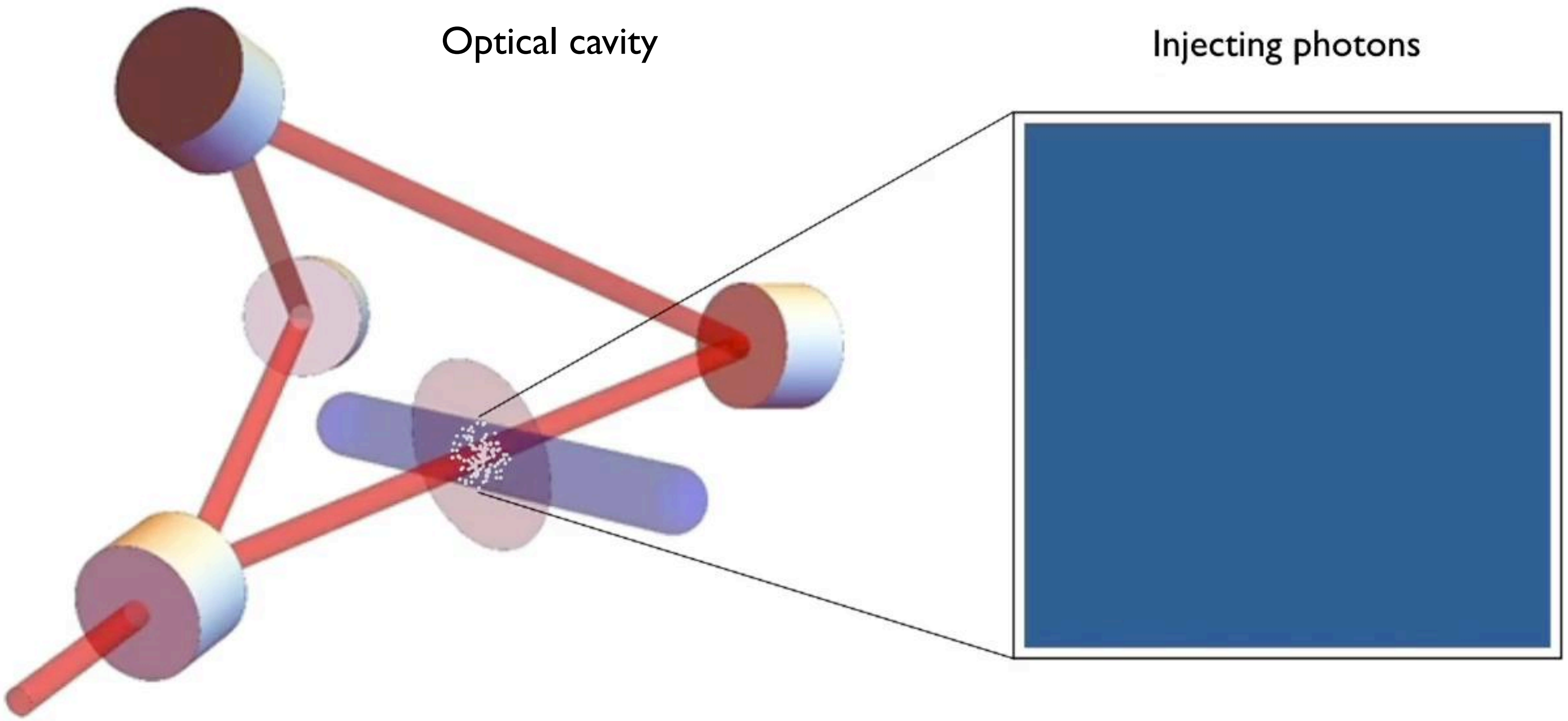
S.D. and Erich Mueller
PRA 97, 033825 (2018)

NSF
ARO - MURI

Creating and Braiding Anyons in an Optical Cavity

Shovan Dutta — Cornell University



Other projects during PhD

Published:

- Kinetics of Bose-Einstein condensation in a "dimple" trap
  - PRA 91, 013601 (2015)
- Dynamics of spin impurities in a Bose lattice gas
  - PRA 88, 053601 (2013)
- Dimensional crossover in a spin-imbalanced Fermi gas
  - PRA 94, 063627 (2016)
- Collective modes of a soliton train in a Fermi superfluid
  - PRL 118, 260402 (2017)
- Engineering FFLO-superfluid phases in a Fermi gas
  - PRA 96, 023612 (2017)

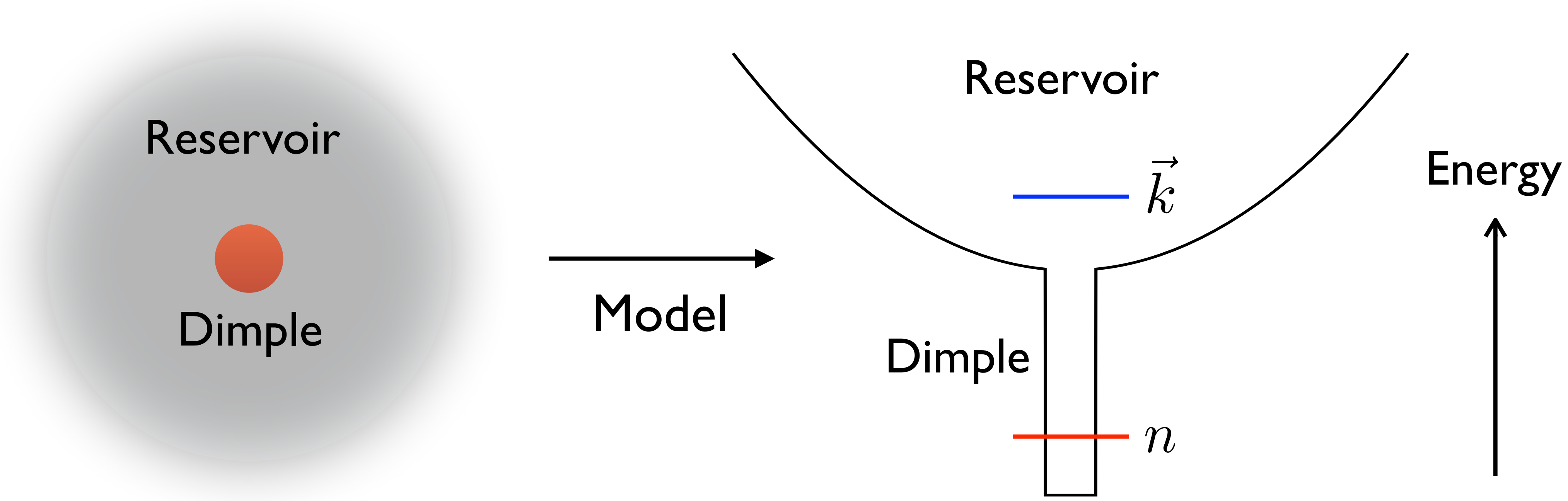
Others:

- Thermalization in a quasi-1D trap
- Spin-imbalanced Fermi gas in an array of coupled tubes
- Nucleation of superfluid-B phase in liquid Helium-3

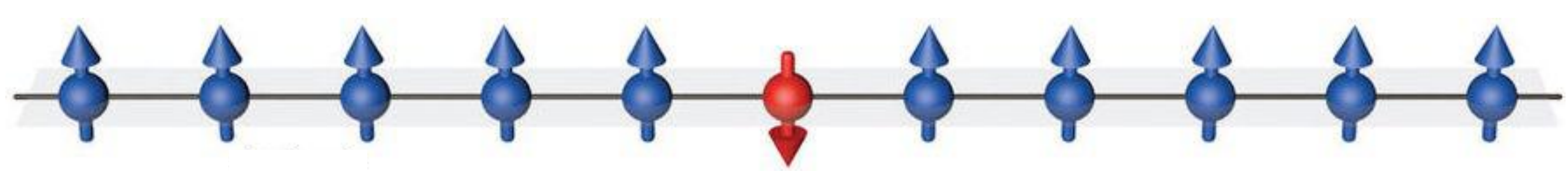
Other projects during PhD

Dynamics in Bose gases

- Kinetics of Bose condensation in a dimple trap PRA 91, 013601 (2015)



- Dynamics of spin impurities in a 1D lattice PRA 88, 053601 (2013)



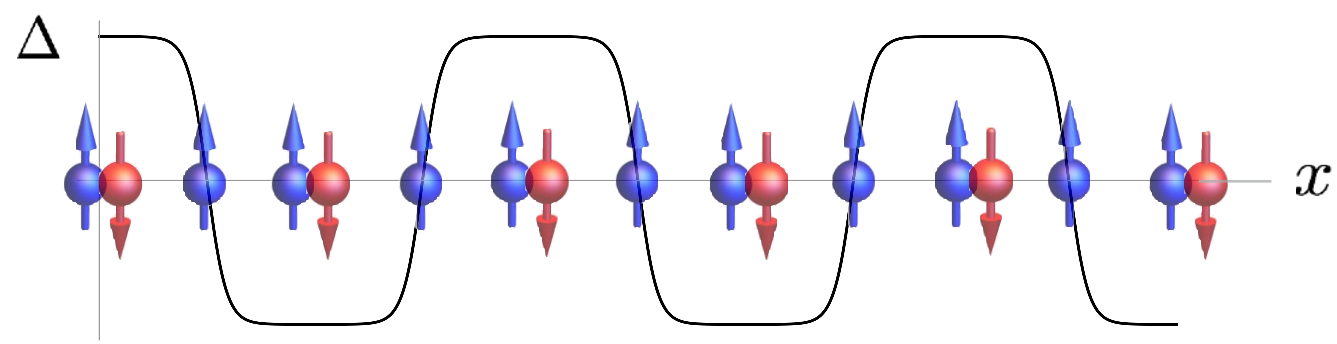
Other projects during PhD

Pu

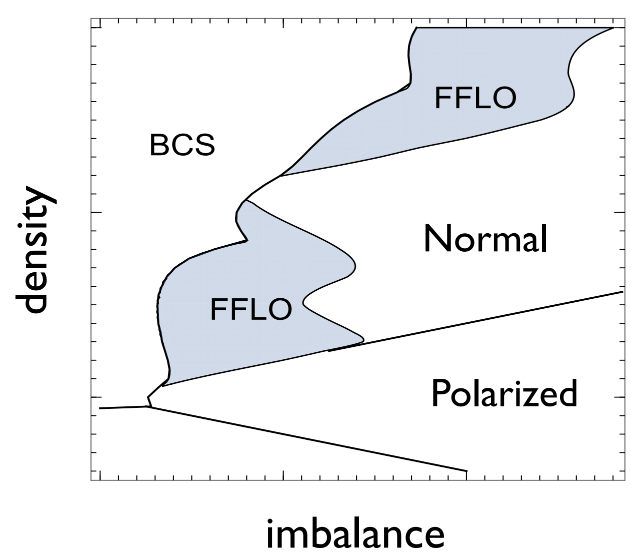
O

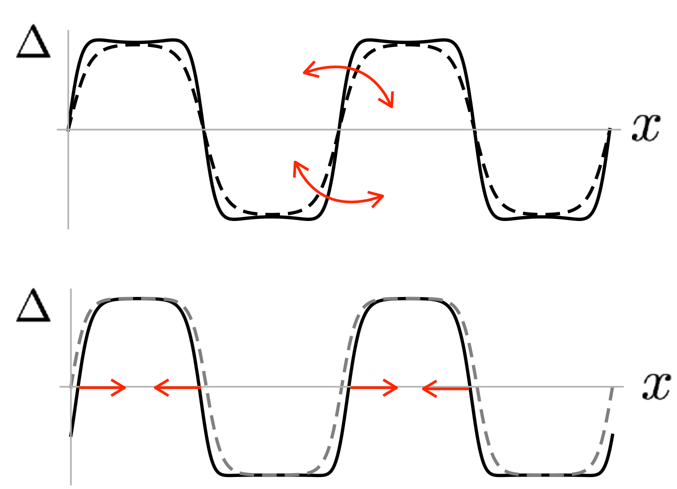
Exotic superfluidity in Fermi gases

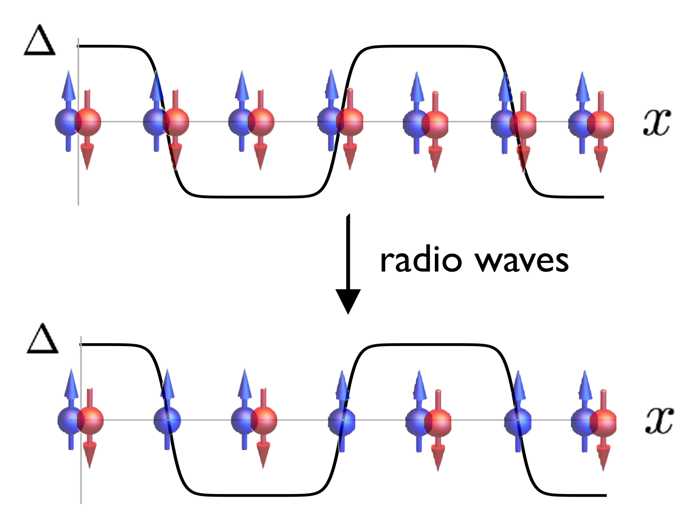
FFLO phase — spin-imbalanced superfluid with a train of domain walls



1D — 3D crossover	Collective modes	Protocol for engineering
PRA 94, 063627 (2016)	PRL 118, 260402 (2017)	PRA 96, 023612 (2017)







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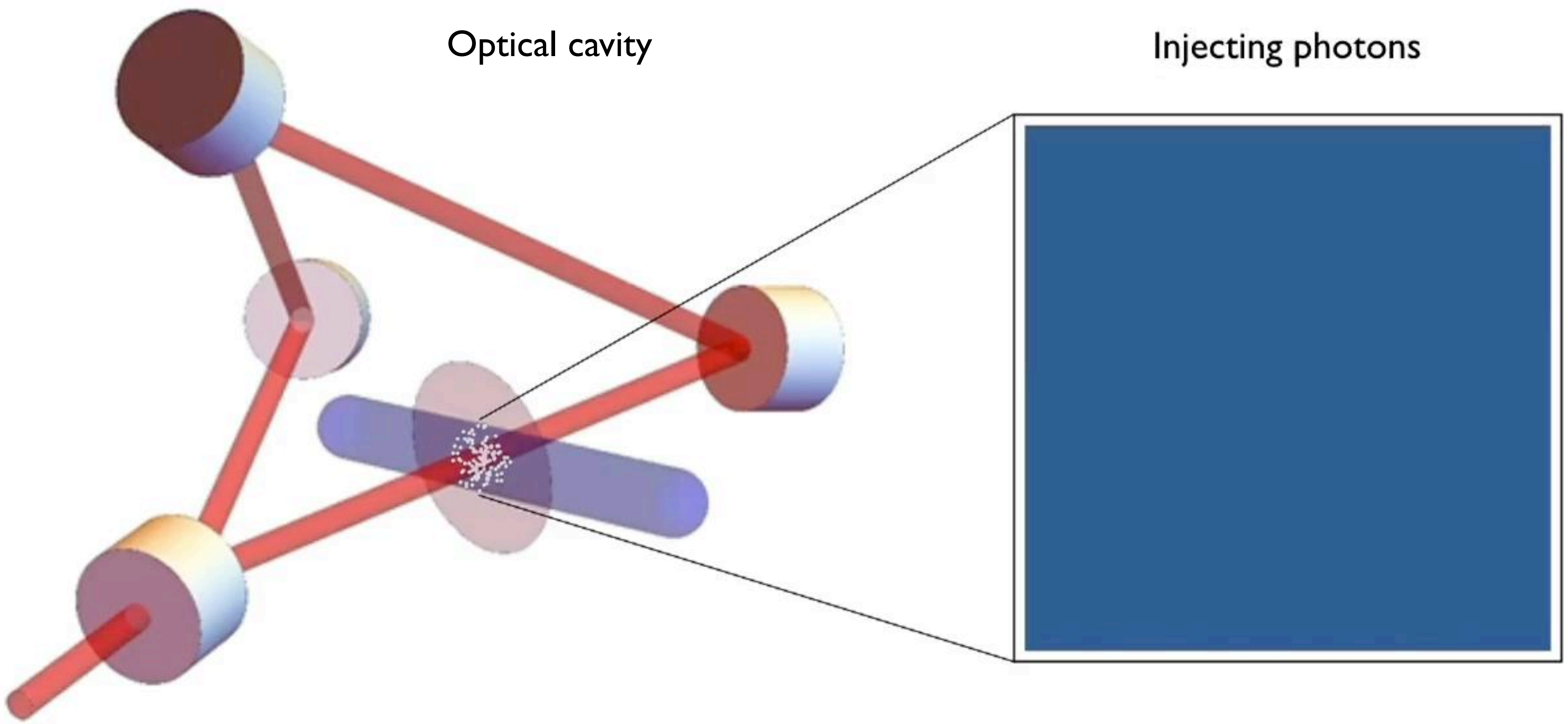
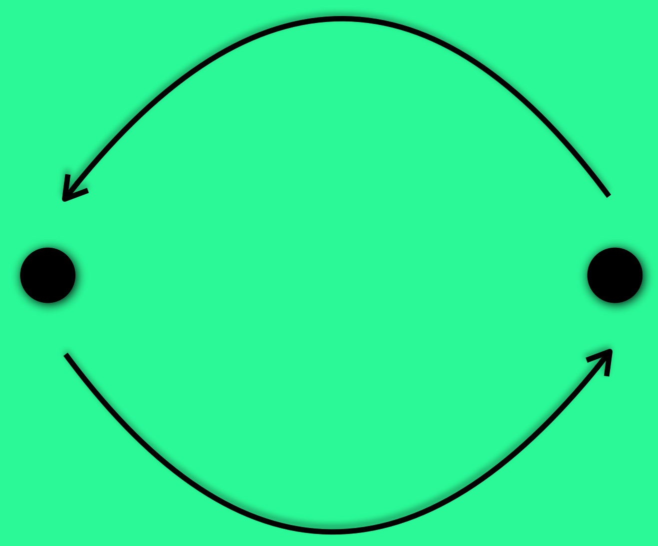


Motivation: Anyons

- Fractional Exchange Statistics: exotic physics in flatland



Bosons: $+1$

Fermions: -1

Anyons: $e^{i\phi_s}$

Non-Abelian: U

- Proposed hardware for topological quantum computation

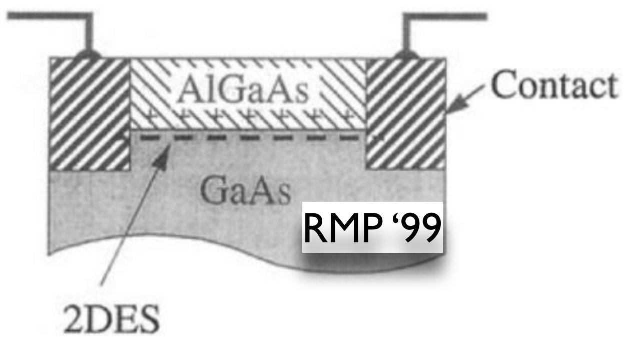
(A. Kitaev '03; C. Nayak et al. '08)

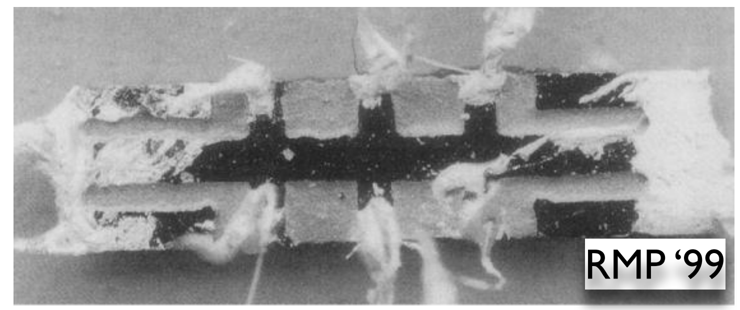
Challenge:

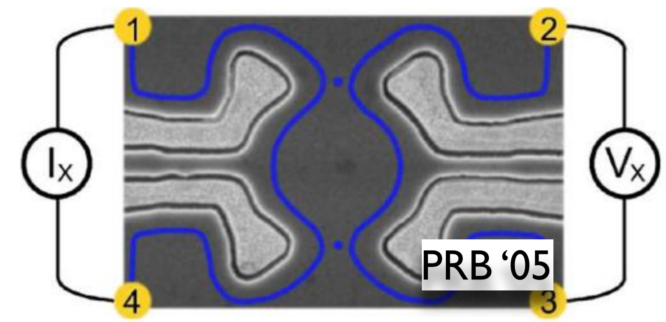
1. Create many-body state with anyonic excitations
   - fractional quantum Hall (FQH) states
2. Create quasiparticles
3. Move them around one another
4. Measure accumulated phase

Platforms

- Interacting 2D fermions in a magnetic field

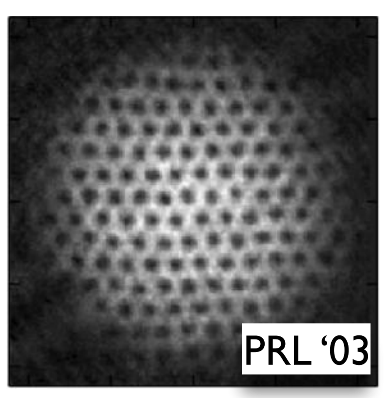


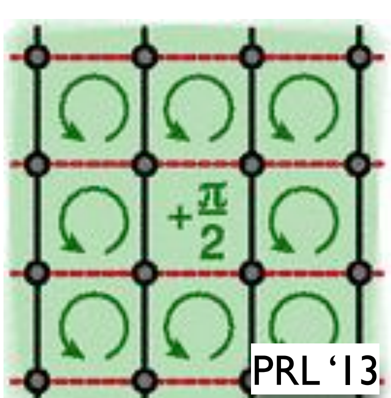




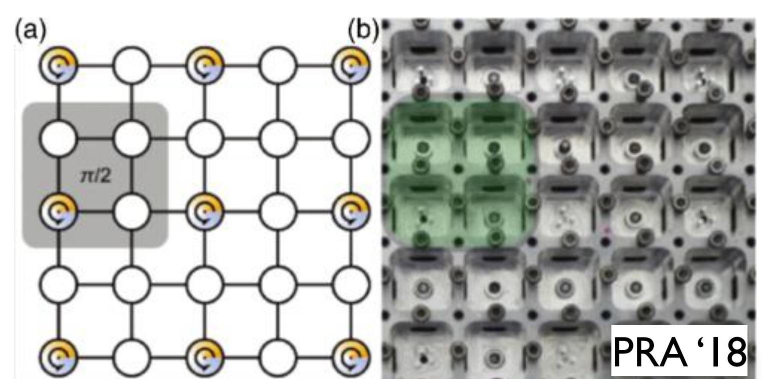
- Interacting 2D bosons in an effective magnetic field

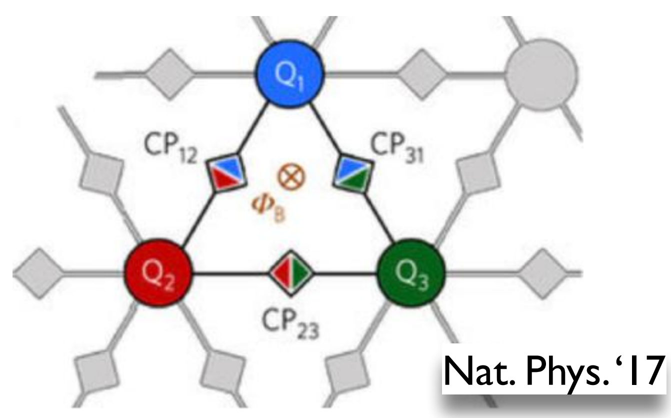
Cold atoms



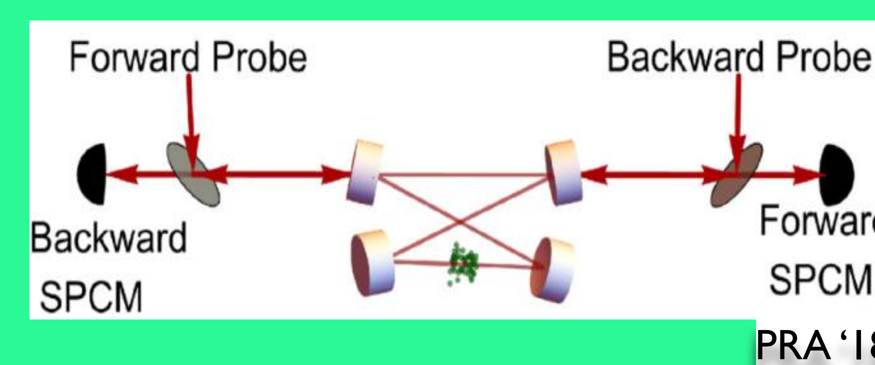


Coupled μ-wave cavities





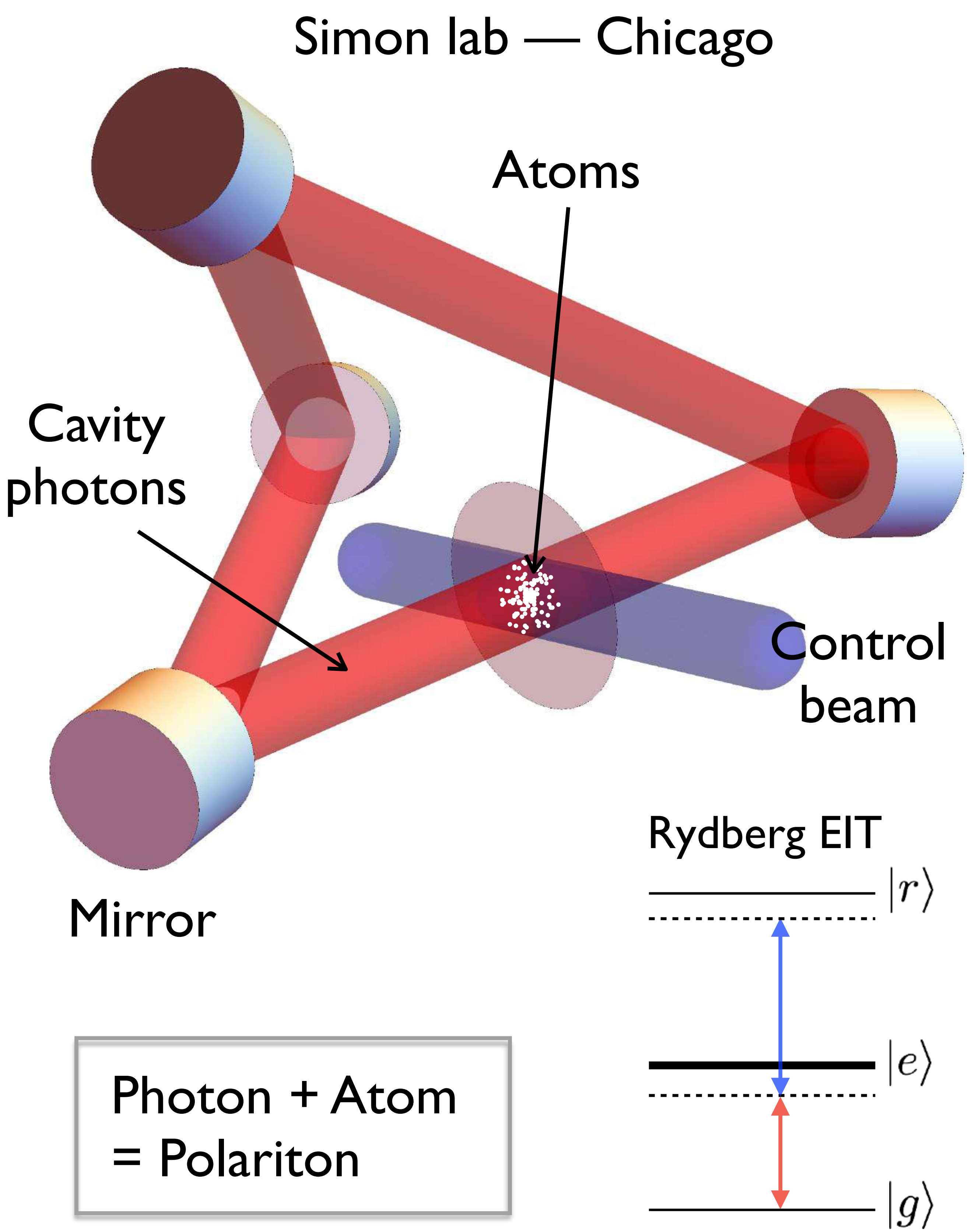
Multimode optical cavity



pros: driven state preparation
versatile in/out coupling

cons: photon loss
weak interaction
(matter mediated)

Experimental setup



Photon + Atom
= Polariton

Near-degenerate cavity

- longitudinal mode number fixed
- 2D dynamics in transverse plane with finite effective mass

Concave mirrors

- transverse harmonic confinement

Non-planar geometry

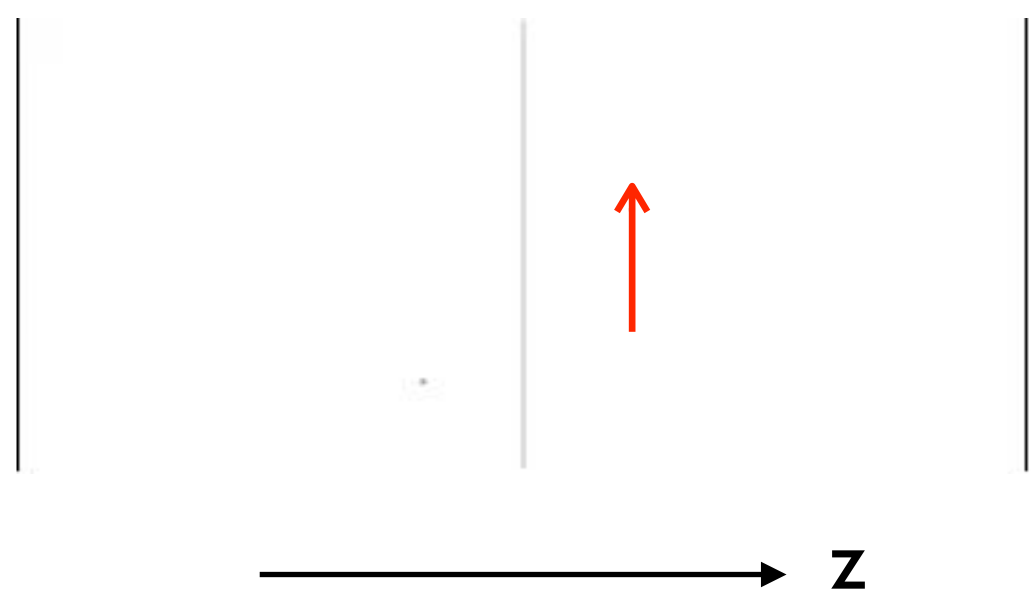
- light field rotated about axis
- effective magnetic field

Atom-photon coupling

- long-lived interacting polaritons
- photon dynamics + Rydberg int.

Experimental setup

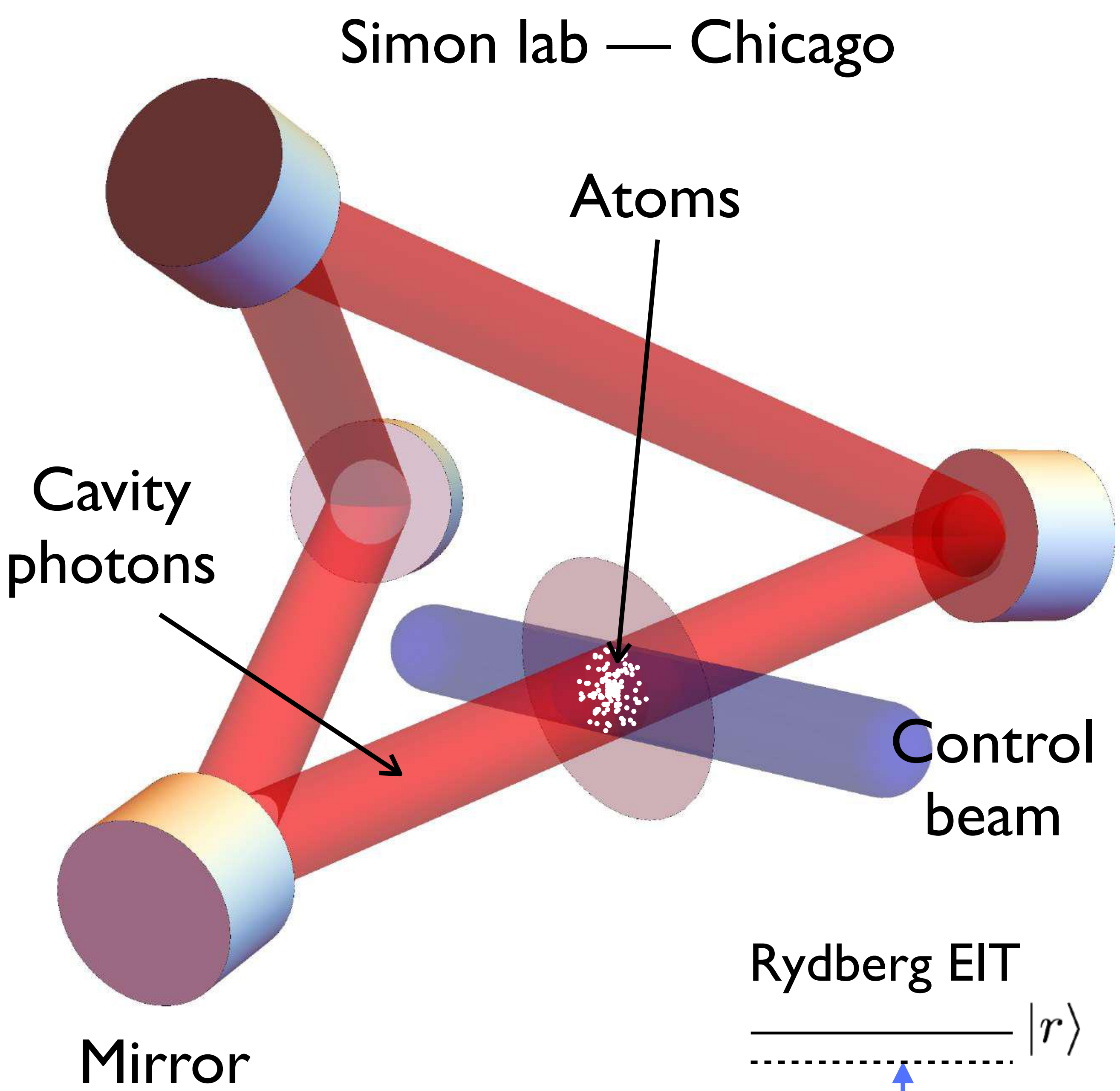
Near-degenerate cavity — effective transverse dynamics



Uniform motion along transverse direction at speed $p_\perp / M_\perp$

$$
\begin{aligned}
E &= \sqrt{p_z^2 + p_\perp^2}\, c \\
&\approx p_z c + \frac{p_\perp^2 c}{2 p_z} \\
&= E_z + \frac{p_\perp^2}{\underbrace{2(E_z/c^2)}_{M_\perp}}
\end{aligned}
$$

Experimental setup



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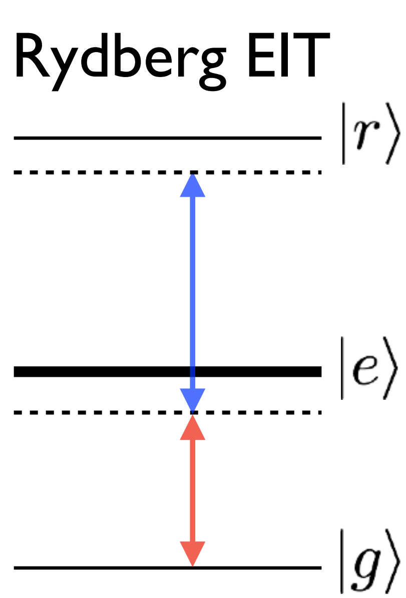

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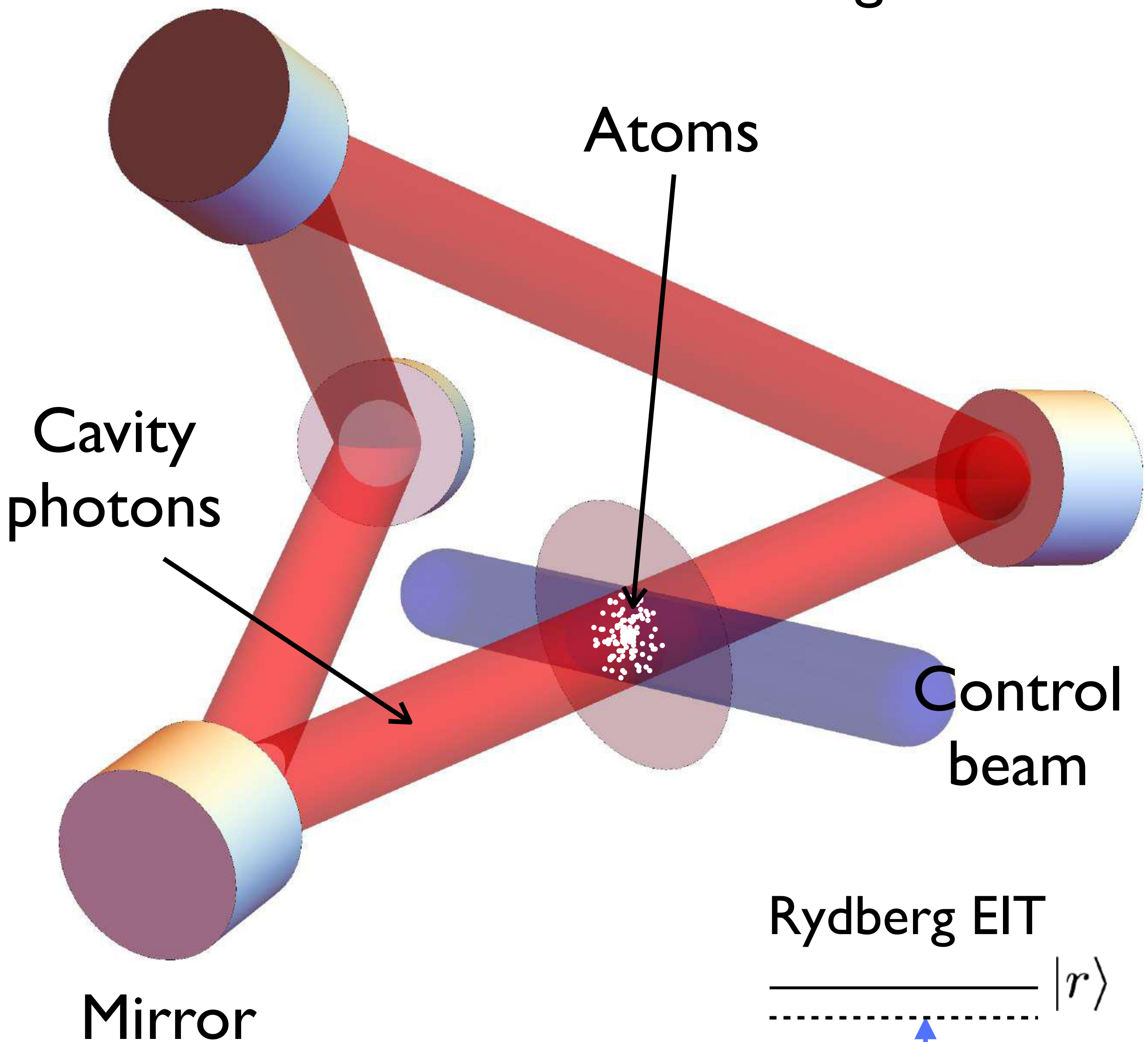
Simon lab — Chicago



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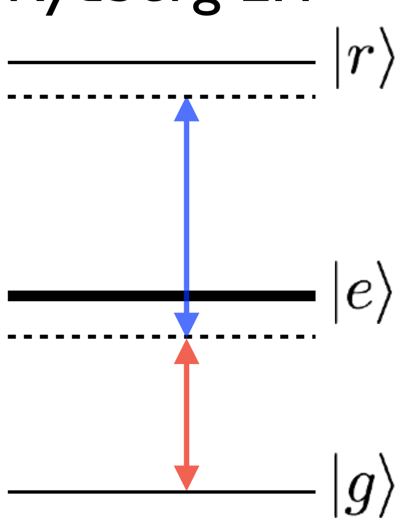
Rydberg EIT



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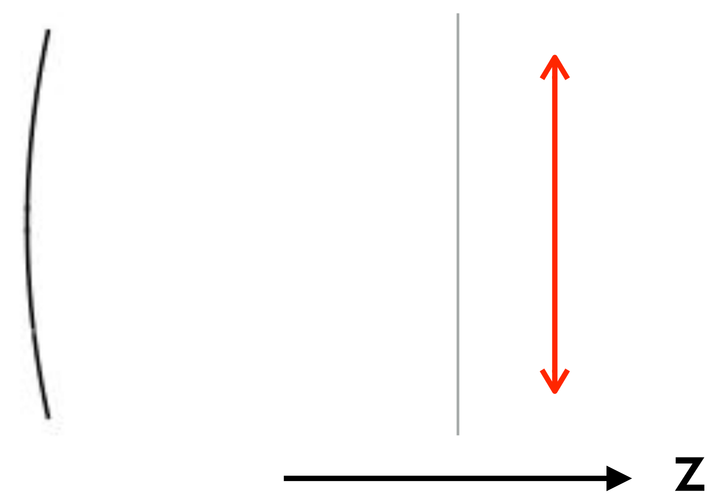
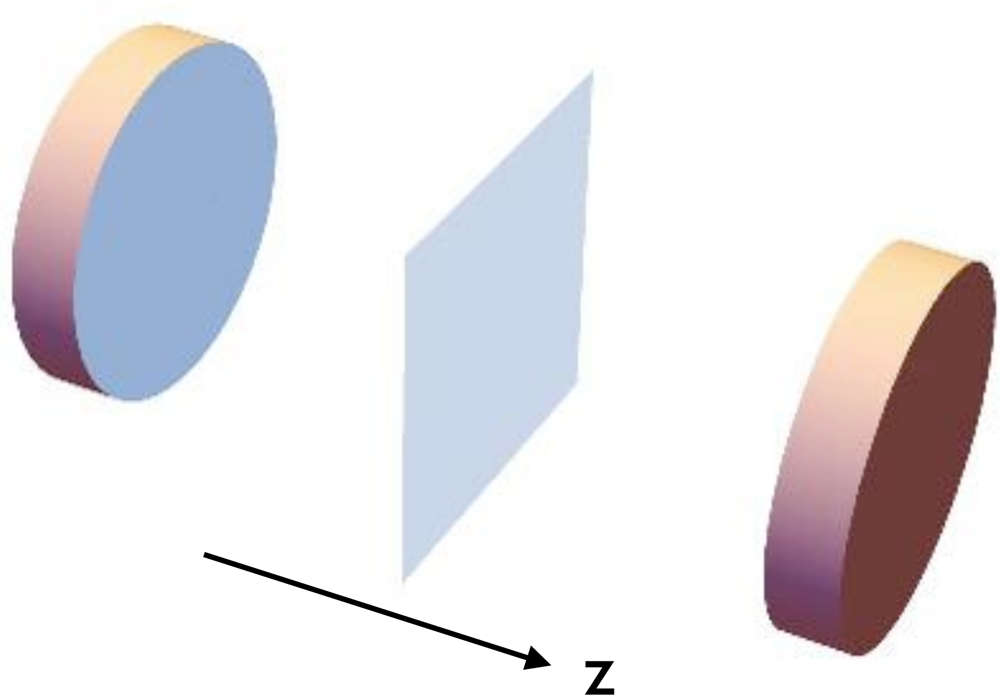
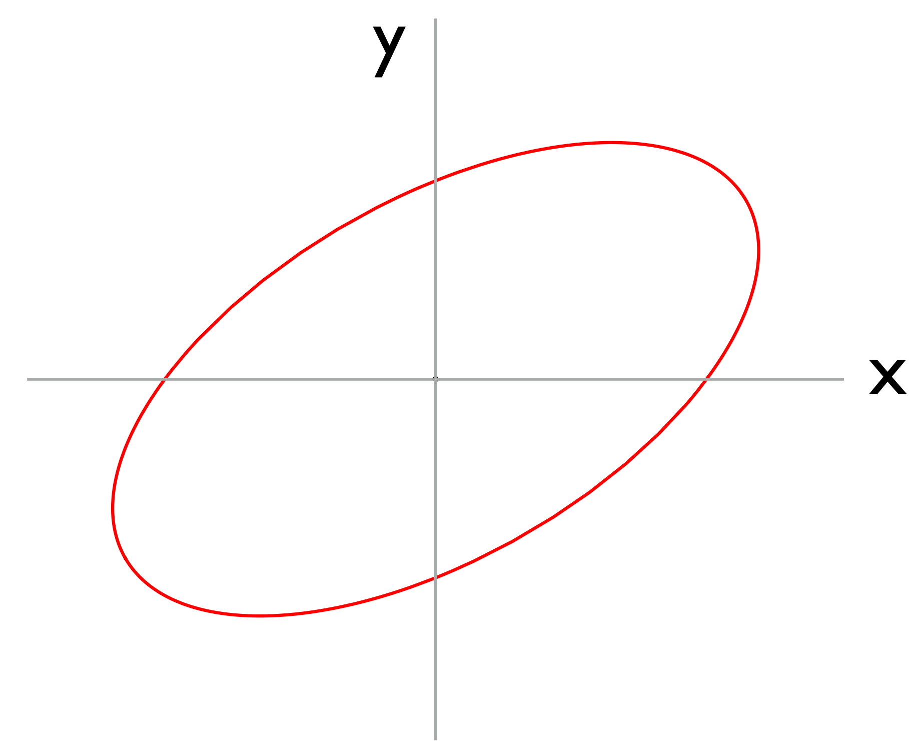
Concave mirrors — transverse confinement



$$E \approx \sqrt{(p_z(r_\perp))^2 + p_\perp^2}\, c$$

$$\approx E_z + \frac{p_\perp^2}{2M_\perp} + \frac{1}{2} M_\perp \omega_\perp^2 r_\perp^2$$





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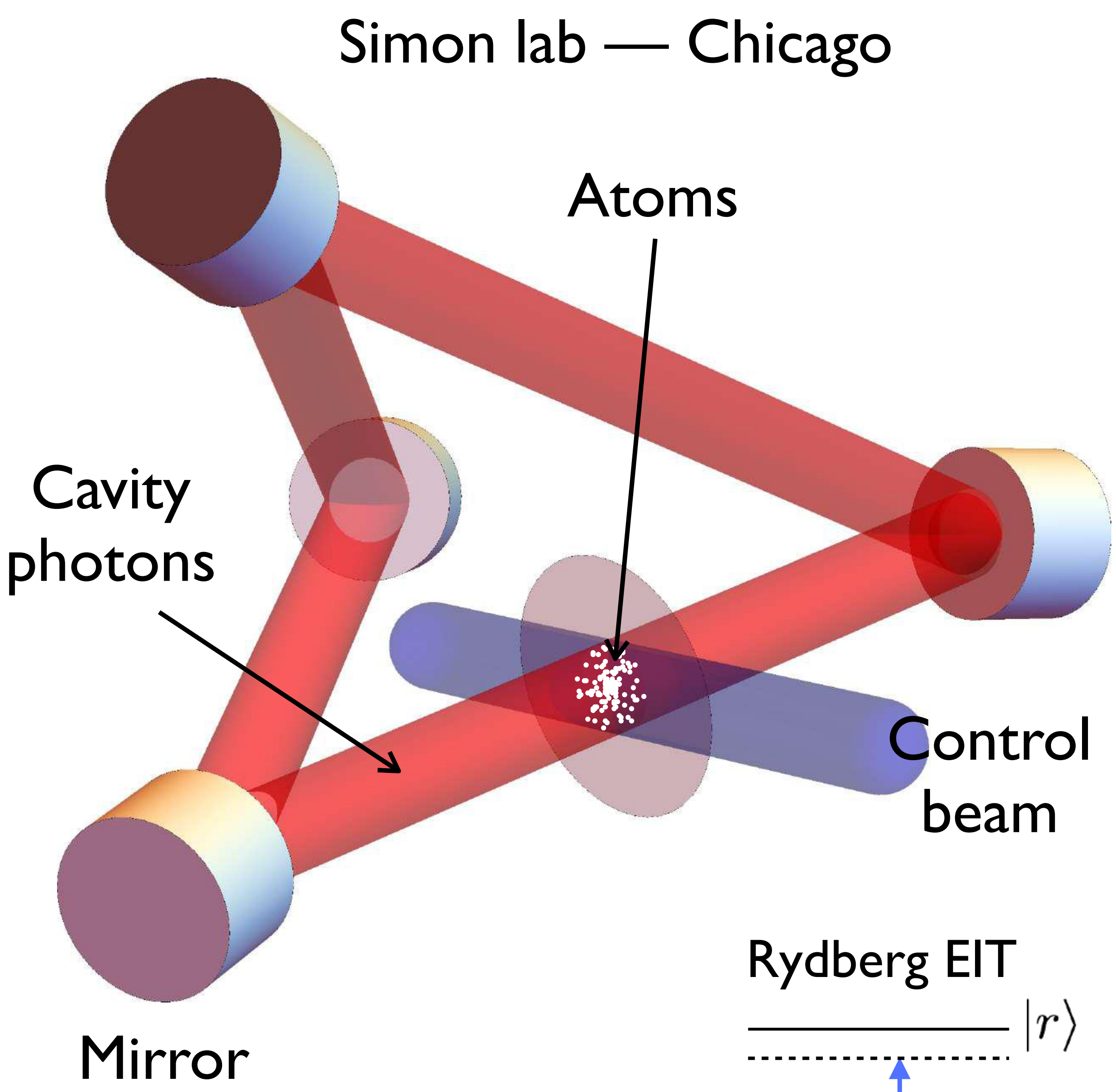

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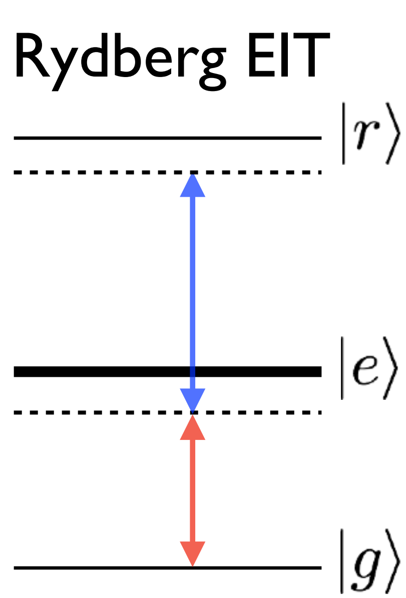

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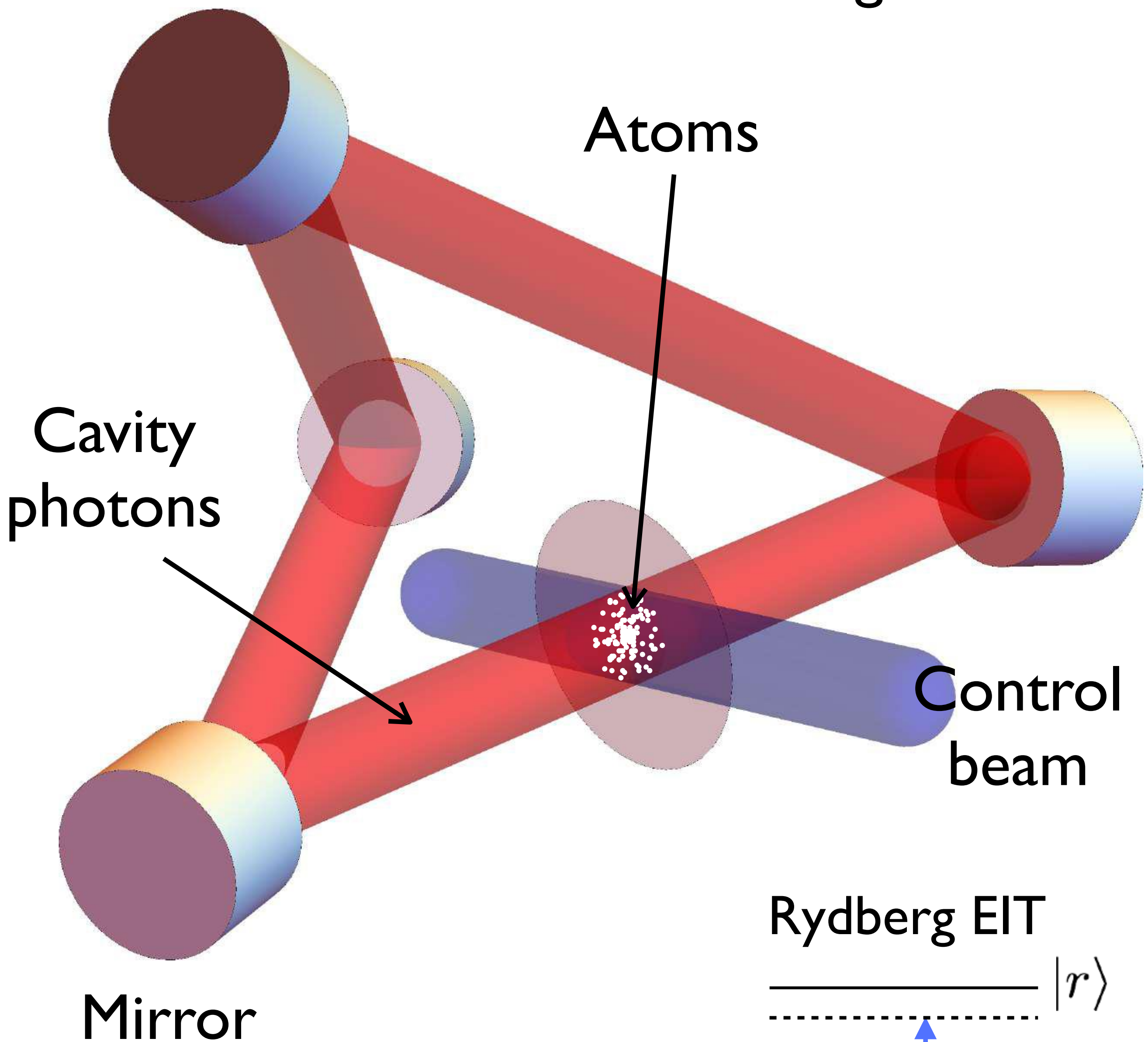

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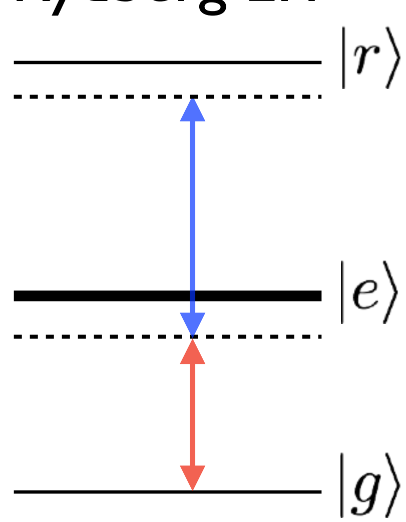
Rydberg EIT



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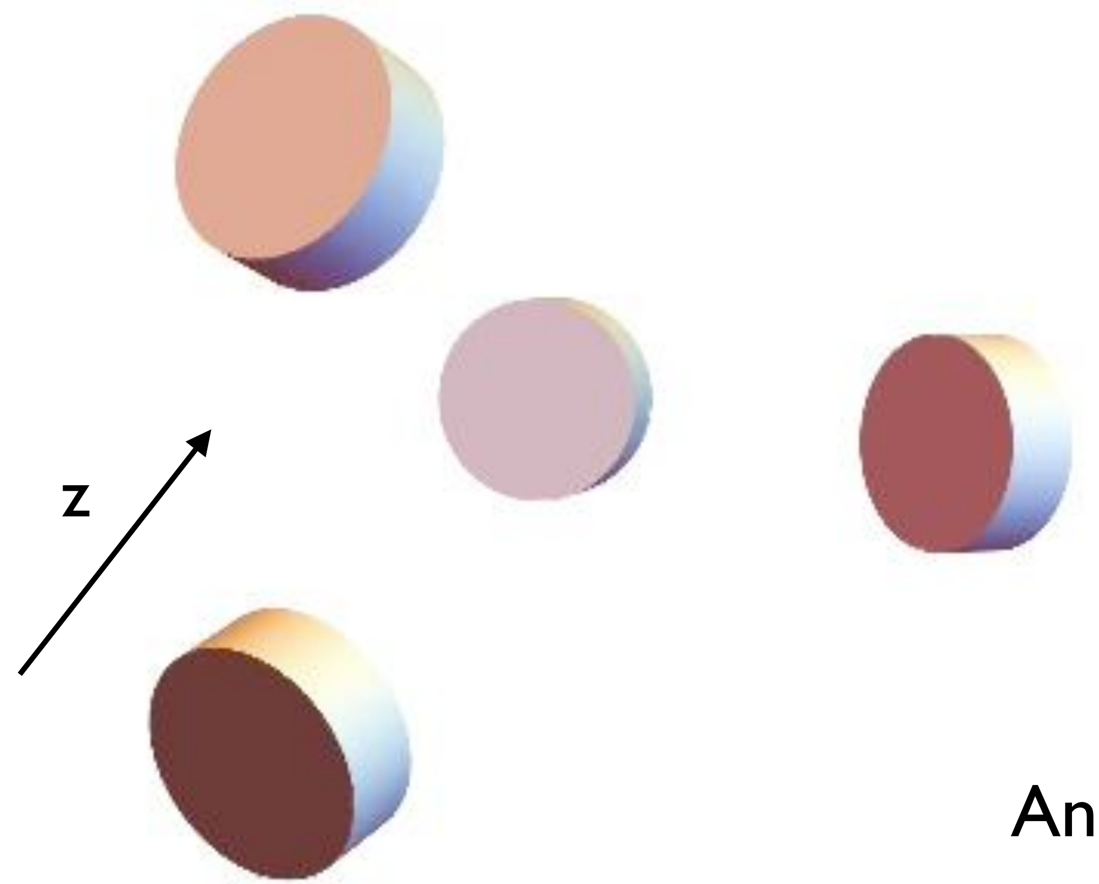
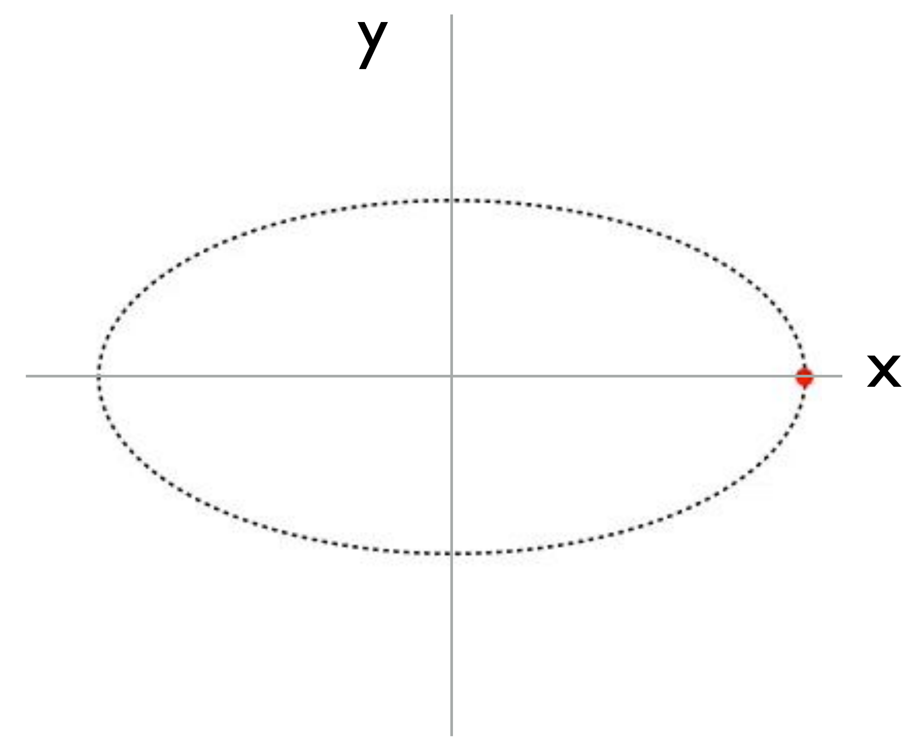
Non-planar geometry — effective magnetic field



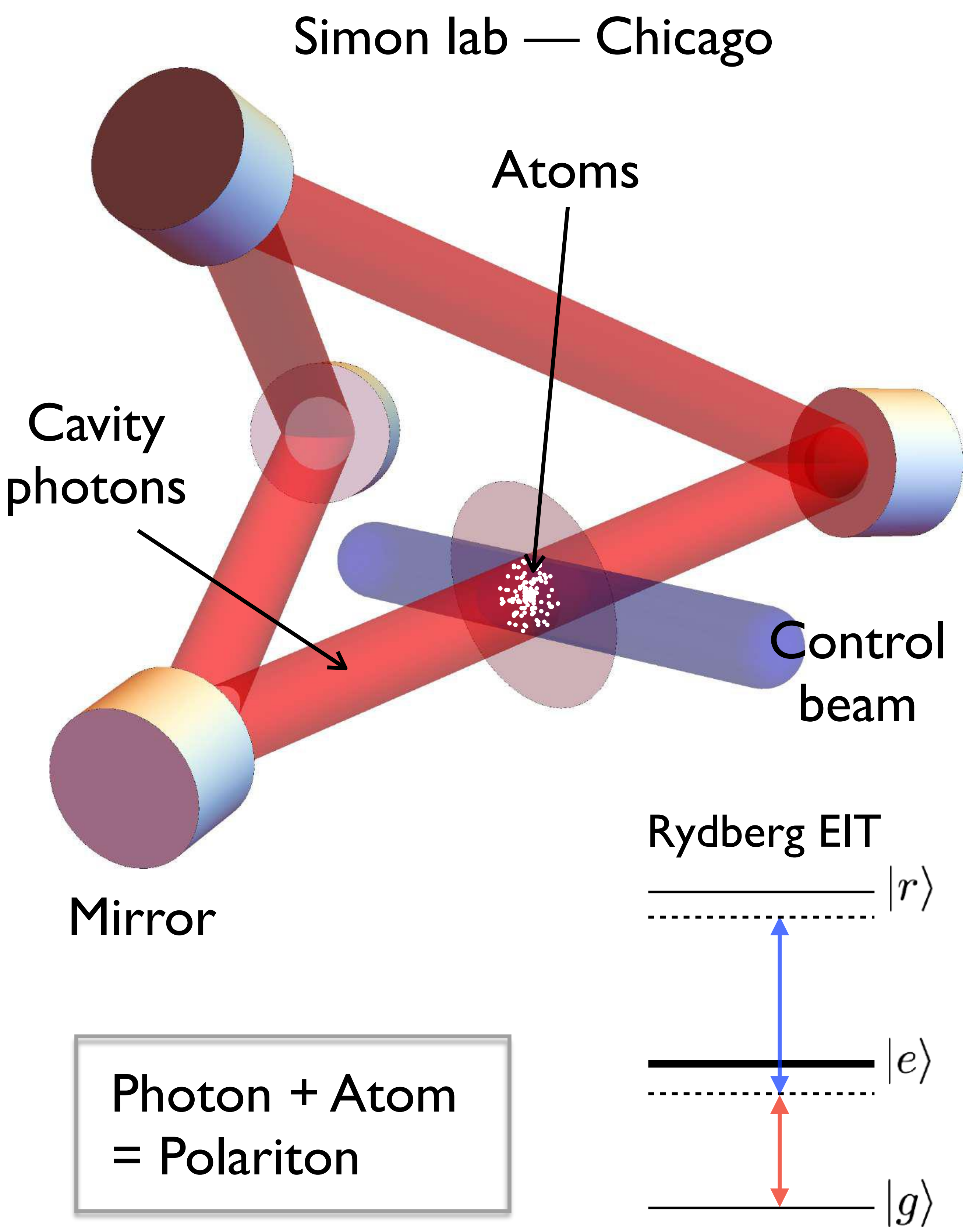


Analogous to a uniformly rotating frame

$$\vec{F}_{\text{Coriolis}} = \vec{v}_\perp \times (2M_\perp \vec{\omega}_{\text{rot}}) \equiv \vec{v}_\perp \times (q\vec{B}_{\text{eff}})$$

$$\vec{F}_{\text{Centrifugal}} = -\vec{\nabla}_\perp \left(-\frac{1}{2} M_\perp \omega_{\text{rot}}^2 r_\perp^2 \right)$$

Experimental setup



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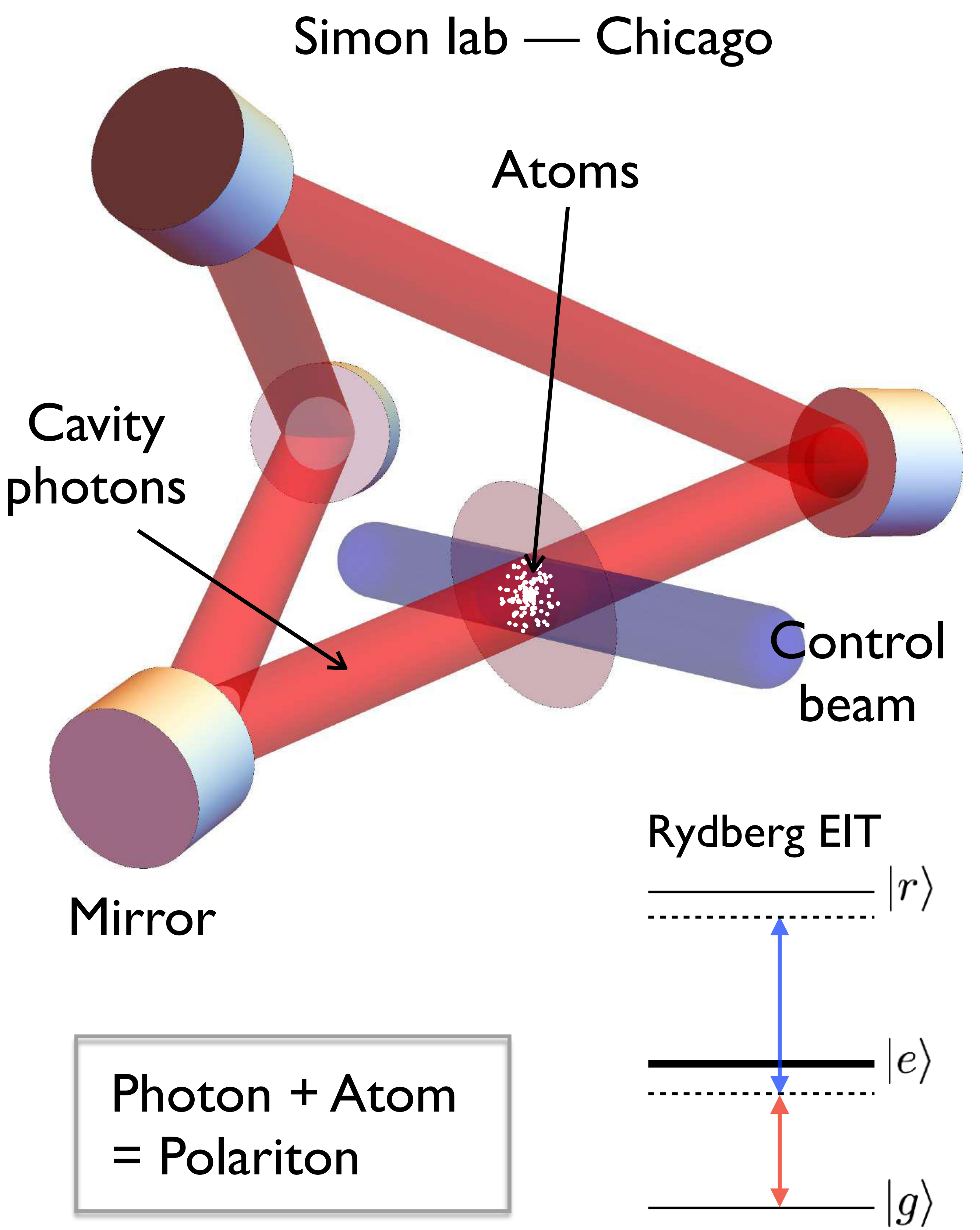

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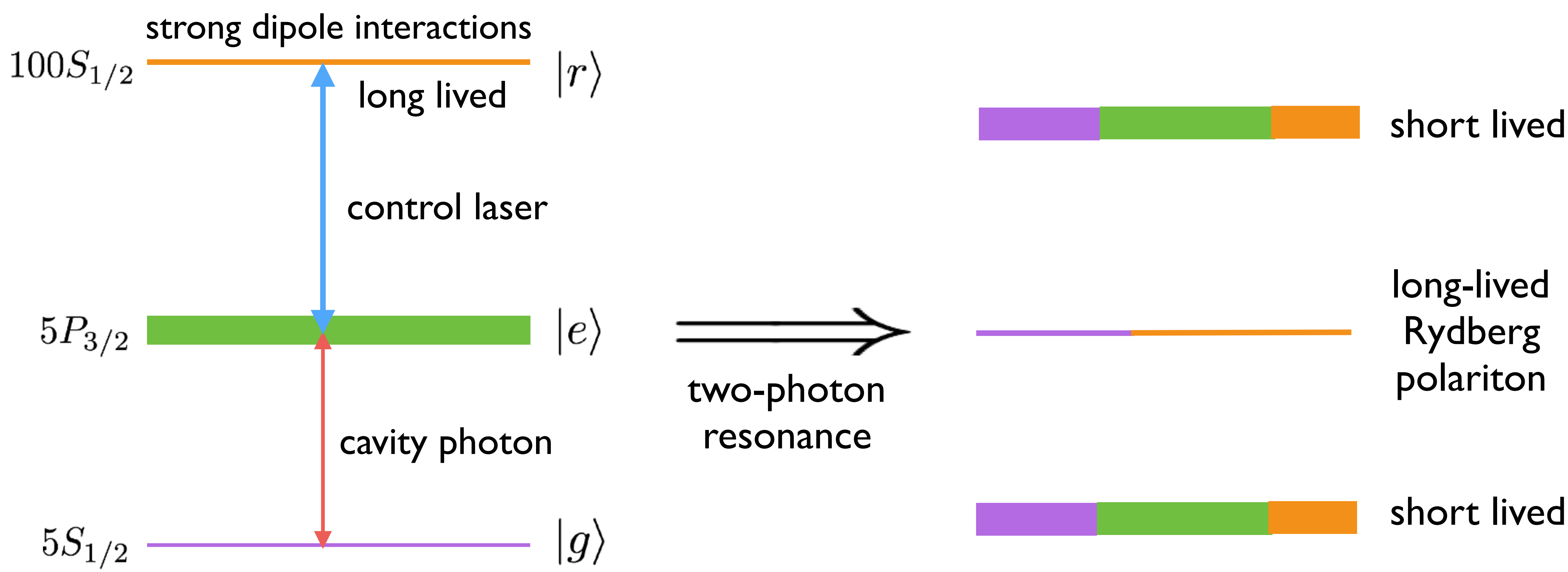
Atom-photon coupling — Rydberg polaritons

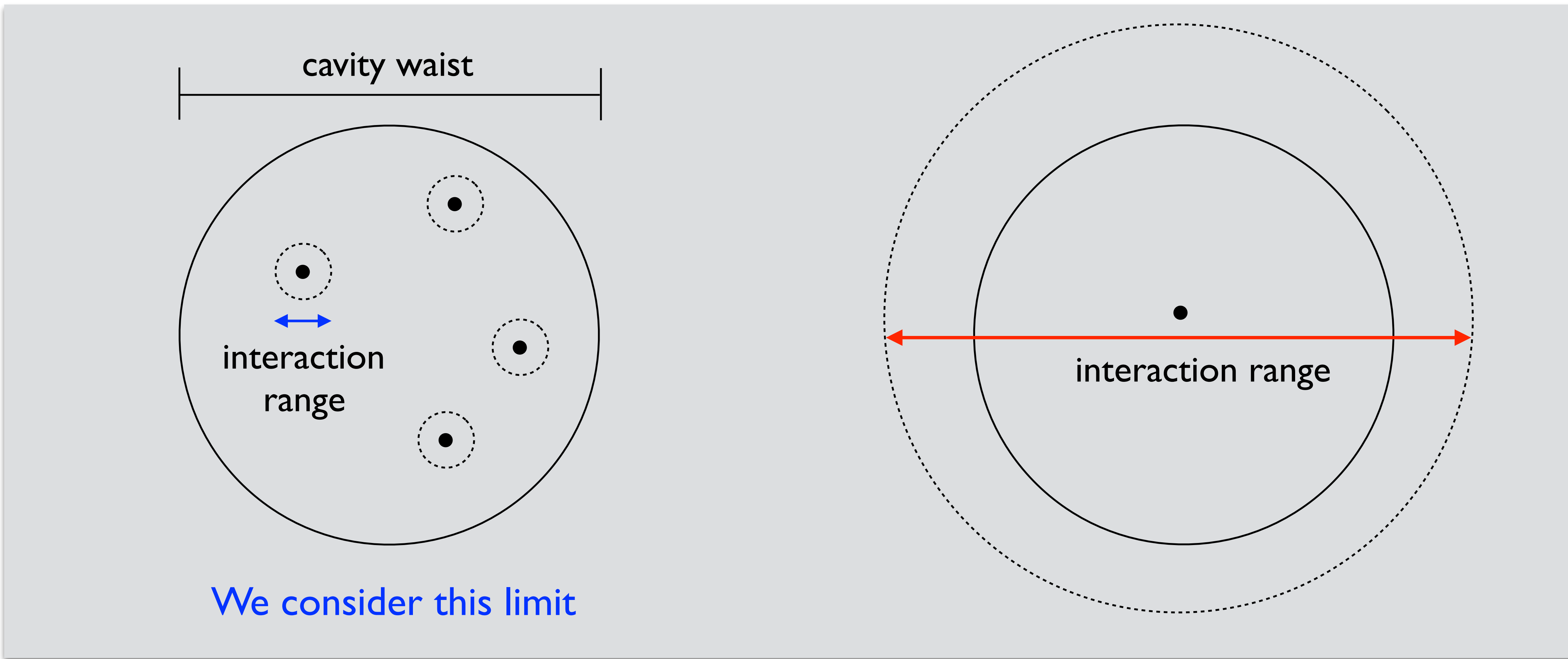


Project dynamics onto polariton mode

- atom-photon hybrid
- dynamics of photons
- interactions of Rydberg

Experimental setup

Atom-photon coupling — Rydberg polaritons



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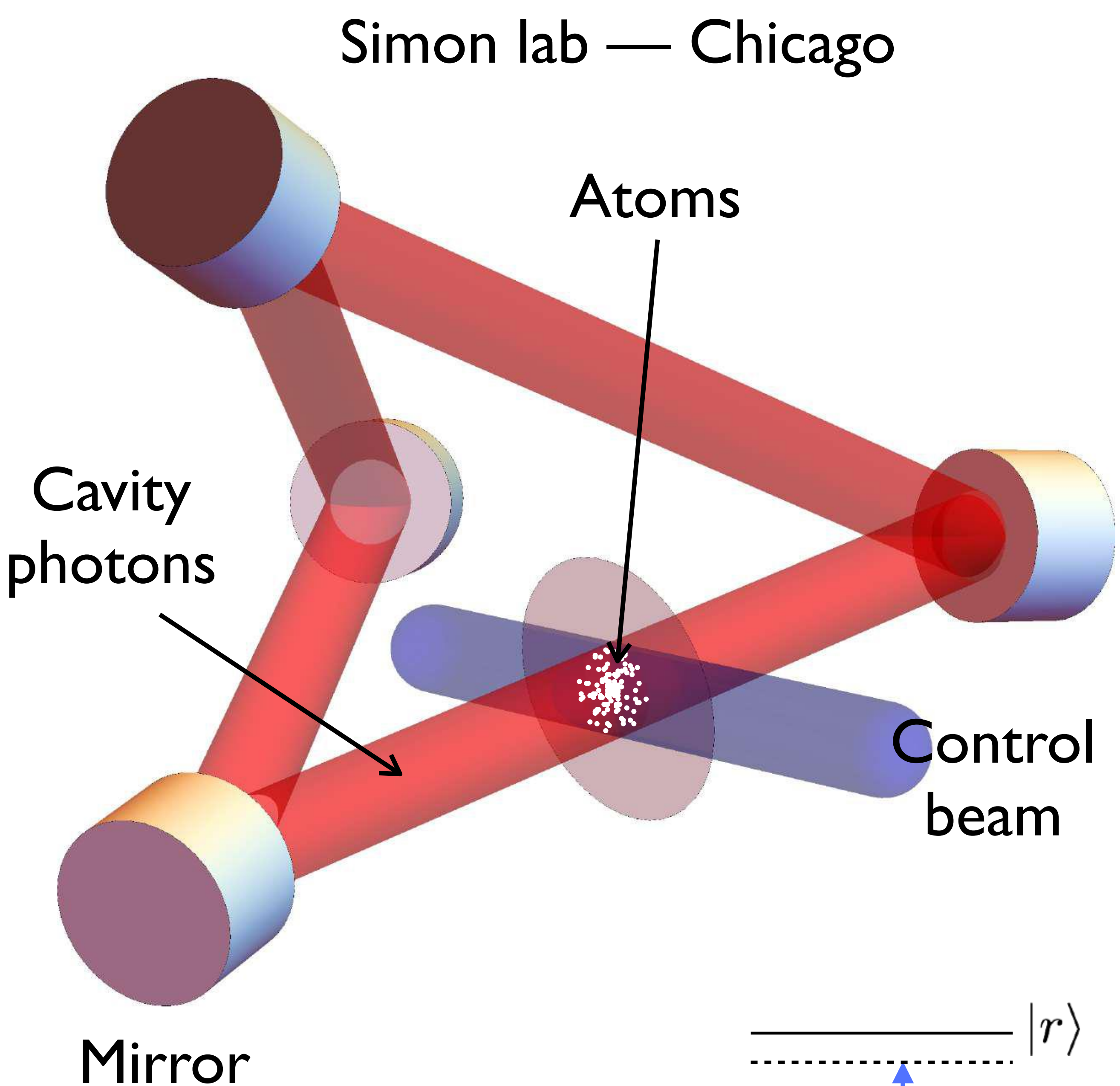

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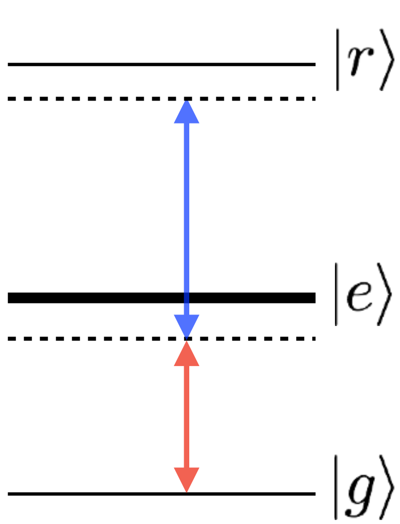

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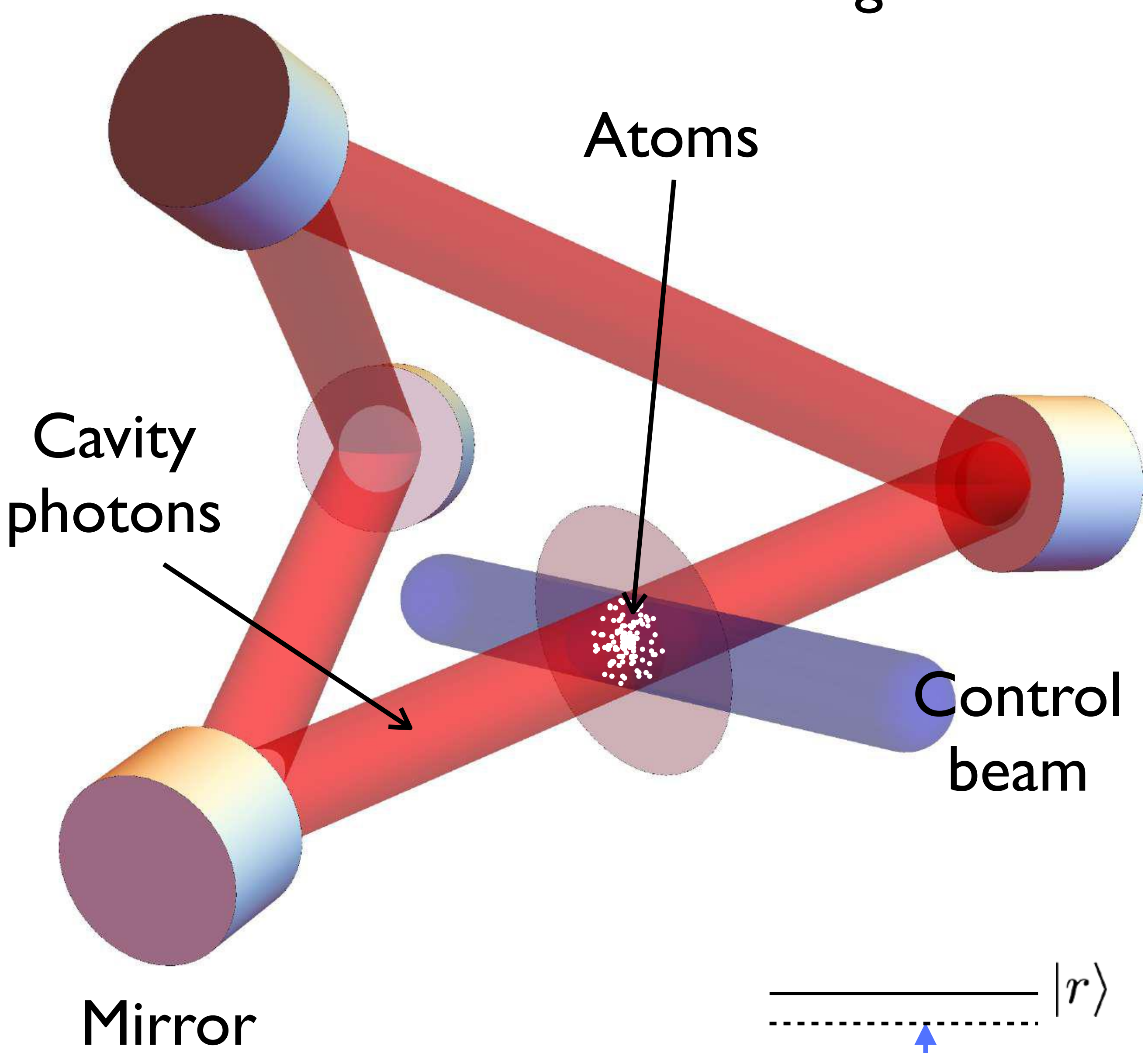

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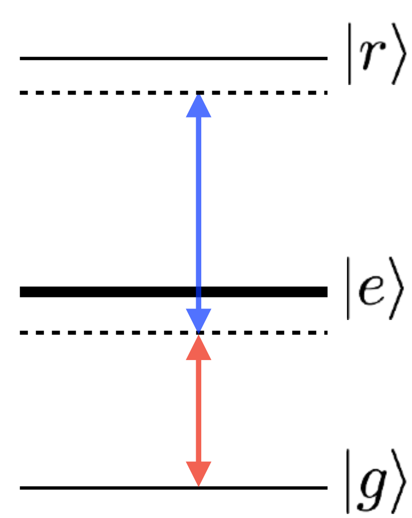

Near-degenerate cavity

- longitudinal mode number fixed

2D massive harmonically trapped interacting bosons in magnetic field

Atom-photon coupling

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- photon dynamics + Rydberg int.

Experimental setup

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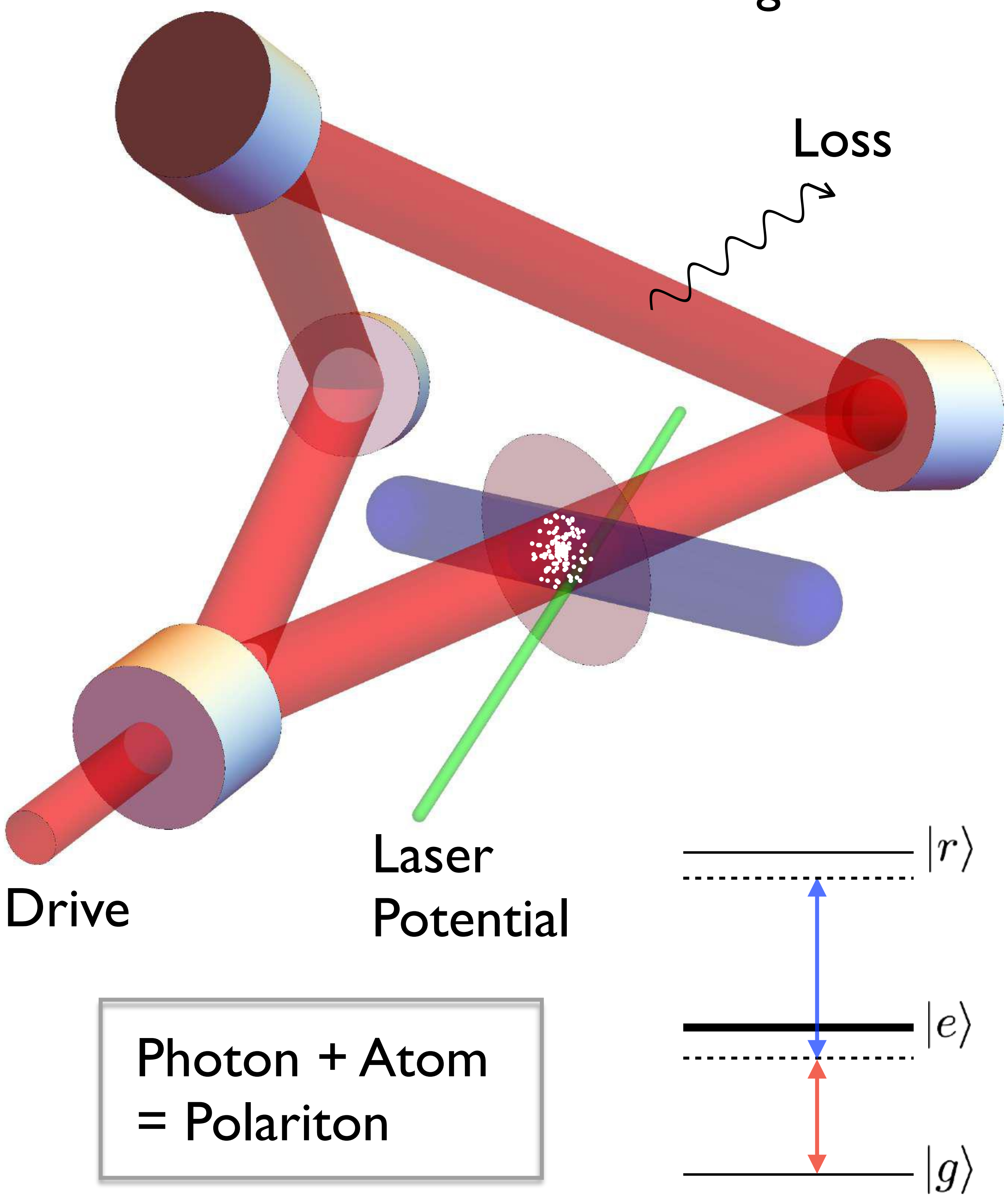

Photon + Atom
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2D massive
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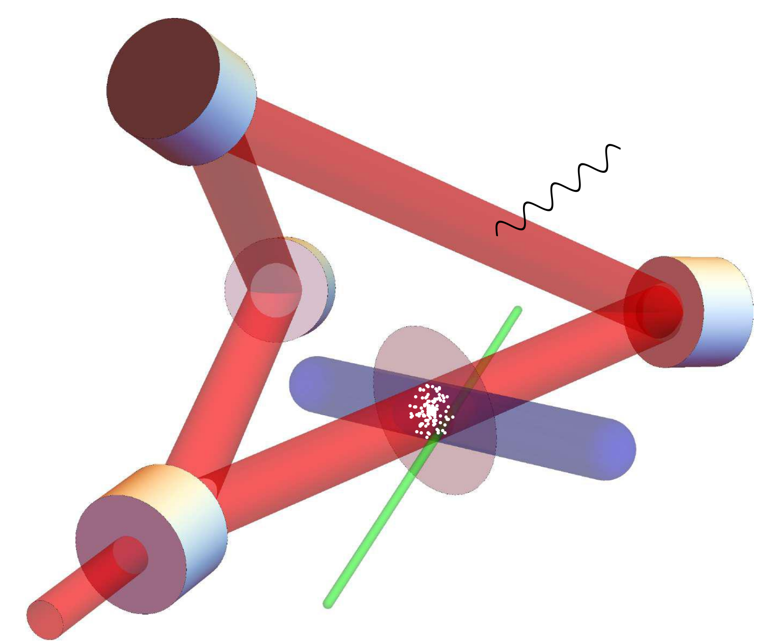
+

Drive, Potential, Loss

Model

$$\hat{H} = \int d^2r\, \hat{\psi}^\dagger \Bigg[\underbrace{\frac{(-i\vec{\nabla} - M\omega_B r\hat{\varphi})^2}{2M}}_{\text{kinetic}} + \underbrace{\frac{1}{2}M\omega_T^2 r^2}_{\text{trap}} \Bigg] \hat{\psi} + \underbrace{\pi l^2 V_0\, \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi}\hat{\psi}}_{\text{interaction}}$$

$$+ \underbrace{F(\vec{r},t)\,\hat{\psi}^\dagger + F^*(\vec{r},t)\,\hat{\psi}}_{\text{drive}} + \underbrace{U(\vec{r},t)\,\hat{\psi}^\dagger\hat{\psi}}_{\text{potential}}$$



M — polariton mass $(M \sim 10^{-4} m_e)$

$2\omega_B$ — cyclotron frequency $(\omega_B \sim 1\ \mathrm{GHz})$

ω_T — trap frequency $(\omega_T \sim 0 - 10\ \mathrm{MHz})$

l — magnetic length $(l \sim 20\ \mu\mathrm{m})$

V_0 — two-particle interaction energy $(V_0 \sim 5\ \mathrm{MHz})$

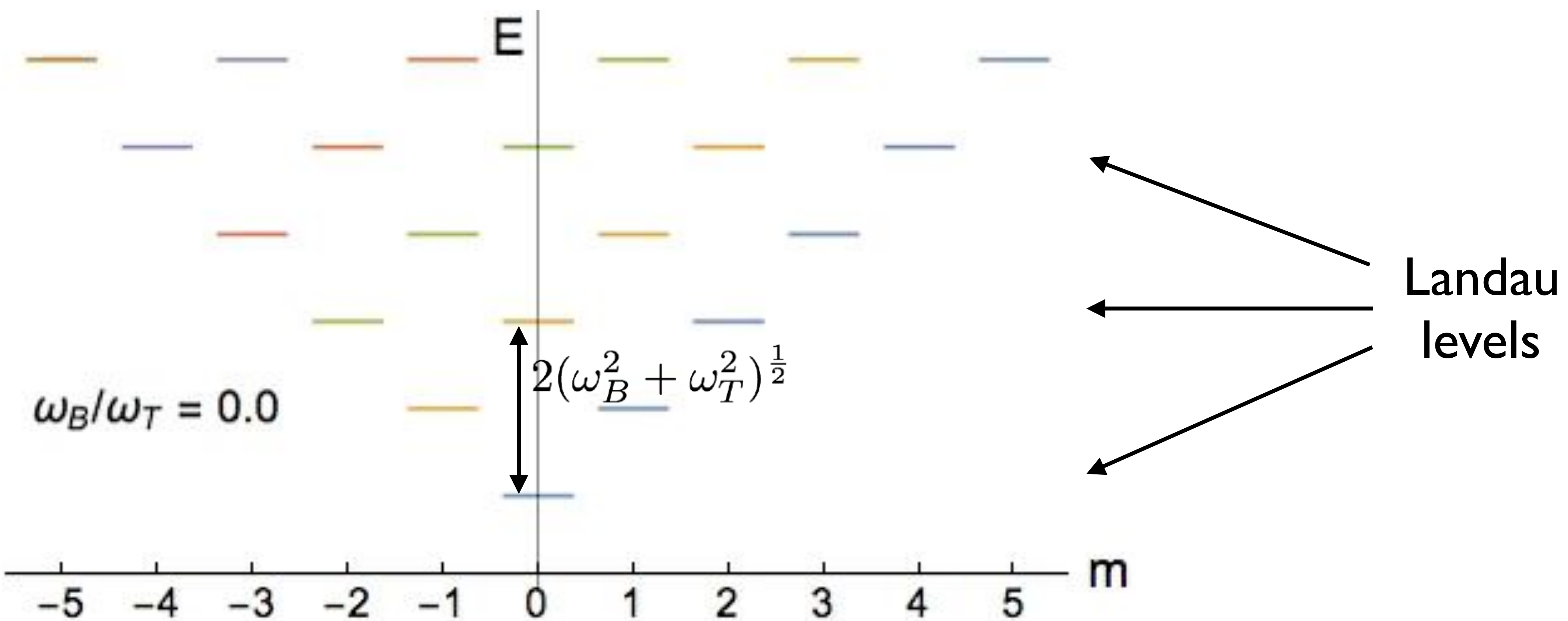
γ — inverse polariton lifetime $(\gamma \sim 0.1\ \mathrm{MHz})$

$\omega_T, V_0, \gamma \ll \omega_B \implies$ Lowest Landau Level physics

Single-particle spectrum

- Single-particle Hamiltonian:

$$\hat{H}_0 = \frac{(p_x + M\omega_B y)^2 + (p_y - M\omega_B x)^2}{2M} + \frac{1}{2}M\omega_T^2(x^2+y^2)$$
$$= \frac{p^2}{2M} + \frac{1}{2}M(\omega_B^2+\omega_T^2)r^2 - \omega_B L$$



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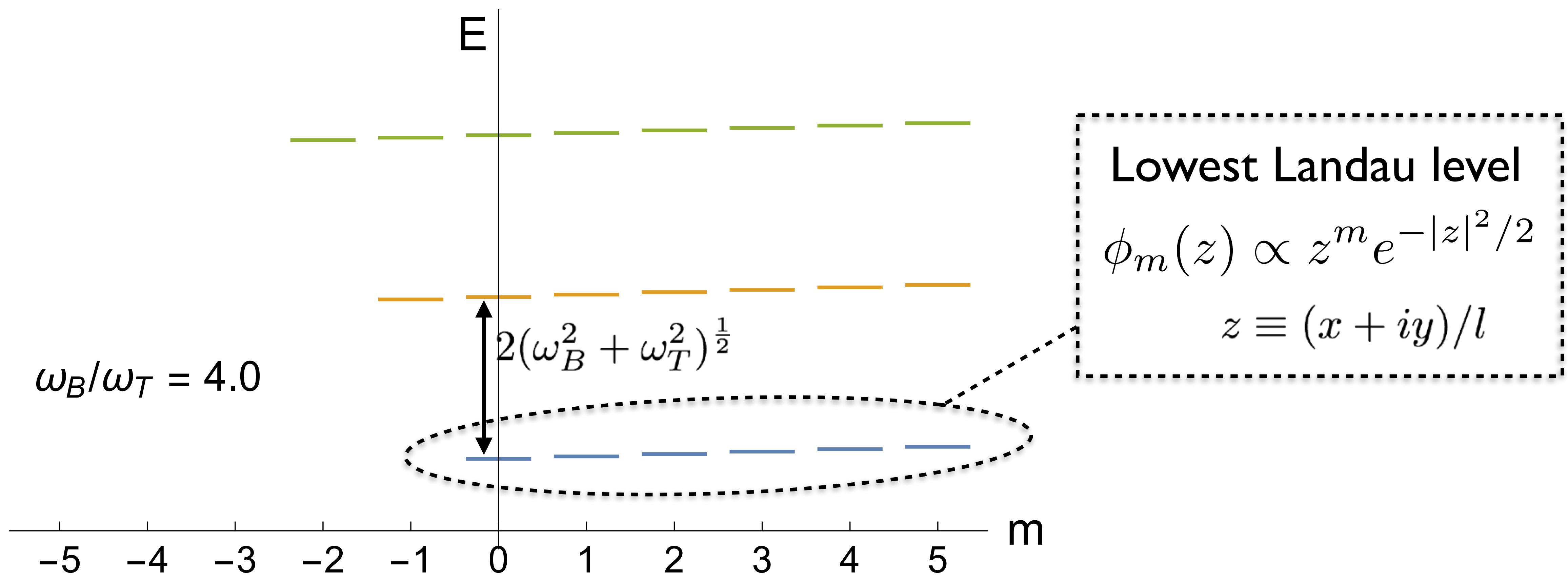
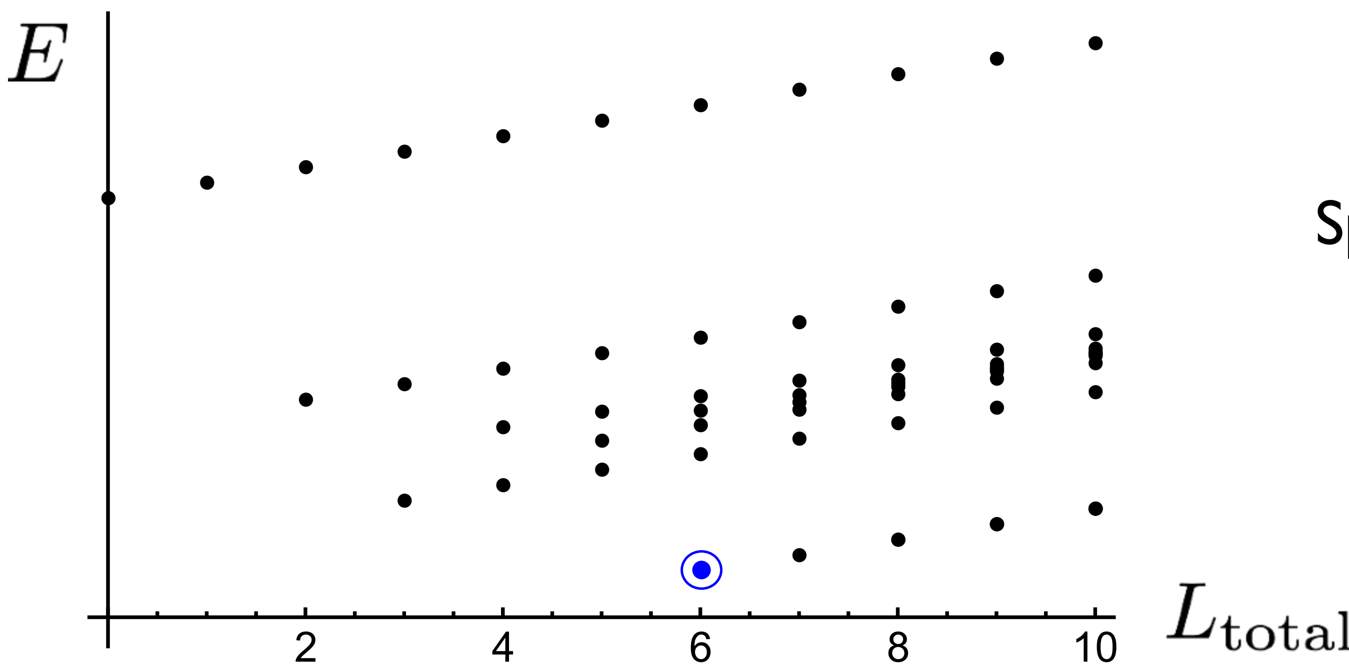


Laughlin states

- Many-body spectrum:



Spectrum for $N = 3$

- Many-body eigenstate: ν=1/2 Laughlin state

$$\Phi_N(z_1, z_2, \ldots, z_N) \propto \prod_{j<k} (z_j - z_k)^2 \, e^{-\sum_i |z_i|^2/2}$$

- has total angular momentum $L_N = N(N-1)$

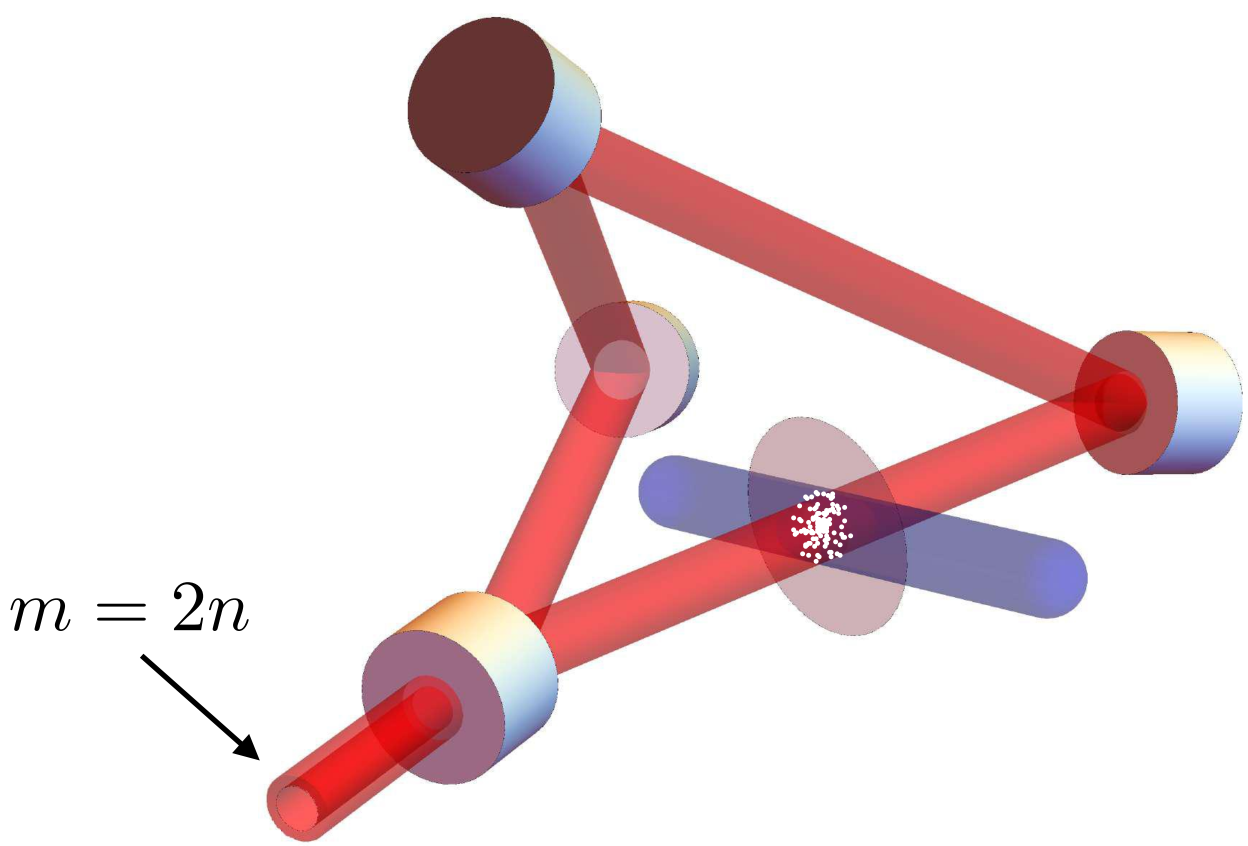
- Drive $|\Phi_N\rangle \to |\Phi_{N+1}\rangle$ by pumping at $m = L_{N+1} - L_N = 2N$

Creating Laughlin state

Idea: inject photons one-by-one such that $|\Phi_0\rangle \to |\Phi_1\rangle \to \ldots |\Phi_N\rangle$

Going from $|\Phi_n\rangle$ to $|\Phi_{n+1}\rangle$:

- Pump photons with angular momentum 2n : $\Delta L = L_{n+1} - L_n = 2n$ (Laguerre-Gauss laser beams: $r^{2n} e^{-r^2/2} e^{i2n\varphi}$)



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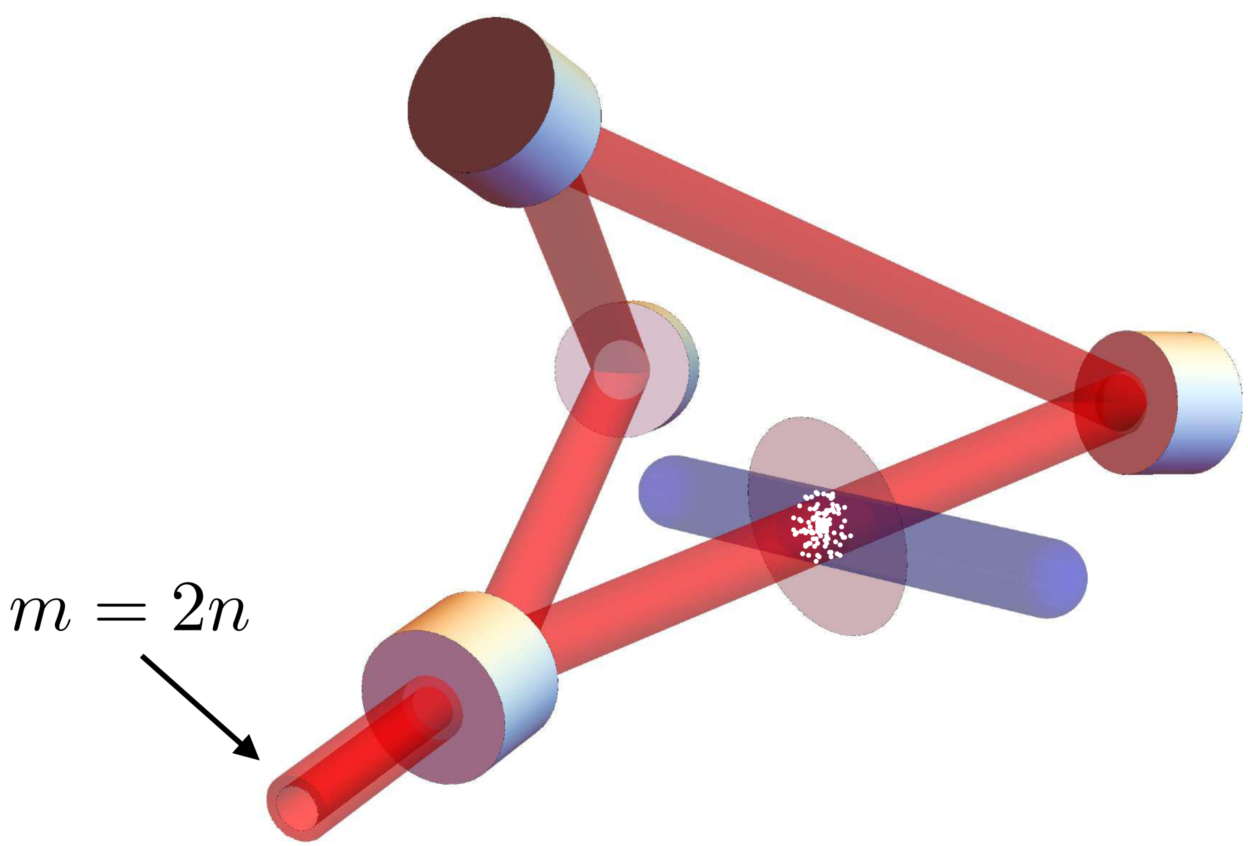
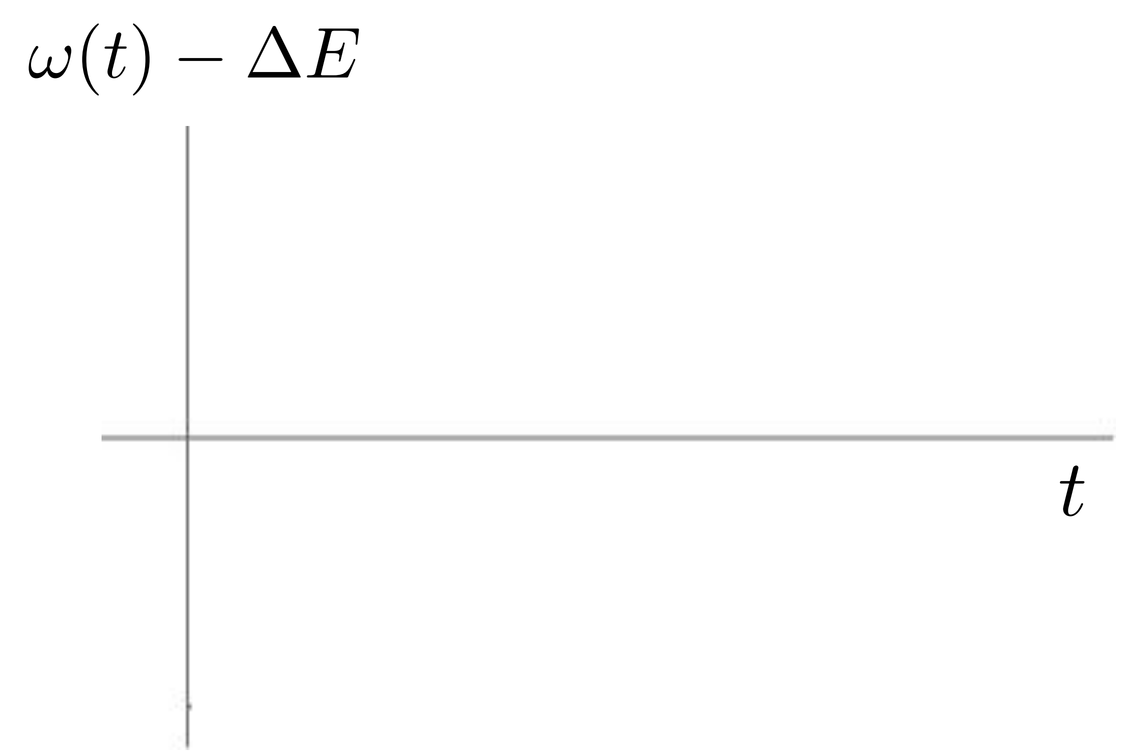
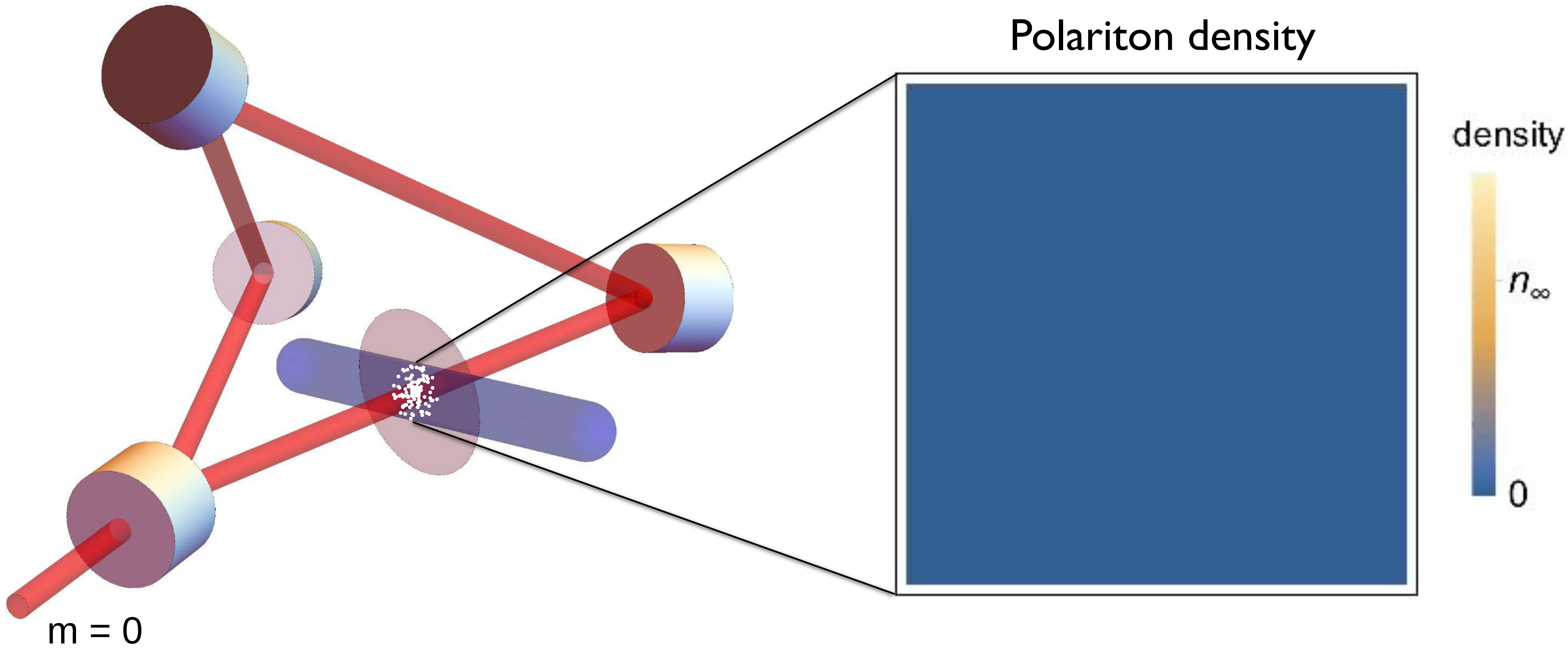
- Pump photons with angular momentum 2n : $\Delta L = L_{n+1} - L_n = 2n$
  (Laguerre-Gauss laser beams: $r^{2n} e^{-r^2/2} e^{i2n\varphi}$)
- Sweep frequency thru resonance : $\Delta E = E_{n+1} - E_n = \omega_B + n\omega_T^2/\omega_B$
  (Rapid Adiabatic Passage)





Creating Laughlin state



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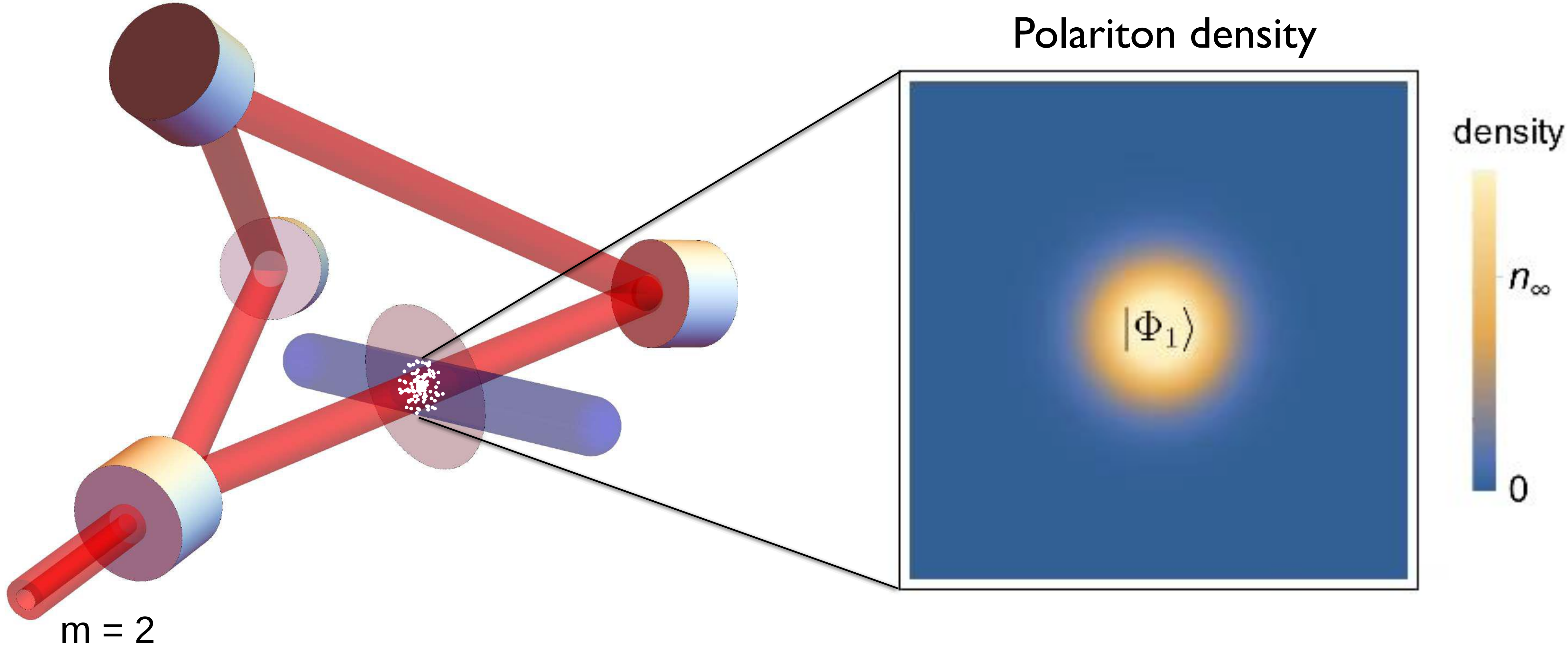

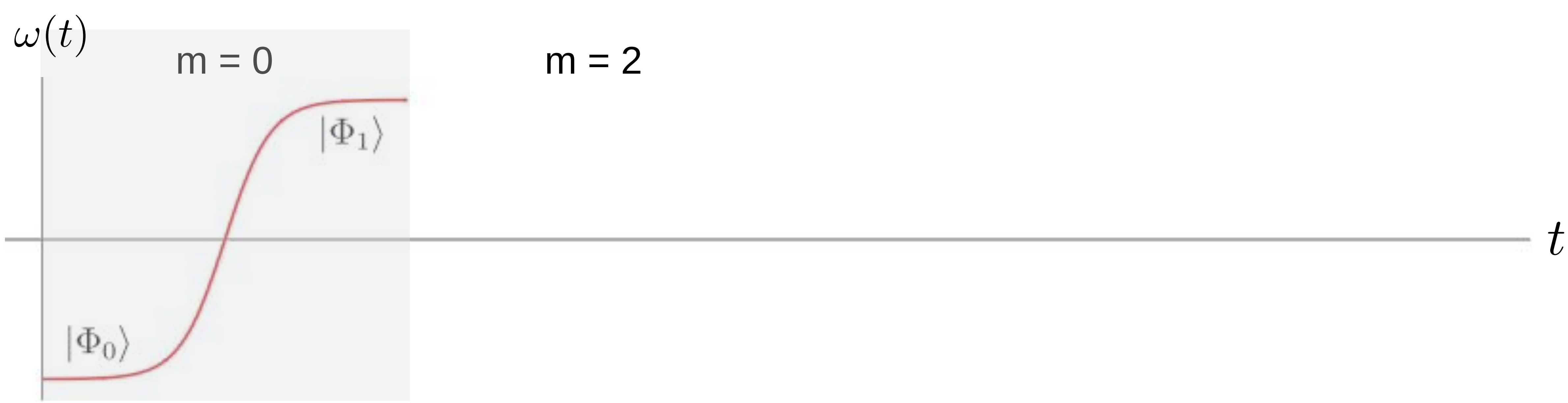

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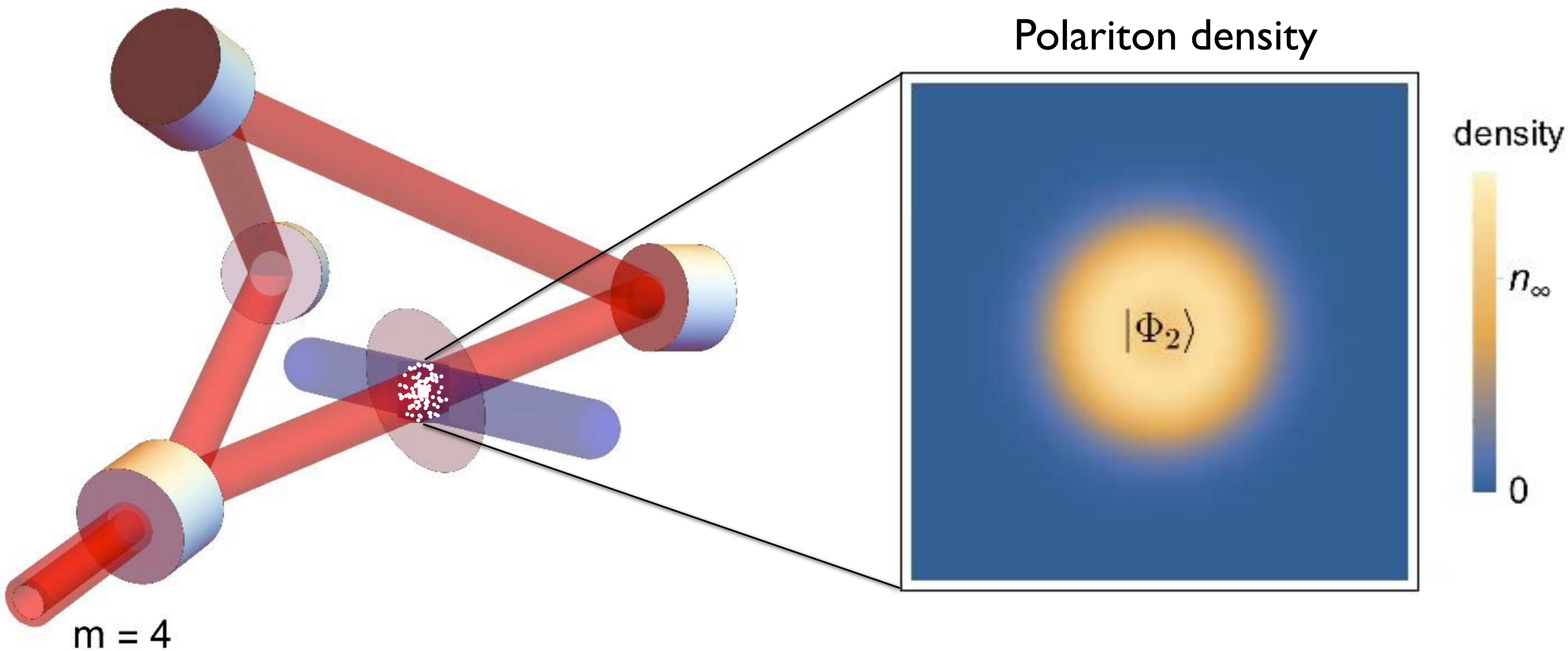
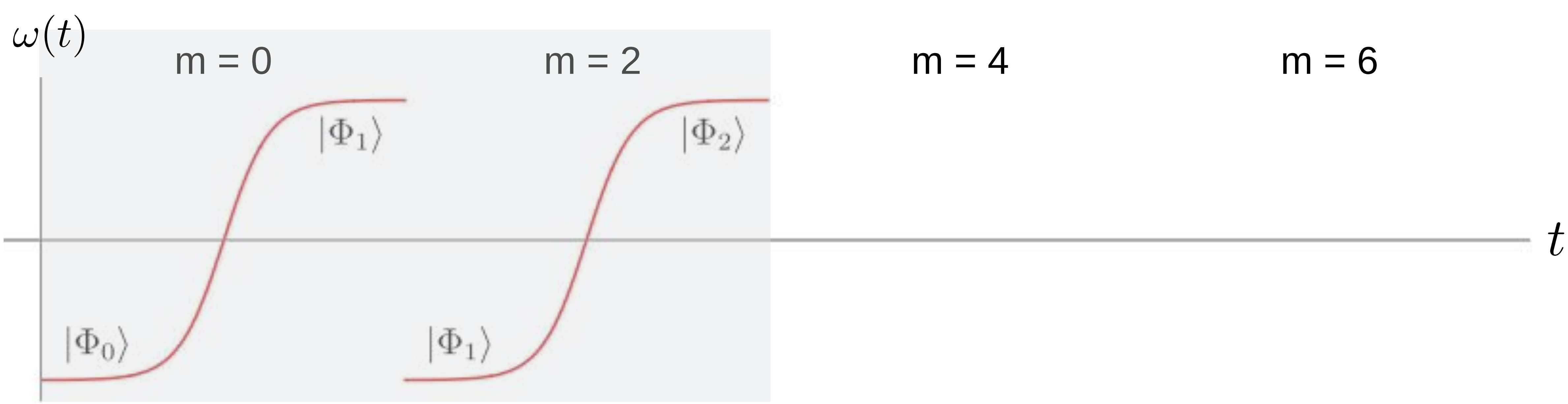


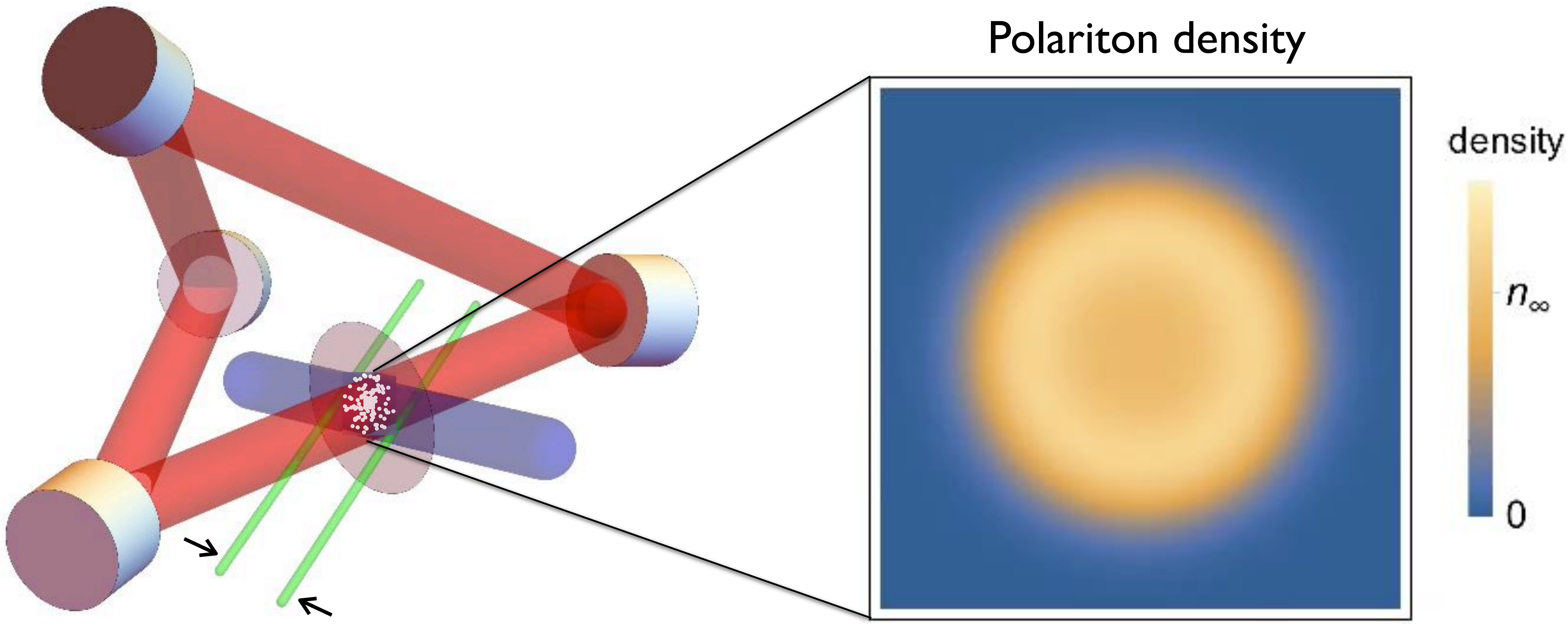


Creating Quasiholes

Idea: insert strong localized potentials adiabatically to bind holes

Potentials: $\hat{U}(t) = \pi l^2 U_0 \int d^2 r \, [\delta(\vec{r}-\vec{r}_0(t)) + \delta(\vec{r}+\vec{r}_0(t))] \, \hat{\psi}^\dagger(\vec{r}) \hat{\psi}(\vec{r})$

Quasiholes: $\Phi_N^{\circ\circ}(\{z_j\}) \propto \prod_{i=1}^N (z_i - z_0)(z_i + z_0) \, \Phi_N(\{z_j\}) \quad [z_0 \equiv r_0 e^{i\phi_0}/l]$



Braiding Quasiholes

Idea: move the potentials to drag quasiholes around each other

Potentials: $\hat{U}(t) = \pi l^2 U_0 \int d^2r \, [\delta(\vec{r} - \vec{r}_0(t)) + \delta(\vec{r} + \vec{r}_0(t))] \, \hat{\psi}^\dagger(\vec{r}) \hat{\psi}(\vec{r})$

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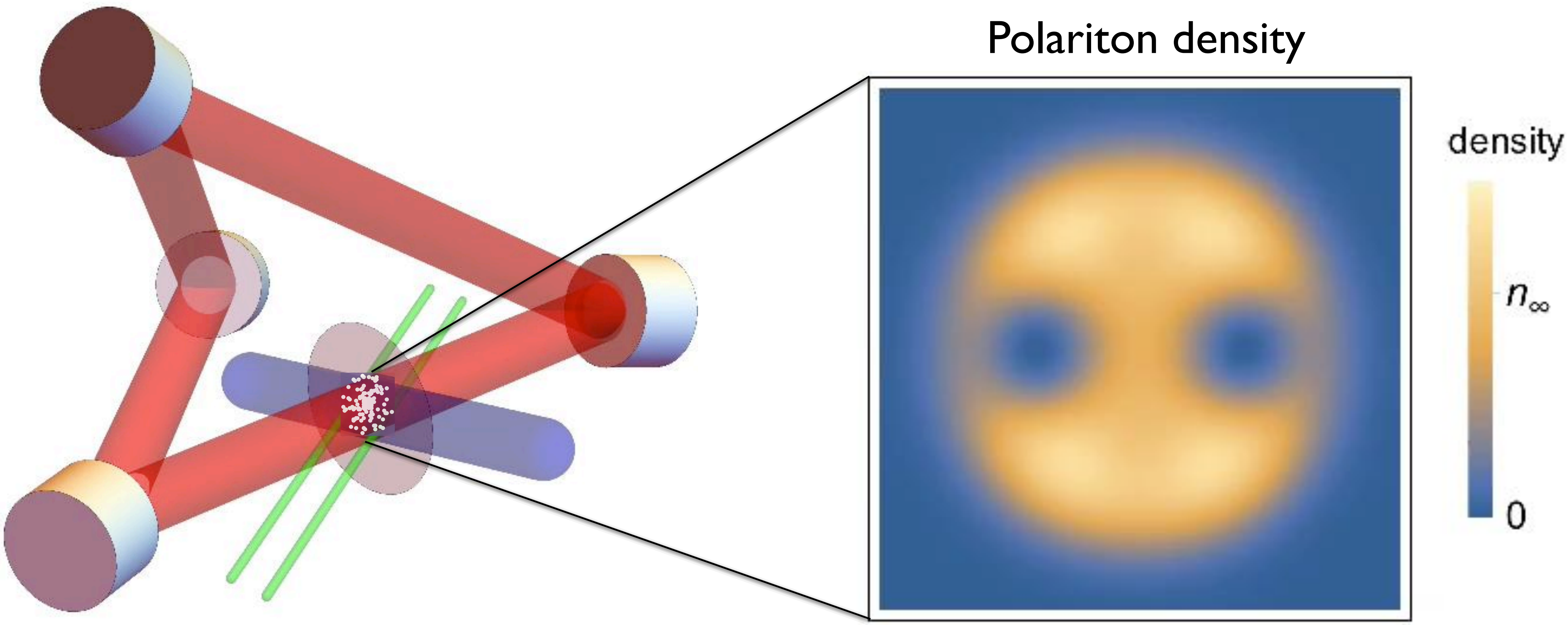
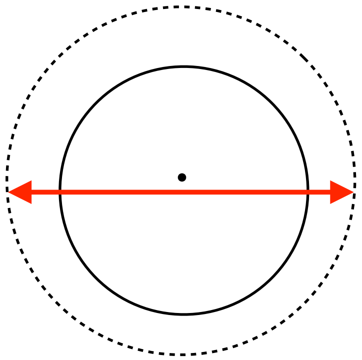


Measuring braiding phase

Idea: compare with a reference $|R\rangle$ which is unaffected by drives

(e.g., a Rydberg excitation with large interaction range)



Steps: 1. Use $\pi/2$-pulse to prepare $|0\rangle + |R\rangle$

$|0\rangle$: zero-polariton state, $|R\rangle$: one atom Rydberg excited

2. Create Laughlin, create holes, braid holes, then repeat backwards

$|0\rangle + |R\rangle \rightarrow e^{i\phi}|0\rangle + |R\rangle$

3. Apply $\pi/2$-pulse to recombine $|0\rangle$ and $|R\rangle$

4. Read out ϕ by measuring ground-state occupation

5. Repeat under different experimental conditions to extract statistical phase ϕ_s from ϕ

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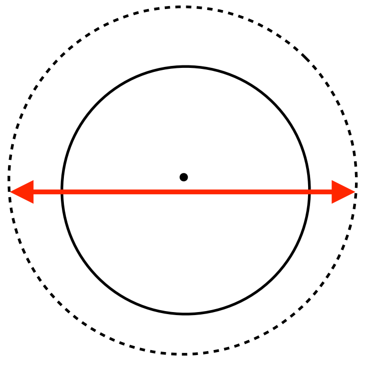

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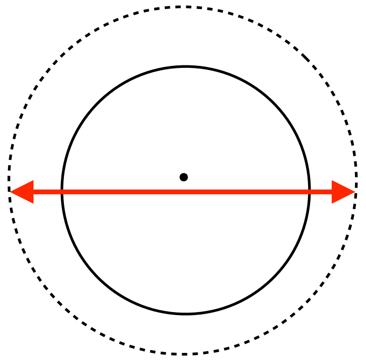

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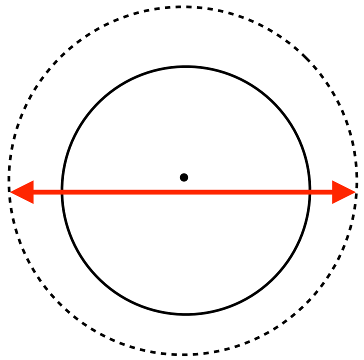

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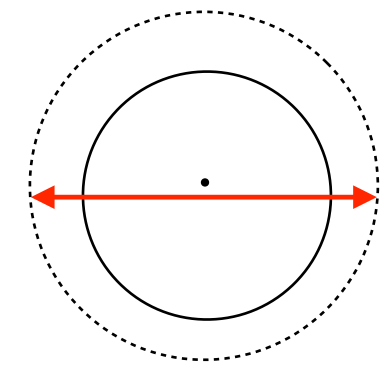

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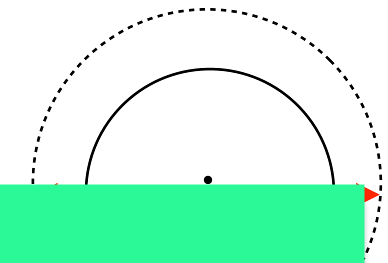


$$\phi = \underbrace{\phi_d}_{\text{dynamical}} + \underbrace{\phi_g}_{\text{geometric}} \qquad \phi_g = \underbrace{\phi_{\mathrm{AB}}}_{\text{Aharonov-Bohm}} + \underbrace{\phi_s}_{\text{statistical}}$$

Repeat experiment at different rates to separate ϕ_g from ϕ_d

5. Repeat under different experimental conditions to extract statistical phase ϕ_s from ϕ

Measuring statistical phase

Idea: compare geometric phases from two different experiments

Experiment I

Rotate a single quasihole by 2π

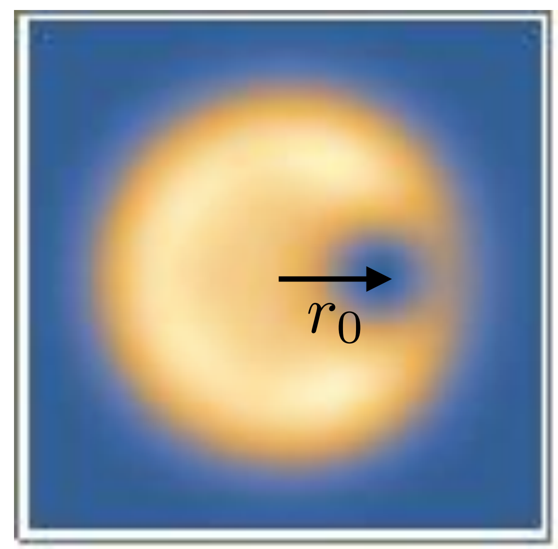
$\phi_g^{\mathrm{I}} = \phi_1 \longrightarrow$ Aharonov-Bohm

Experiment II

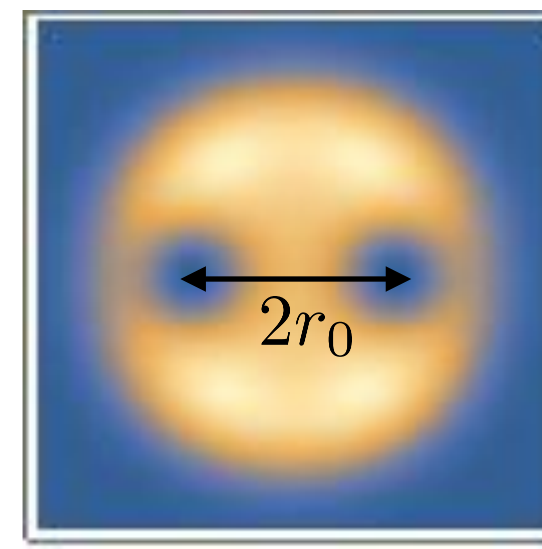
Rotate two quasiholes by π

$\phi_g^{\mathrm{II}} = \phi_1 + \phi_s \longrightarrow$ exchange

Experiment I



Experiment II



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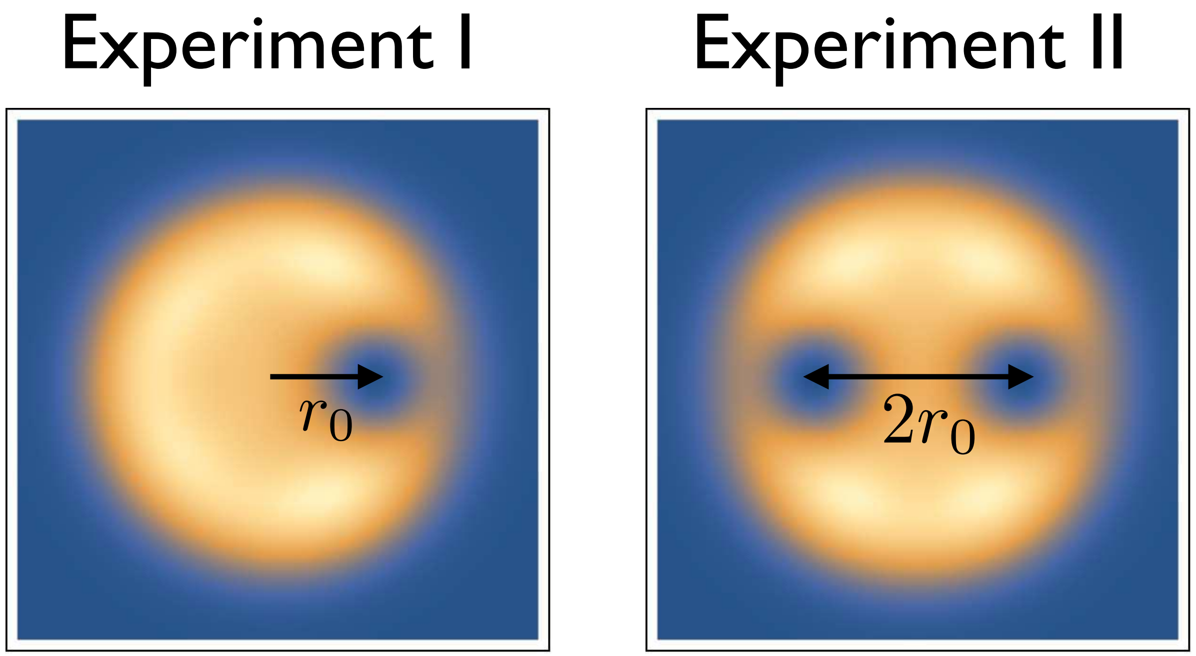
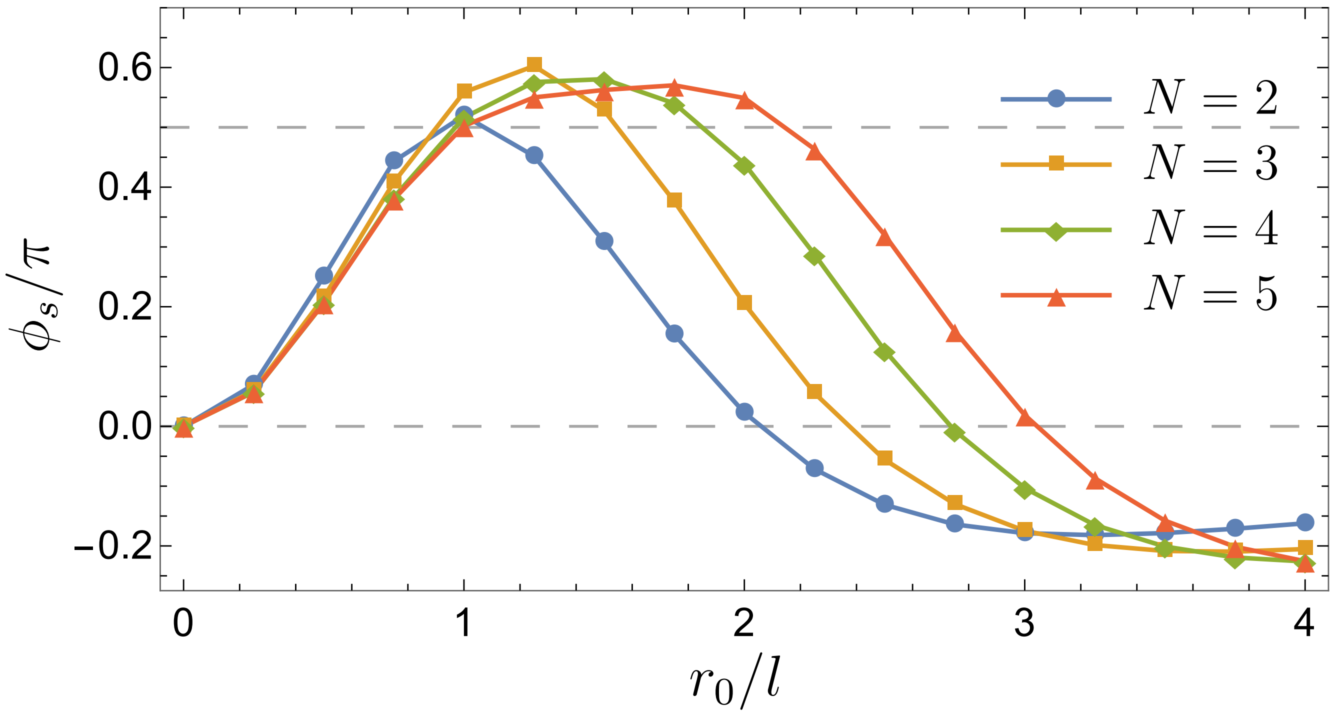
$$\Longrightarrow \quad \phi_s = \phi_g^{\mathrm{II}} - \phi_g^{\mathrm{I}}$$

$\big\downarrow$ thermodynamic limit

$\pi/2$





Measuring statistical phase

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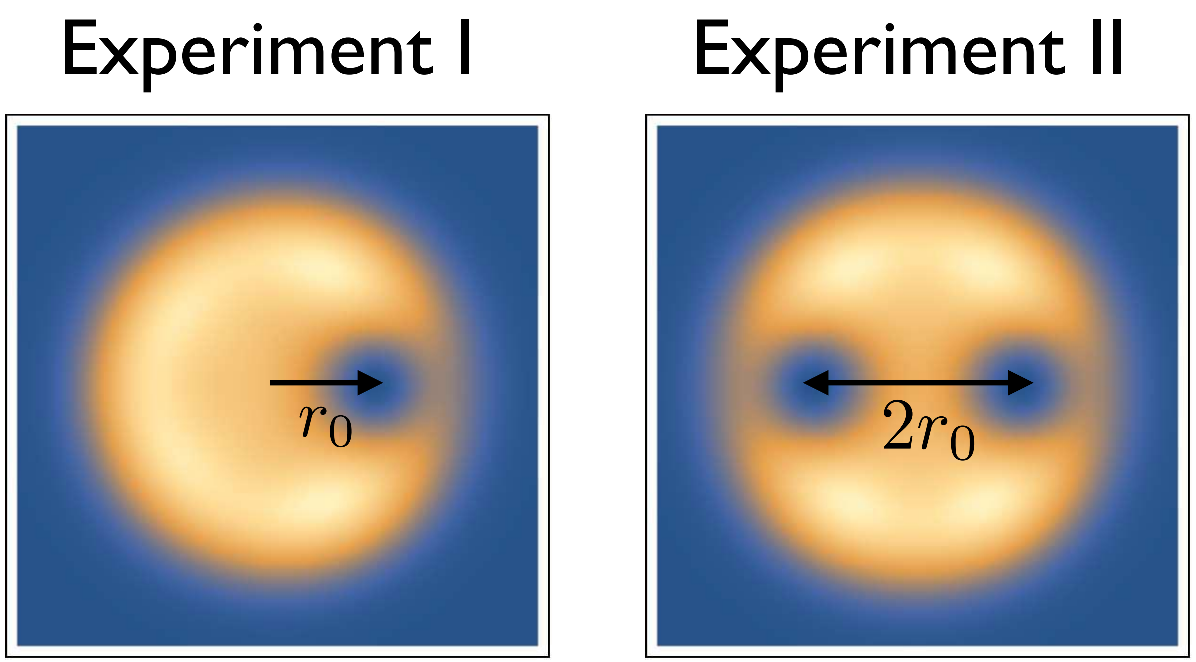
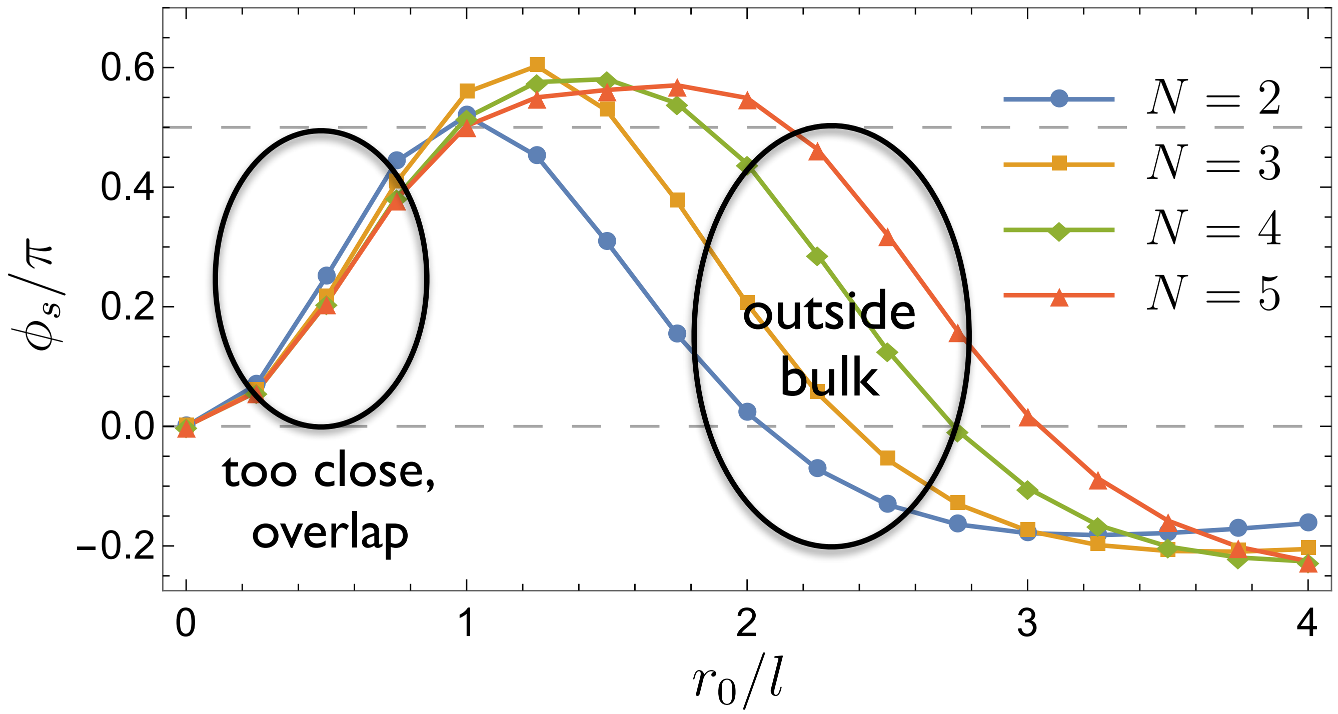
thermodynamic limit

$\pi/2$





Adiabaticity and Coherence

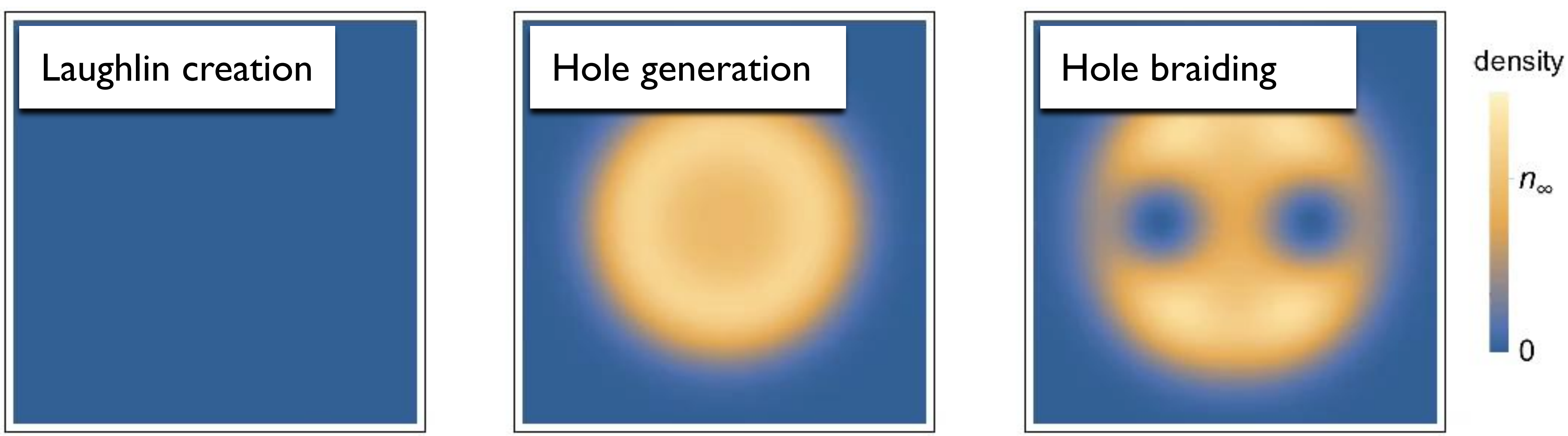
1. Sweeps must be slow enough to prevent unwanted excitations
2. Sweeps must be fast enough to prevent polariton loss

- Adiabaticity: rates limited by excitation gap ~ V_0 ($t \gg 1/V_0$) (Laughlin creation most time consuming)
- Polariton loss set by decay rate γ ($t \ll 1/\gamma$)

requires V_0 much bigger than γ

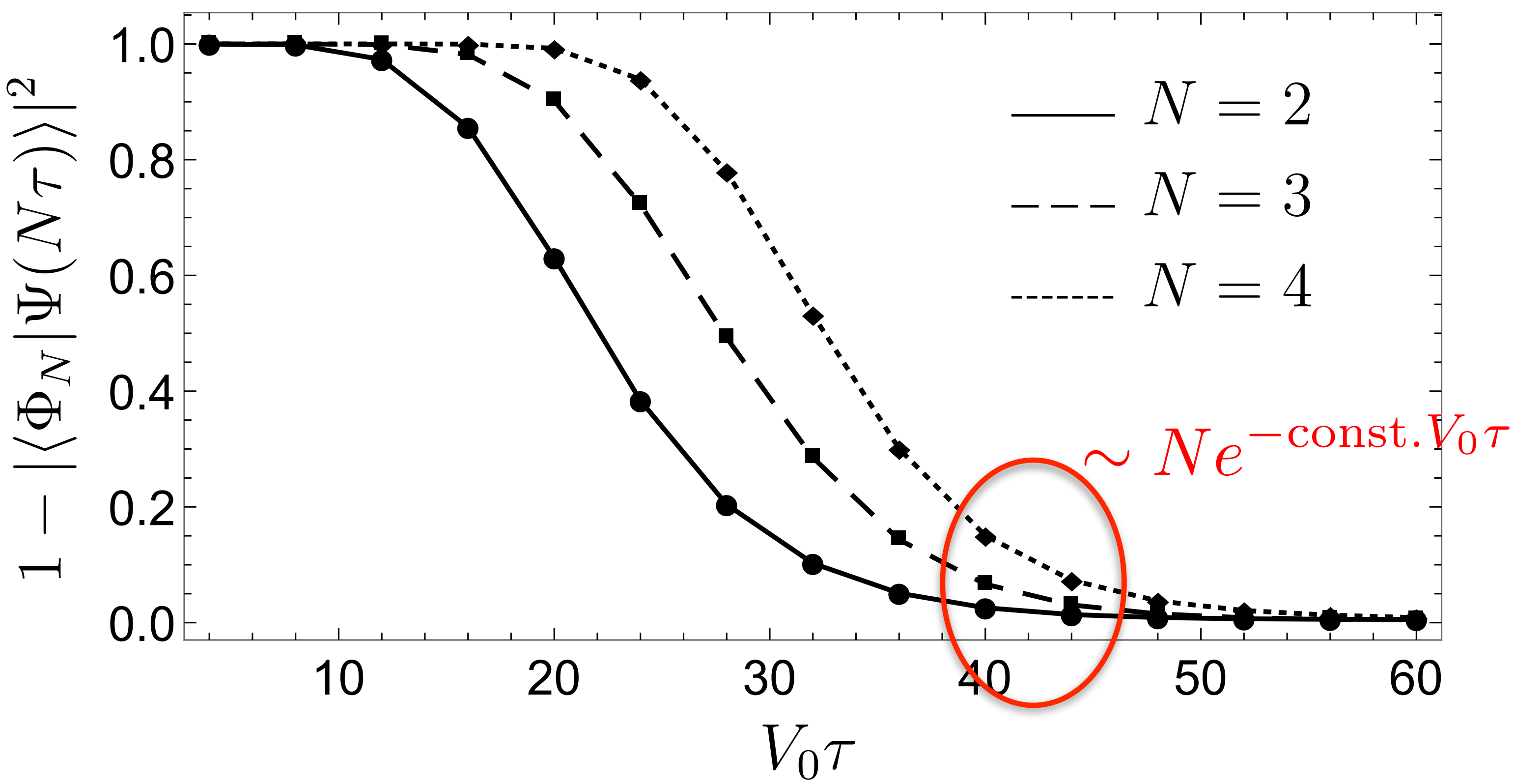
(more precise estimates on next slide)

Examples of non-adiabatic sweeps (no loss) :



Adiabaticity and Coherence

Cumulative error during Laughlin creation:



τ — sweep duration

Error < 1% for

$\tau \gtrsim (40/V_0)\ln N$

Net polariton loss: $N_{\text{loss}} \approx \sum_{n=0}^{N-1}(n\gamma)(\tau) \approx N^2\gamma\tau/2$

$$N_{\text{loss}} \ll 1 \implies V_0/\gamma \gg 10N^2 \ln N$$

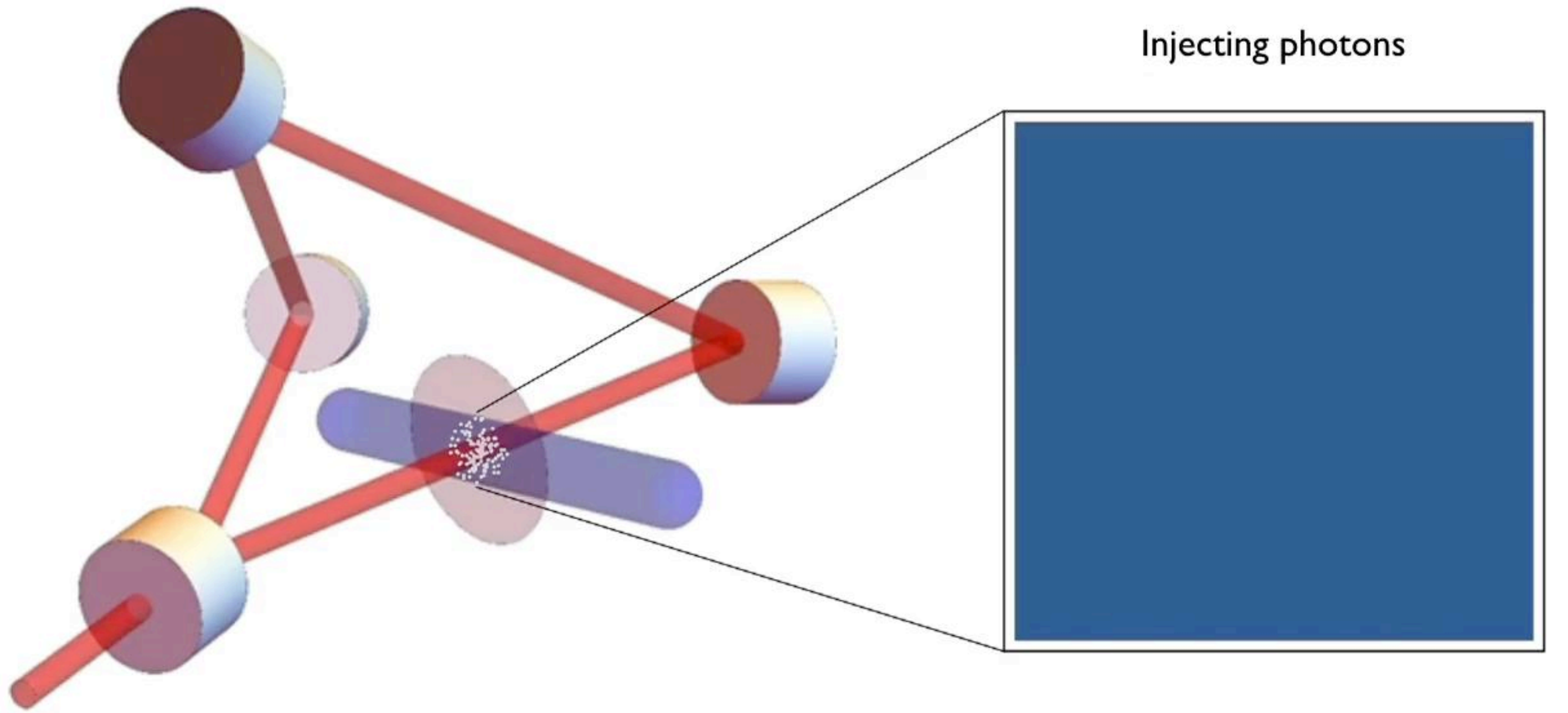
Big improvement over existing protocols (less demanding, high fidelity)

Still very hard to achieve (current experiments have $V_0/\gamma \sim 50$)

Summary

S.D. and Erich Mueller
PRA 97, 033825 (2018)

- Use rapid adiabatic passage to create Laughlin state
- Move pinning potentials to create and braid quasiholes
- Interferometrically measure fractional exchange statistics



- Adiabaticity and coherence require $V_0/\gamma \gg 10N^2 \ln N$
- Realize few-particle Laughlin states and perform externally controlled anyon braiding

Thank you


Nur
Bhuvanesh
Matthew
Yi-Ting

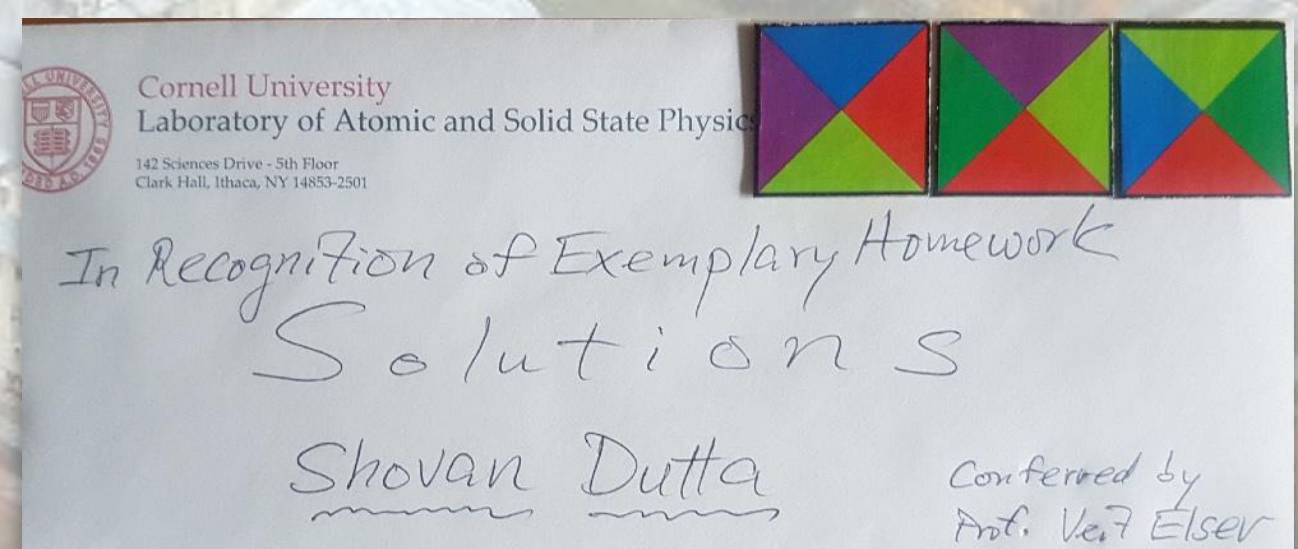

Ran


Junjun


Sayan


Yariv


Daniel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Prof. Veit Elser