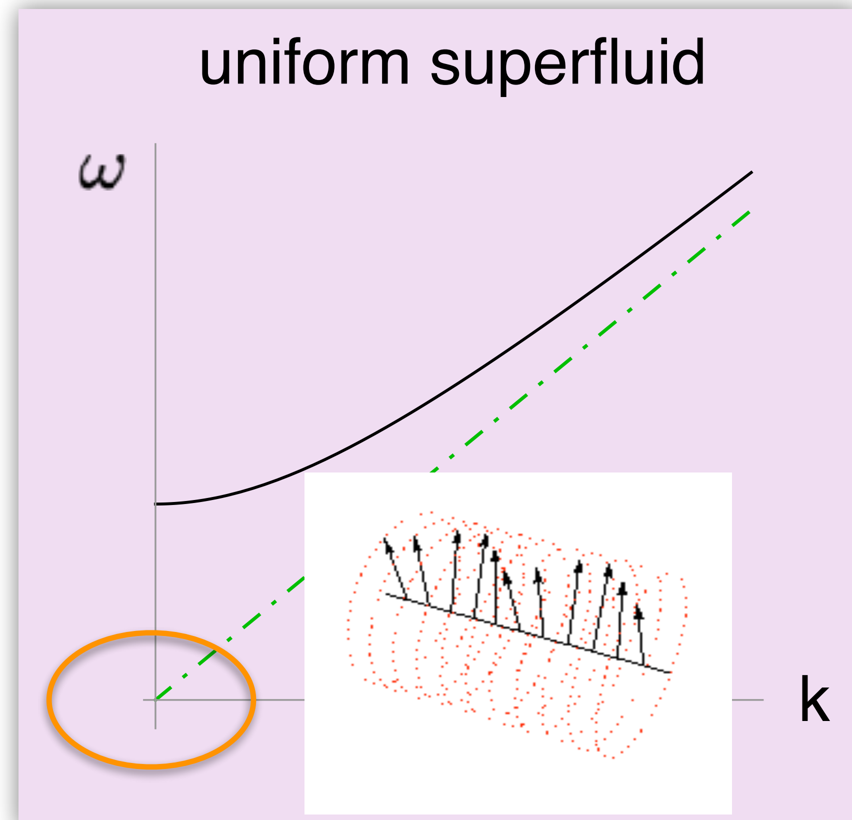


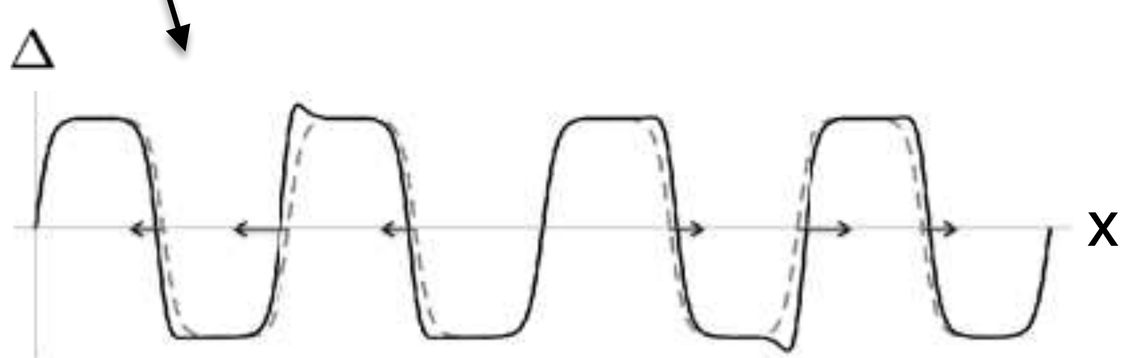
phase twists
("phonons")

Collective modes of a soliton train

Collective-mode spectrum







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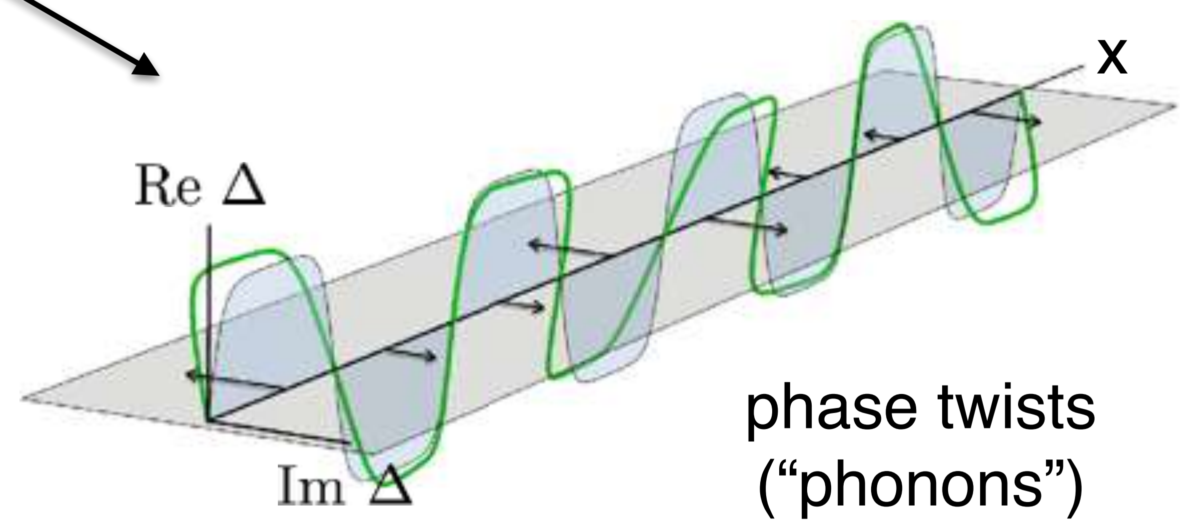

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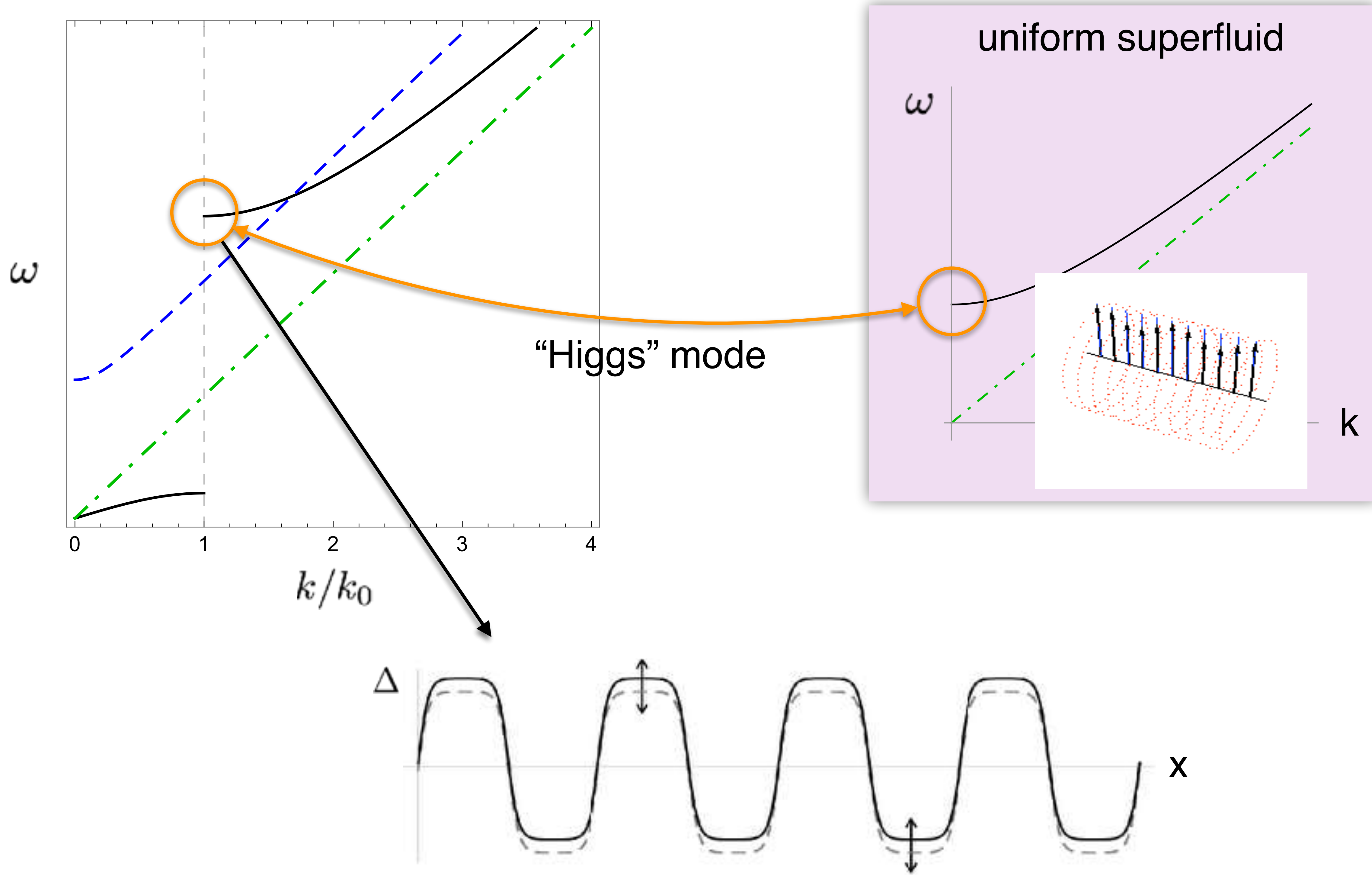
Collective modes of a soliton train

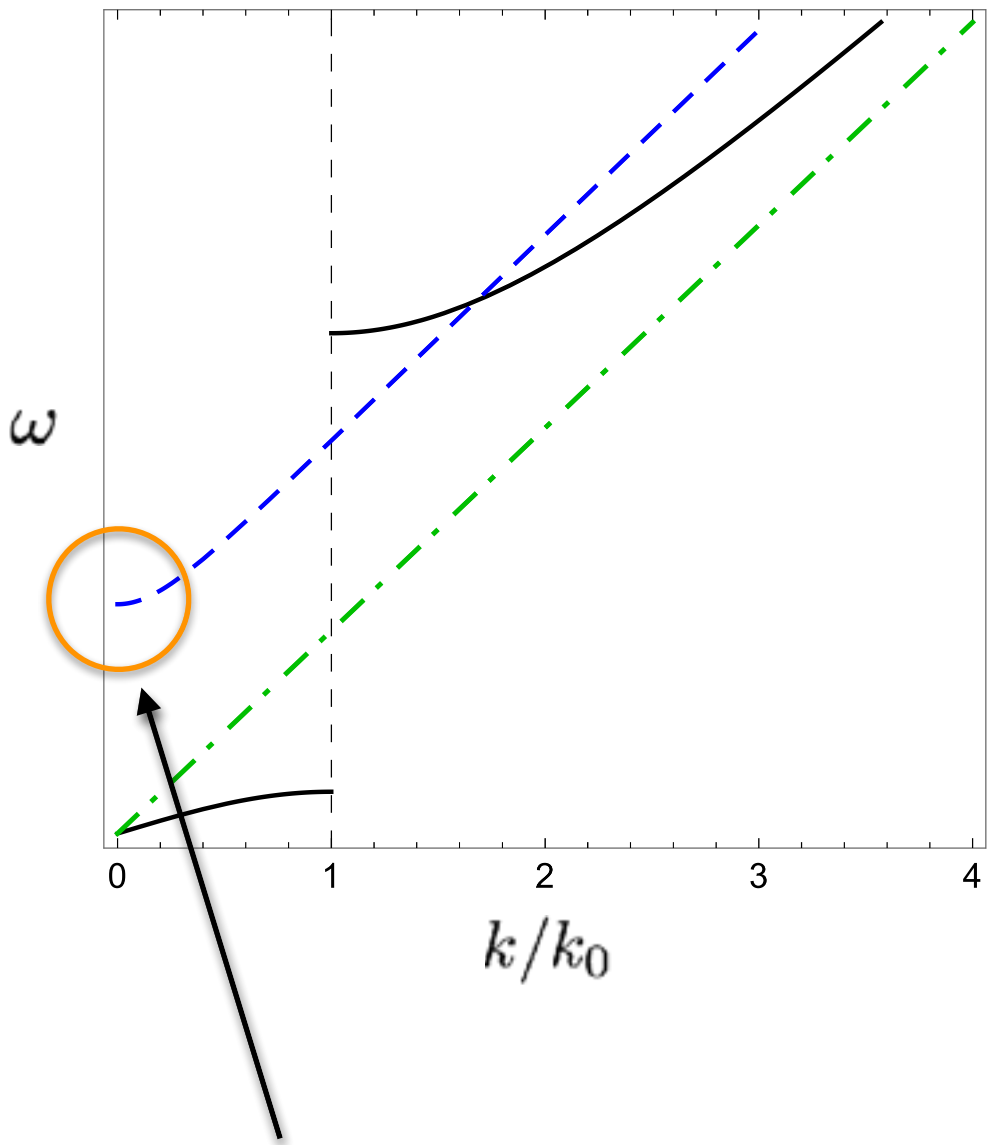
Collective-mode spectrum



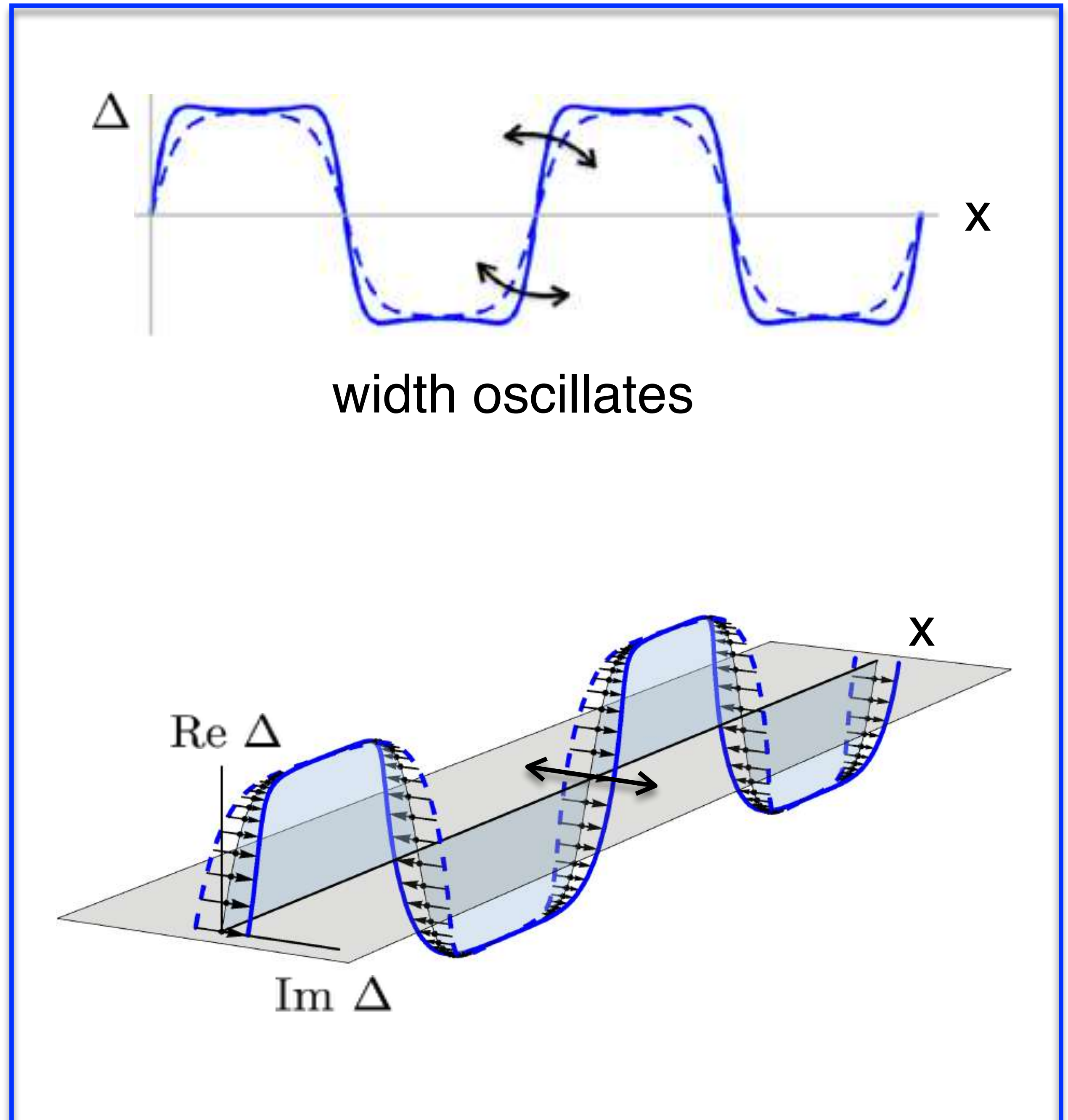
Analog of "Higgs" mode

Collective modes of a soliton train

Collective-mode spectrum

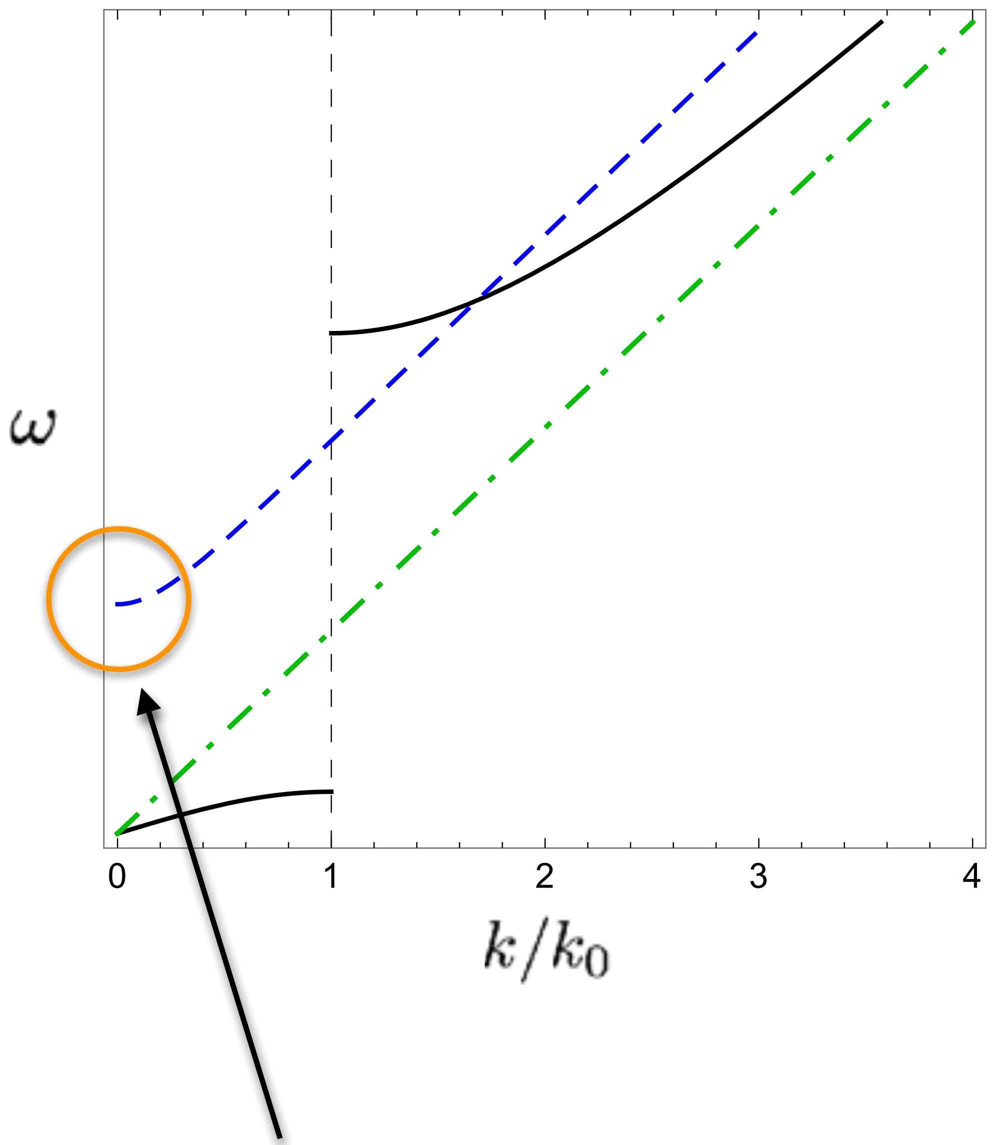


Two new gapped degenerate modes!



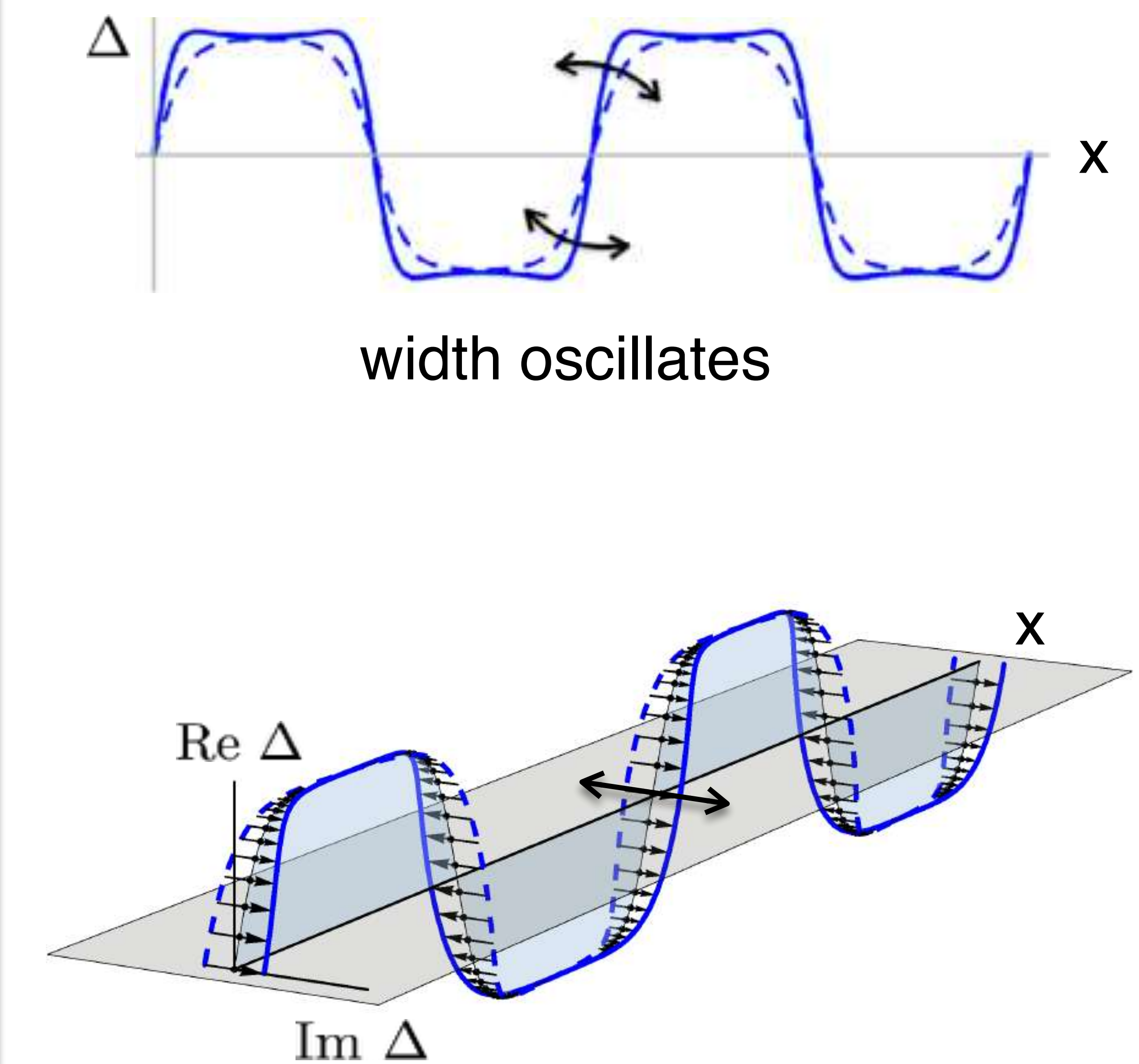
Collective modes of a soliton train

Collective-mode spectrum



Two new gapped degenerate modes!

Look for in experiment!

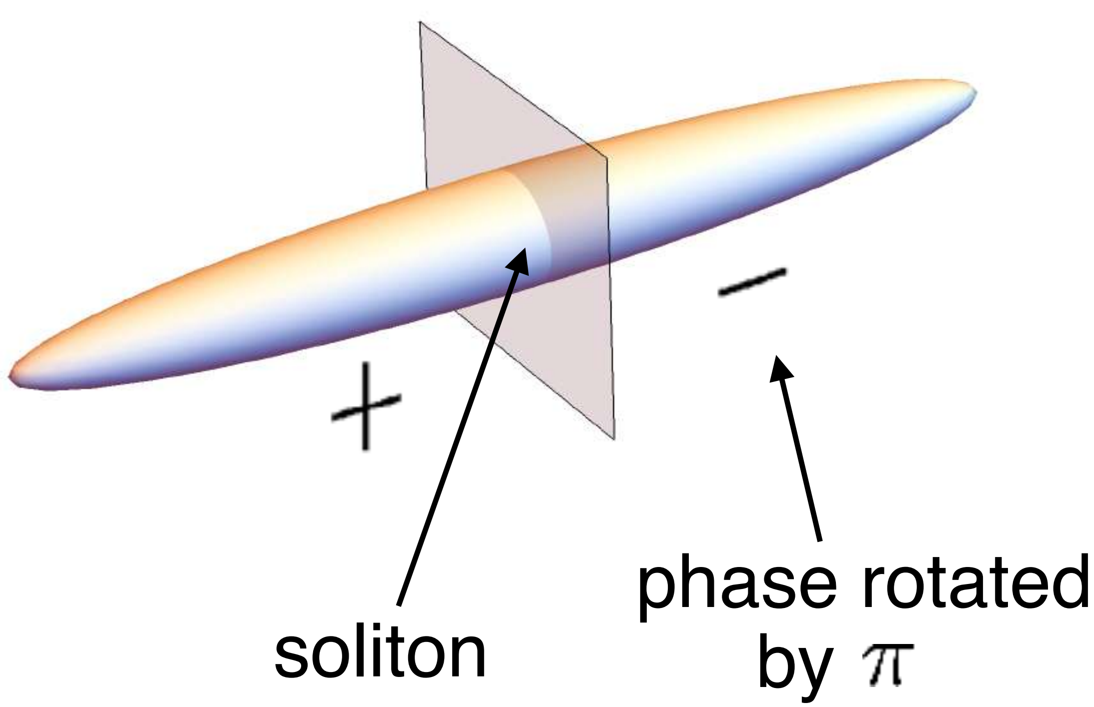


Creating soliton in experiments

Phase imprinting:

- cool atoms in elongated trap
  (forms uniform superfluid)

- shine off-resonant laser on
  one half of the superfluid
  (rotates phase by π)



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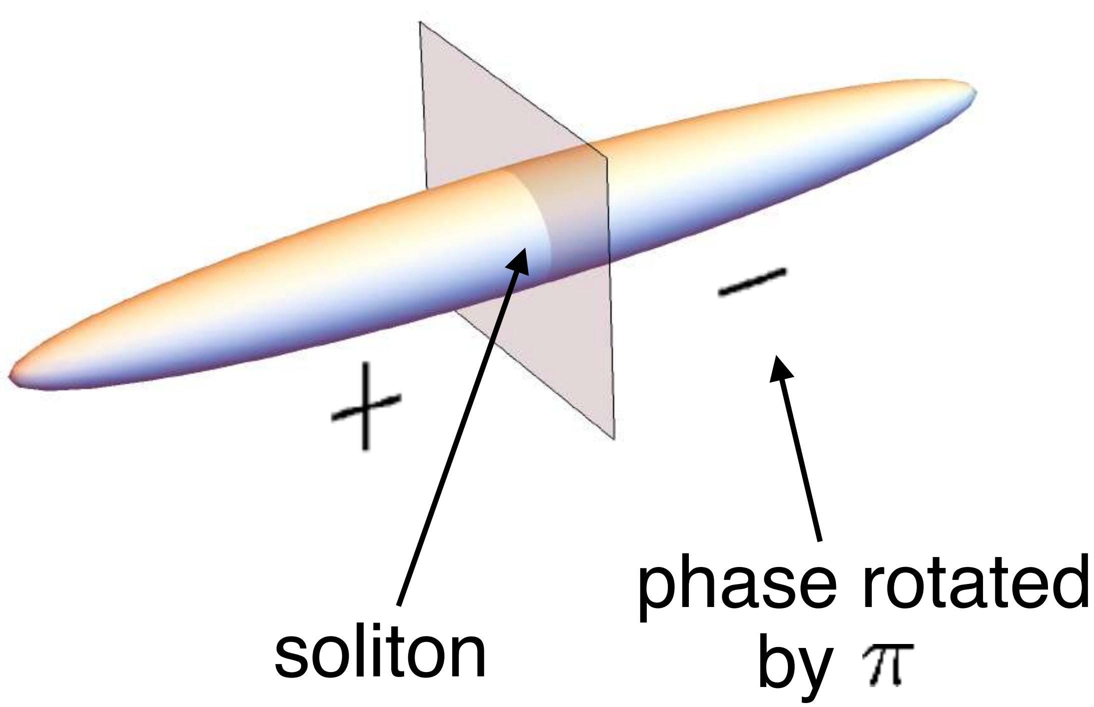
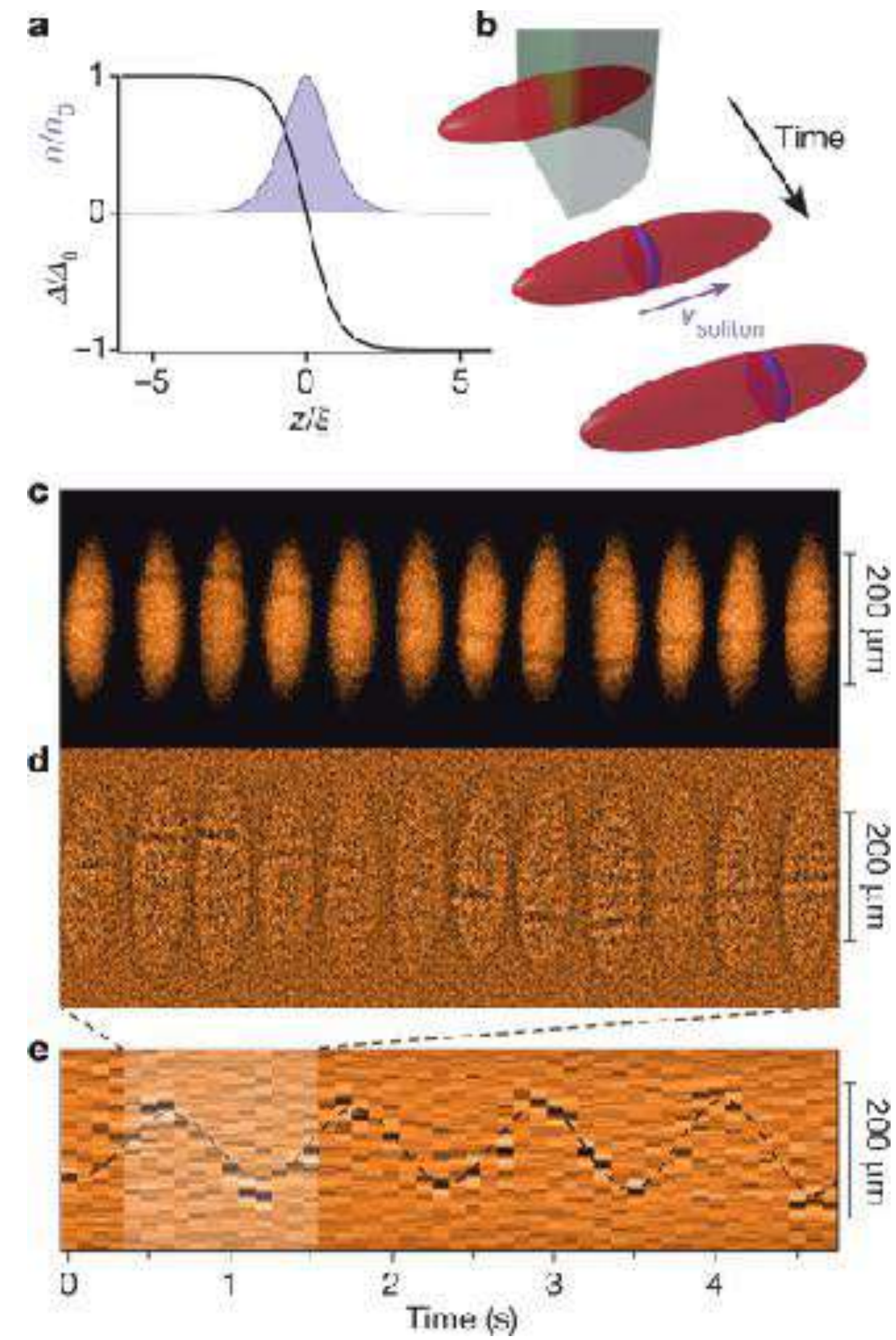
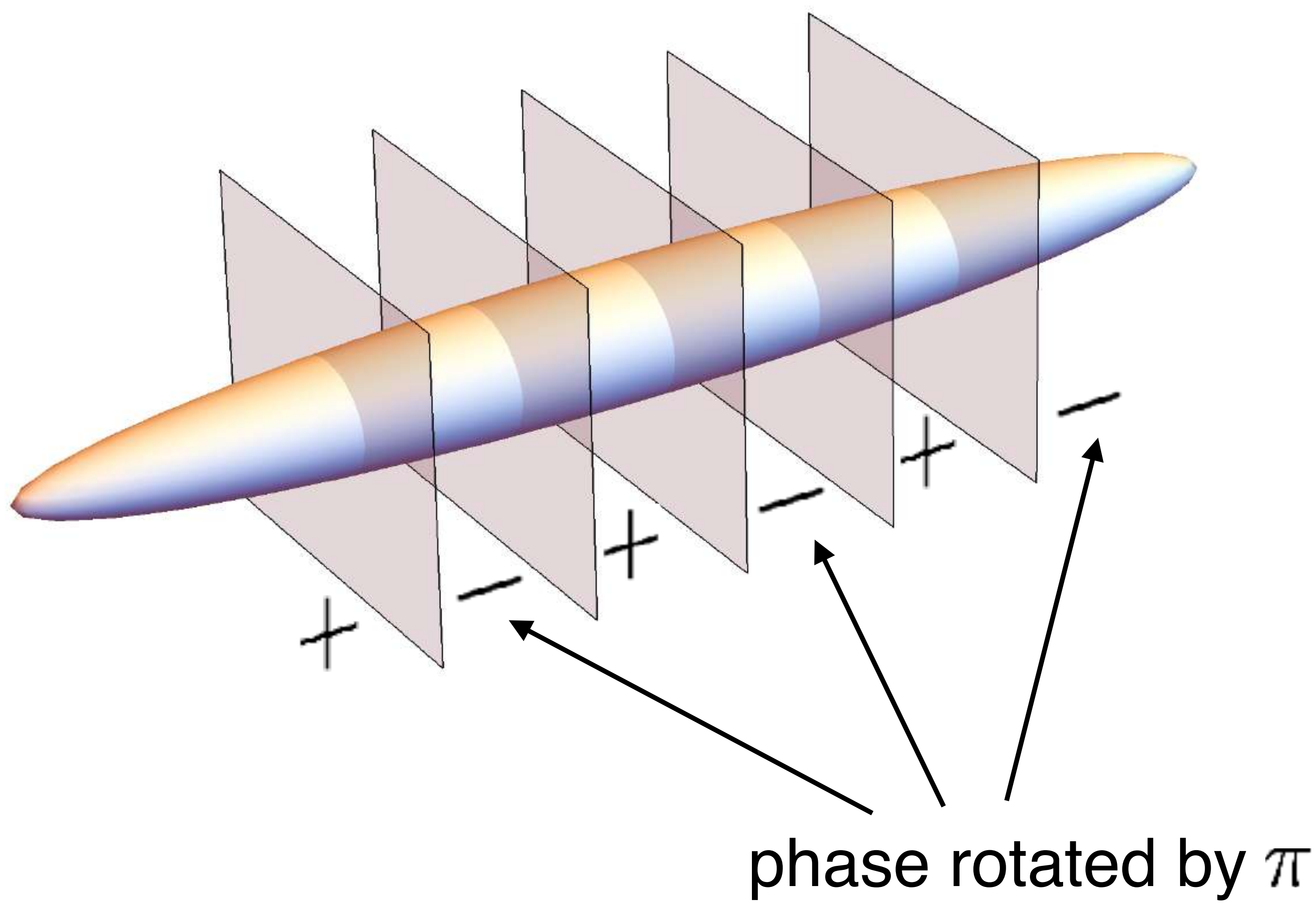




Soliton oscillation observed in Nature 499, 426 (2013)

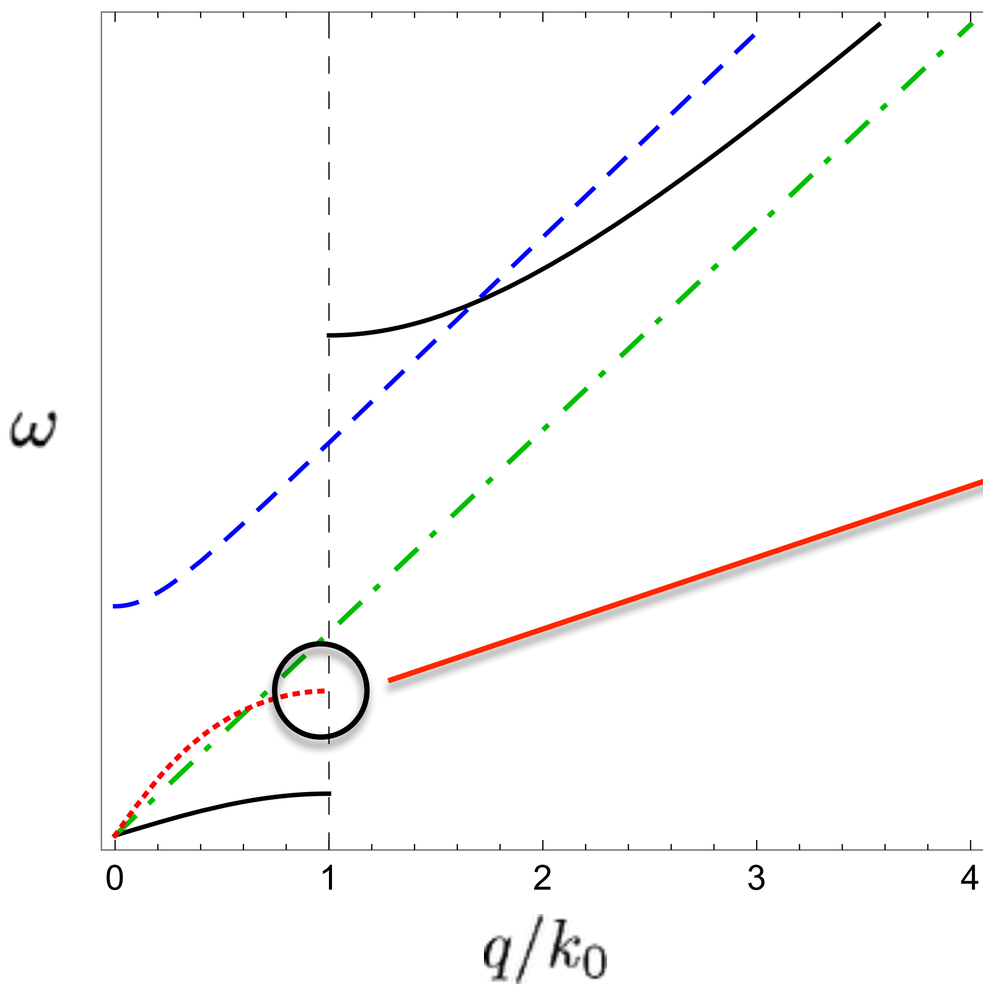
Creating soliton train in experiments

- Phase imprint multiple solitons by shining laser on alternate regions

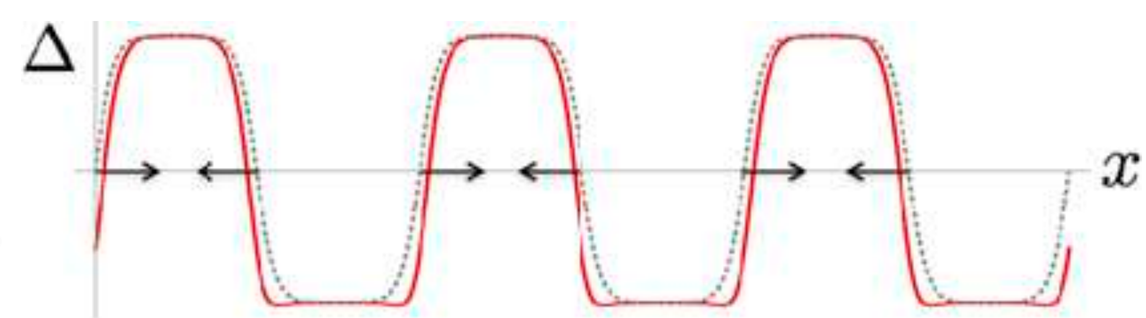


Instability

Collective-mode spectrum:



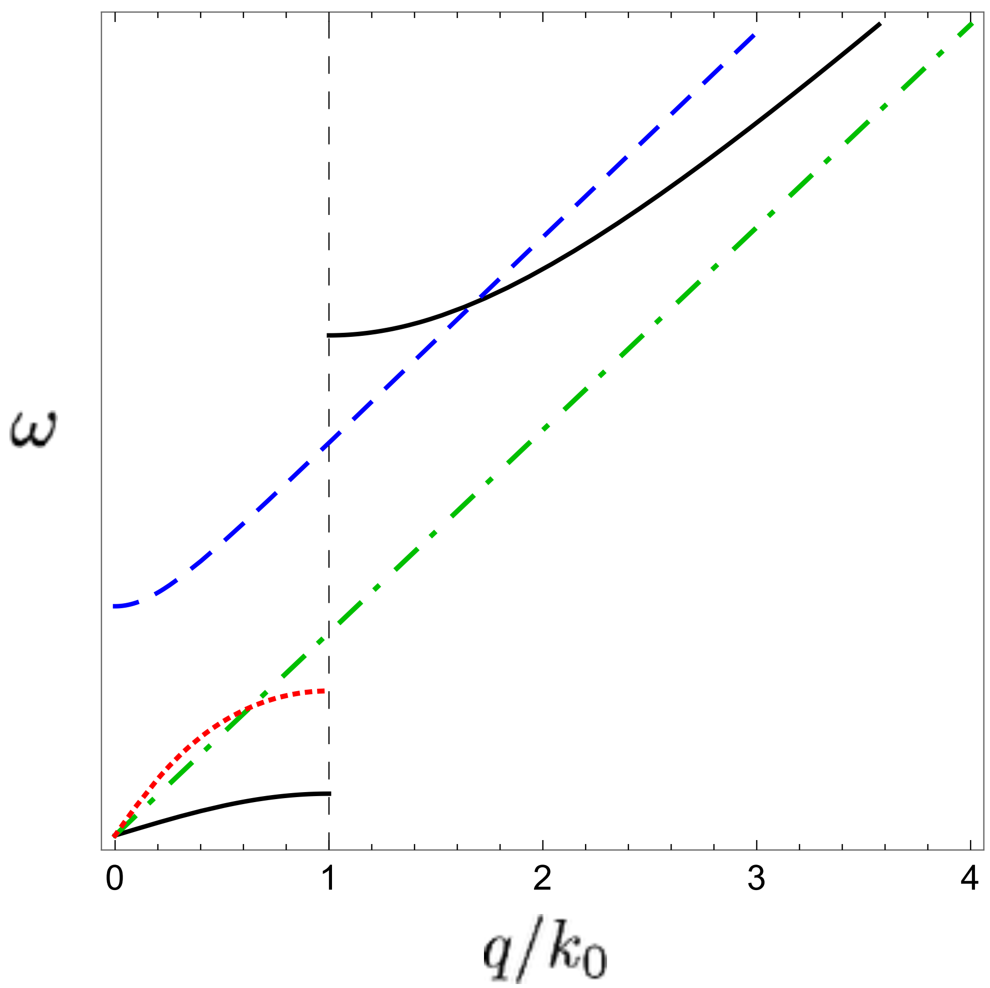
Problem: soliton train has instability!



solitons approach and annihilate each other

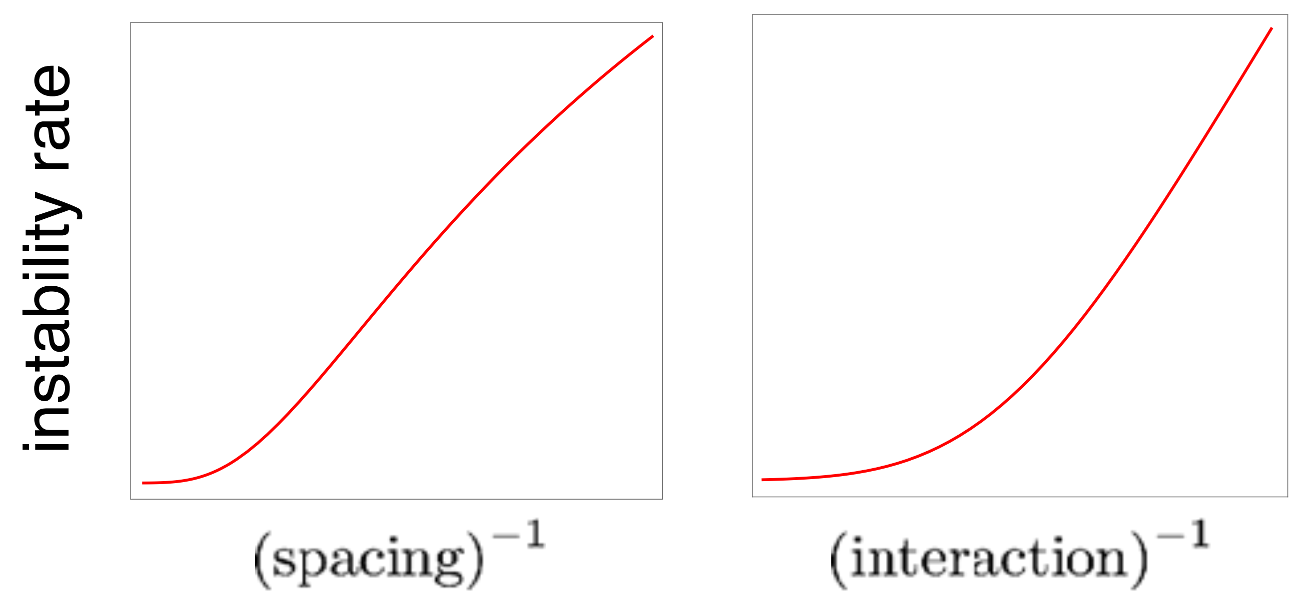
Instability

Collective-mode spectrum:



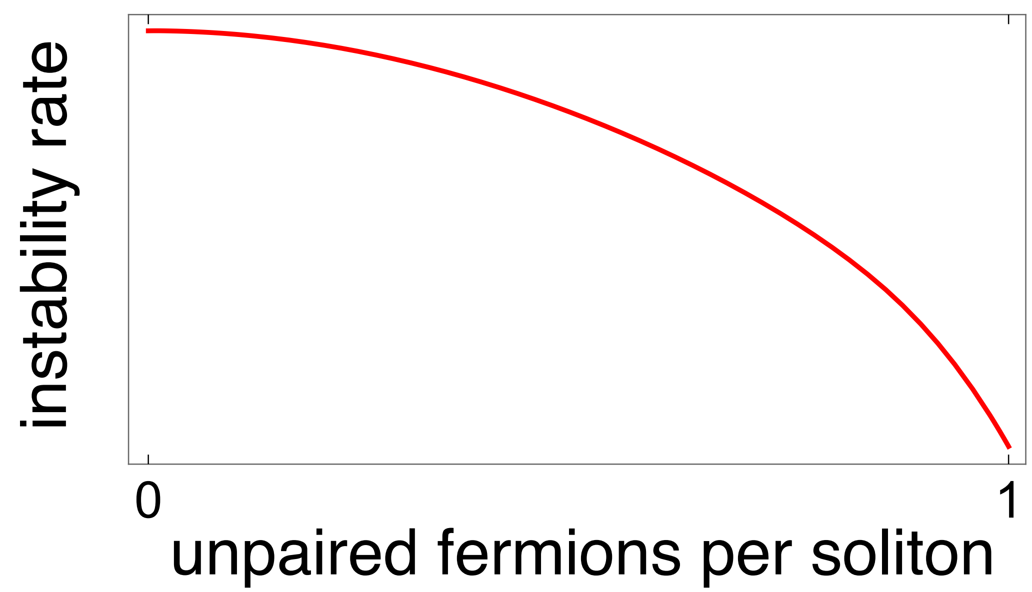
Problem: soliton train has instability!

Resolution: control instability rate by varying soliton spacing or interaction strength



Bonus: create stable FFLO

Introducing unpaired fermions also reduces instability:



Instability vanishes for 1 unpaired fermion per soliton

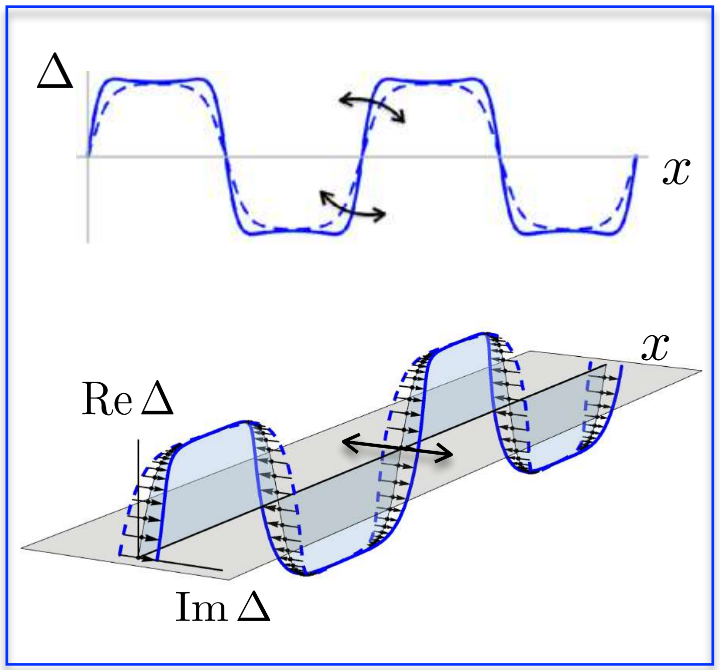
— an example of a partially polarized superfluid (FFLO), long-sought-after in condensed matter physics

$\Longrightarrow$ one can engineer <u>stable</u> FFLO in Fermi-gas experiments!

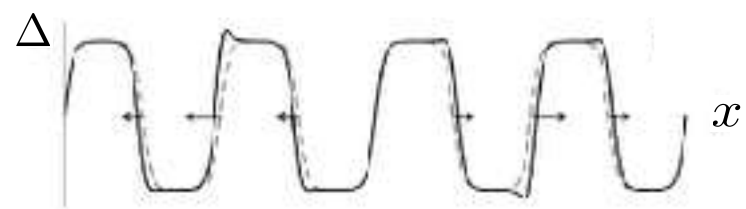
Summary

Shovan Dutta, Erich Mueller
PRL 118, 260402 (2017)

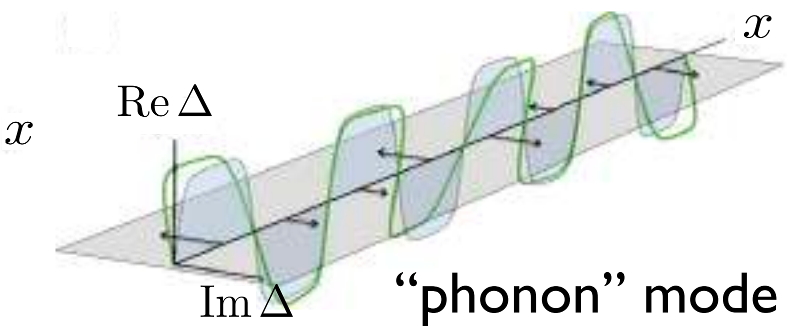
Soliton trains have a rich set of collective modes!



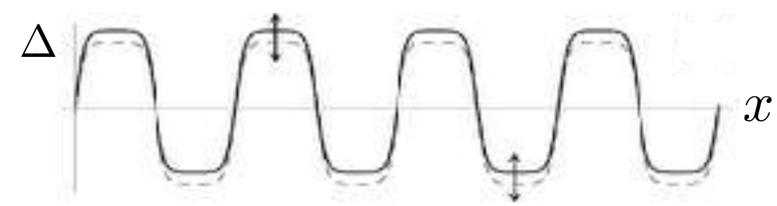
Novel "core" modes —
probe in experiments



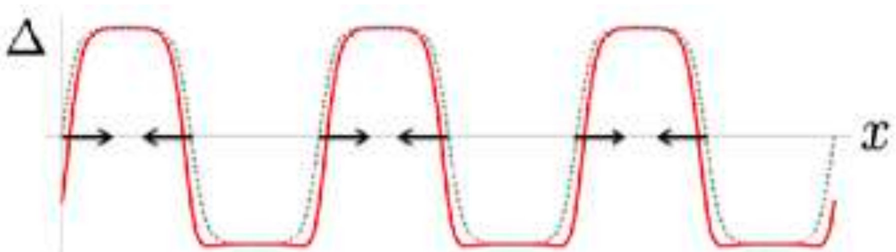
"elastic" mode



"phonon" mode

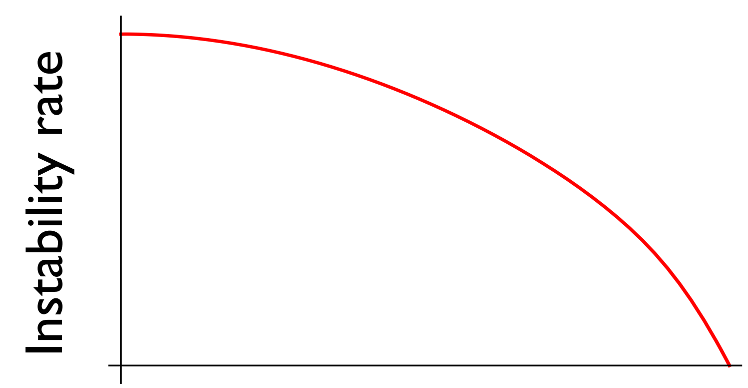


"Higgs" mode



Instability

stabilize by adding
unpaired fermions



Thank You!