Hysteretic quantum phase transitions in a strongly-correlated shaken optical lattice

Shovan Dutta

University of Cambridge
(now at MPIPKS)

B. Song, SD, S. Bhave, J.-C. Yu, E. Carter, N. Cooper, U. Schneider — arXiv:2105.12146
(to appear in Nature Physics)

EPSRC Engineering and Physical Sciences Research Council

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SIMONS FOUNDATION

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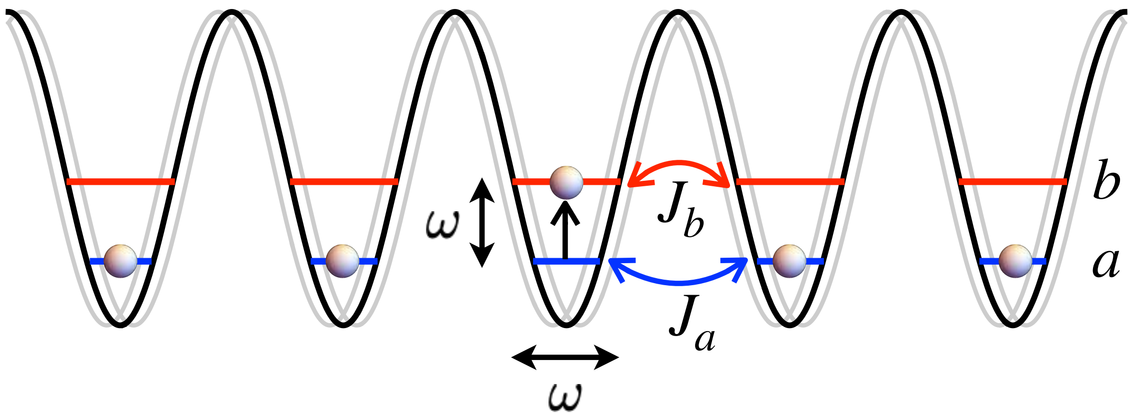
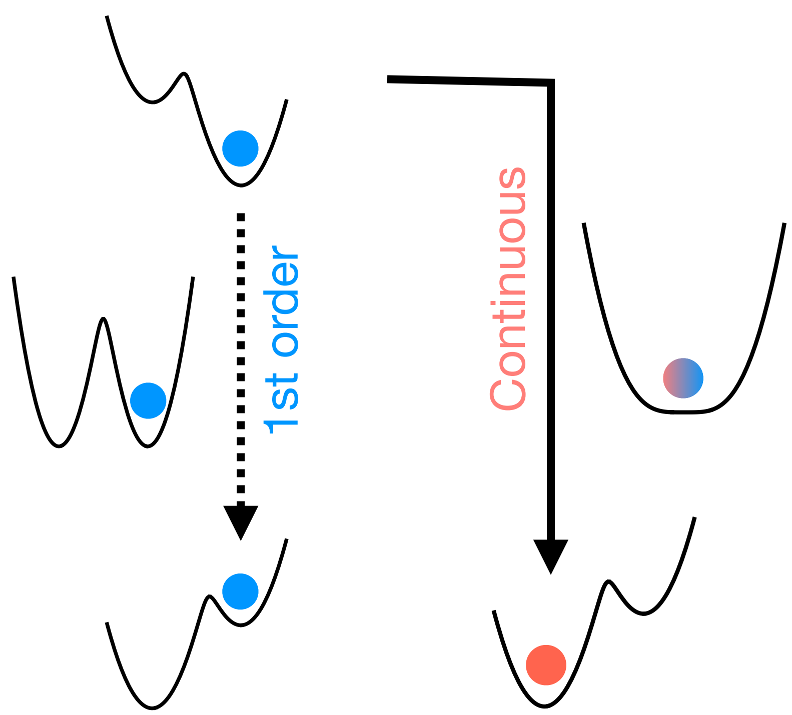
Key takeaway



$J_b \gg J_a \implies$ coupling increases J_{eff}, melting Mott insulator into superfluid

Weak shaking: first-order phase transition — stuck in Mott insulator

Strong shaking: continuous phase transition — evolution toward superfluid



Tuneable metastability and hysteresis in a driven strongly-correlated quantum system

Motivations

Dynamics of 1st-order quantum phase transitions

- Studies have focused on continuous quantum transitions & Kibble-Zurek scaling, now well established

M. Greiner *et al*, Nature '02
J. Simon *et al*, Nature '11
A. Keesling *et al*, Nature '19

- Quantum decay of metastable states ("false vacua") important in cosmology

S. Coleman, PRD '77

- Experiments with 1st-order transitions limited to weakly interacting spinor condensates, where decay is exponentially suppressed

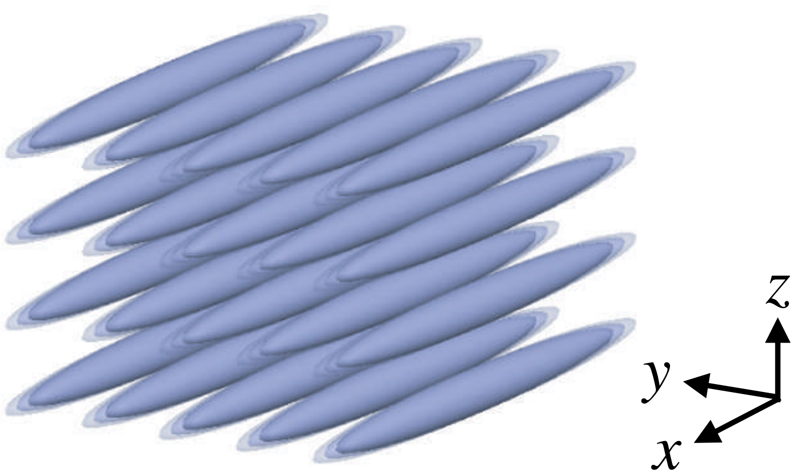
D. Campbell *et al*, Nat Commun '16
A. Trenkwalder *et al*, Nat Phys '16
L.-Y. Qiu *et al*, Sci Adv '20

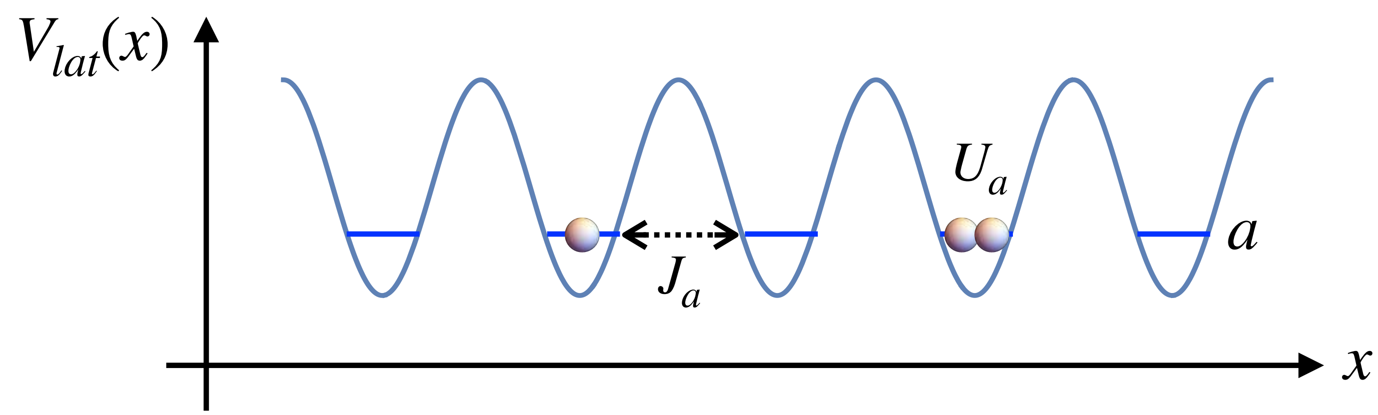
Example of Floquet engineering — can be extended to higher dimensions

A. Eckardt, RMP '17

Undriven system

Array of quasi-1D tubes — transverse motion frozen





Effective quasi-1D Hamiltonian $\hat{H}_0 = \int dx \; \underbrace{\hat{\psi}^\dagger(x) \, [-\partial_x^2 + V_{lat}(x)] \, \hat{\psi}(x)}_{\text{single-particle}} + \underbrace{g \; \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\psi}(x)}_{\text{interaction}}$

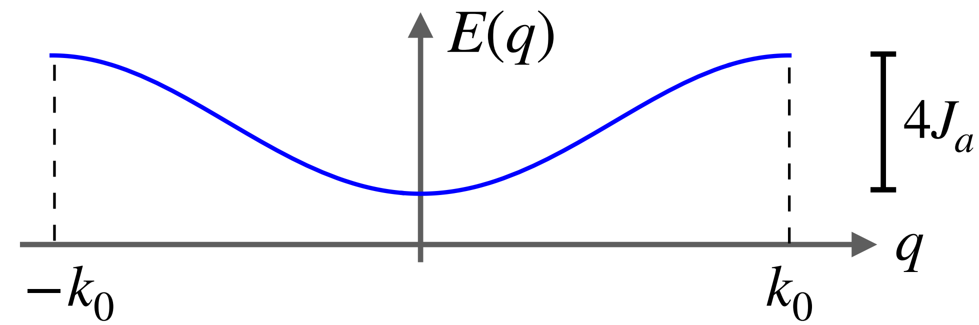
Low-energy physics confined to lowest band $\hat{\psi}(x) = \sum_j \underbrace{w_a(x - x_j)}_{\text{Wannier function}} \, \hat{a}_j$

$$\Longrightarrow \; \hat{H}_0 = -\underset{\substack{\uparrow \\ \text{tunnelling}}}{J_a} \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} \underset{\substack{\uparrow \\ \text{interaction}}}{U_a} \sum_j \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j$$ Bose-Hubbard model

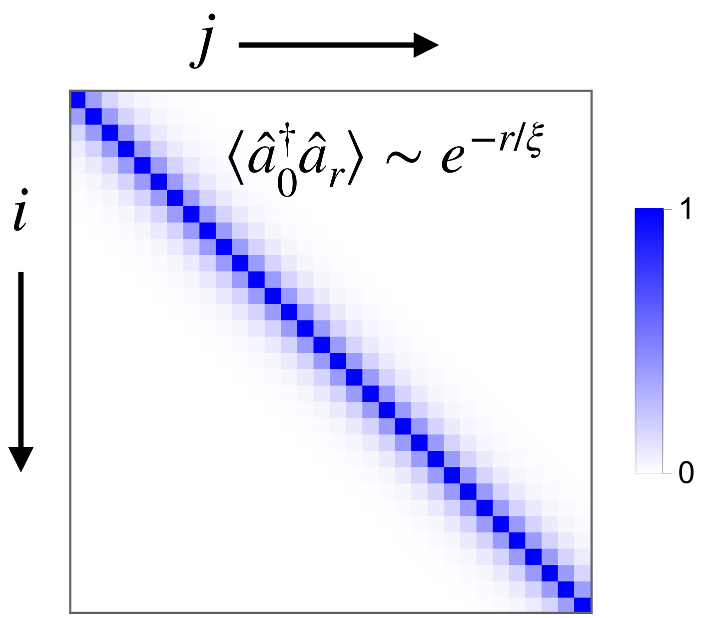
Single-band ground states

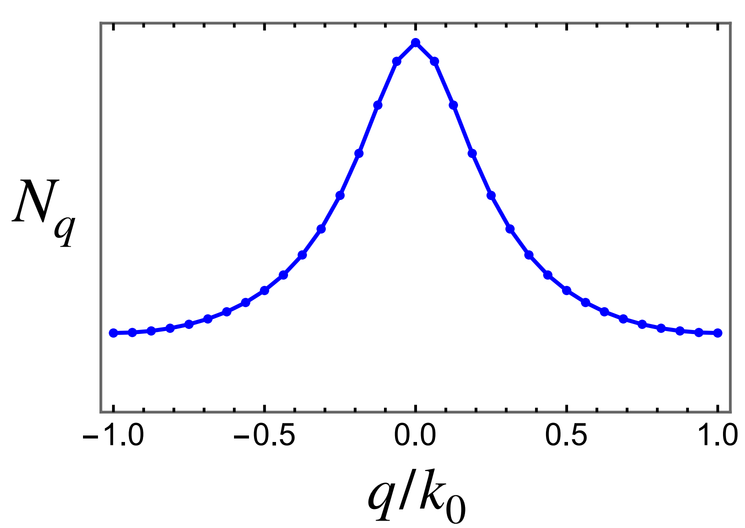
Bose-Hubbard model

$$\hat{H}_0 = -J_a \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U_a \sum_j \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j$$



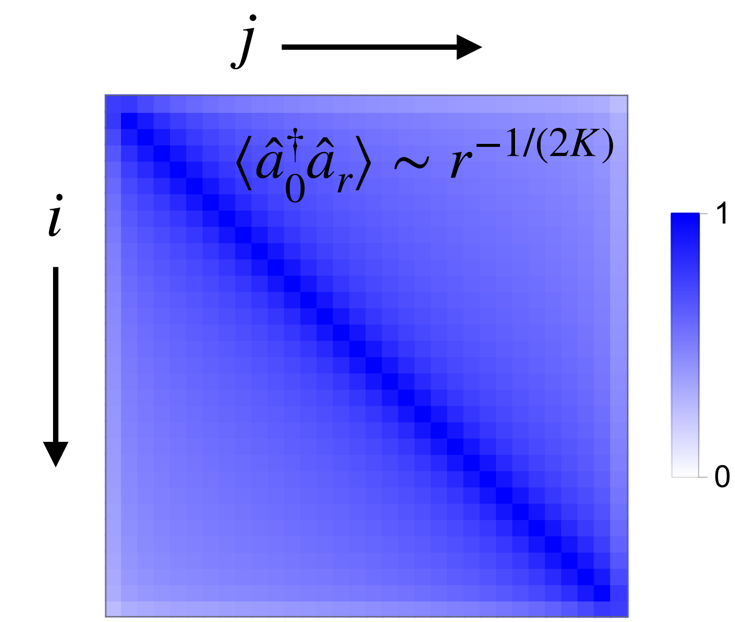
Small tunnelling: Mott insulator

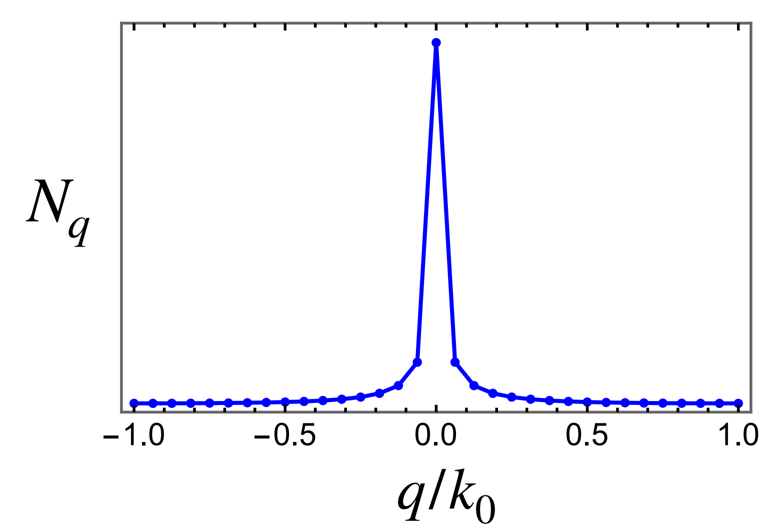




Continuous phase transition at $J_a/U_a \approx 0.3$

Large tunnelling: superfluid

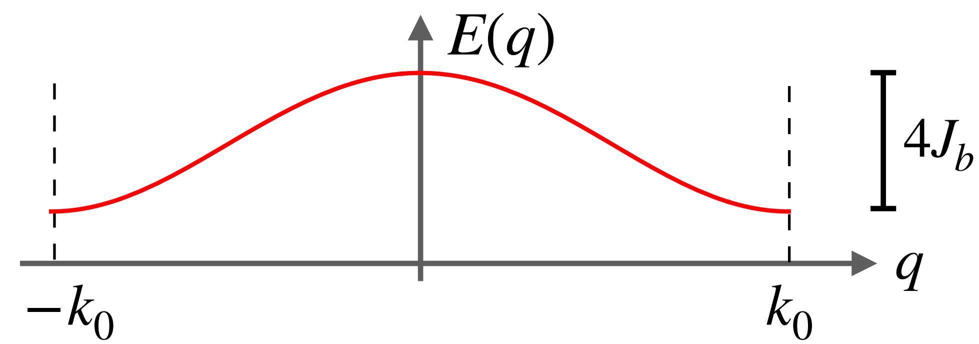




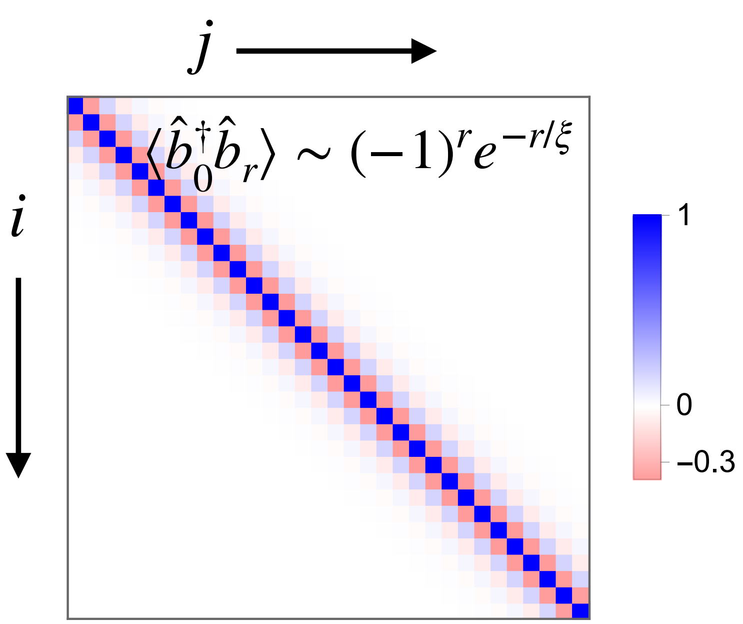
Single-band ground states: staggered phases

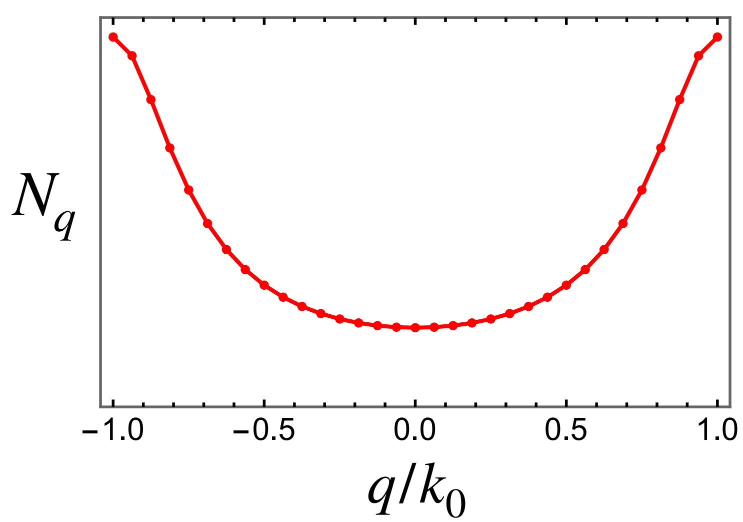
Bose-Hubbard model

$$\hat{H}_0 = +J_b \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \tfrac{1}{2} U_b \sum_j \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j$$



Small tunnelling: staggered Mott

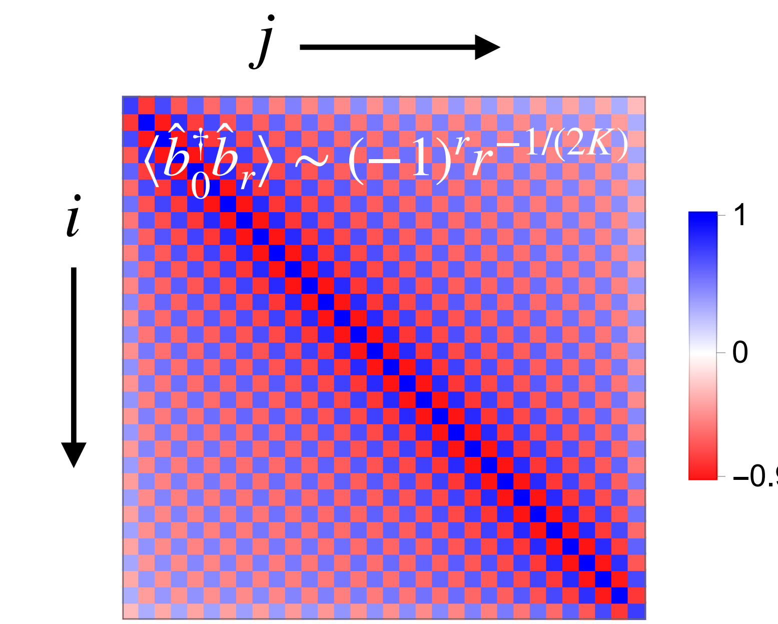


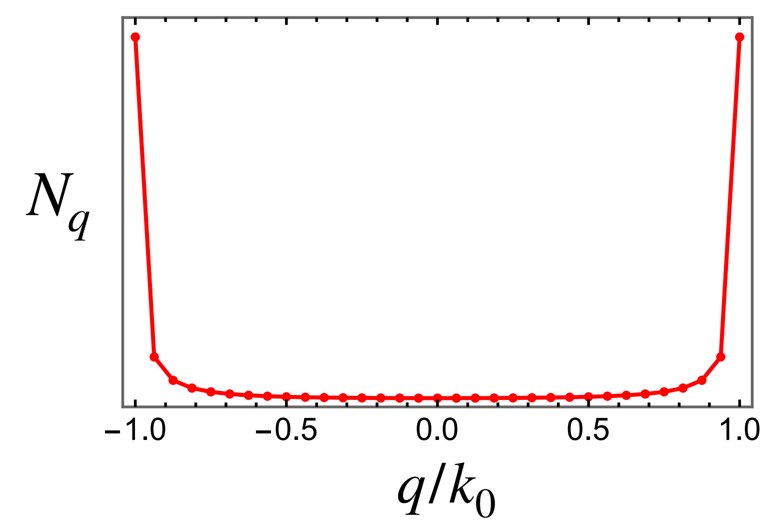


Continuous phase transition

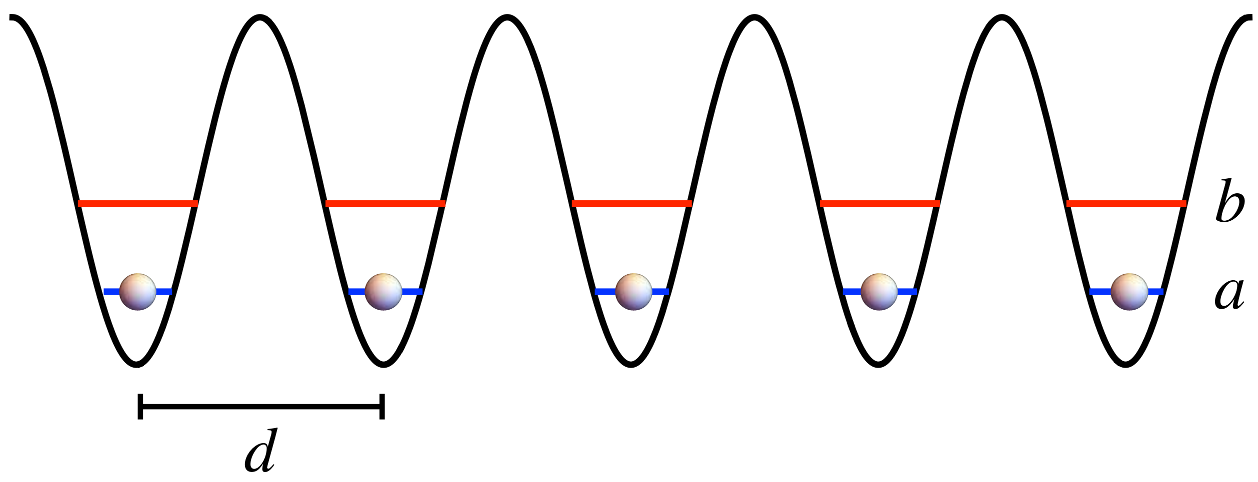
at $J_b/U_b \approx 0.3$

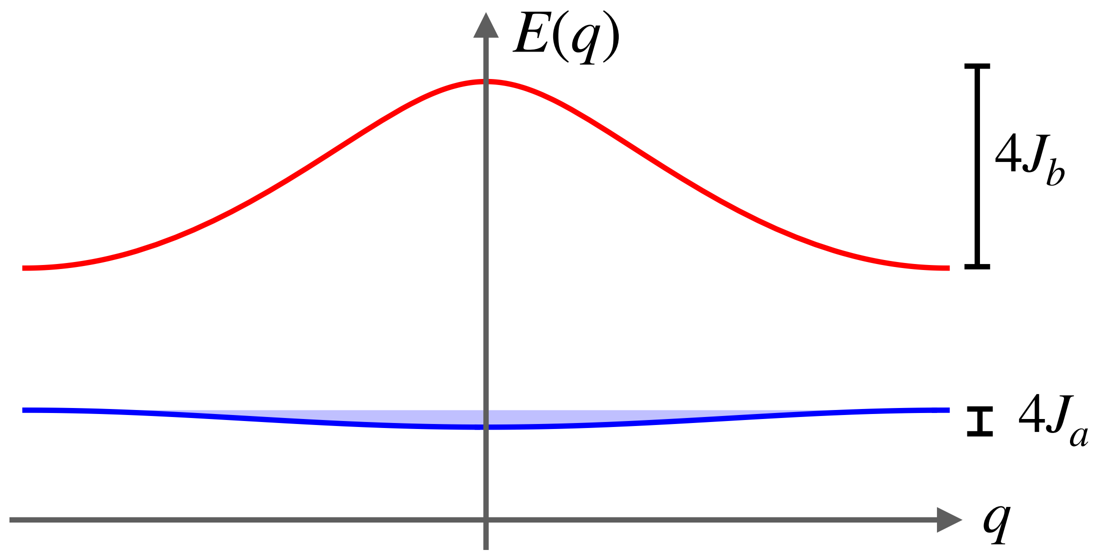
Large tunnelling: π-superfluid





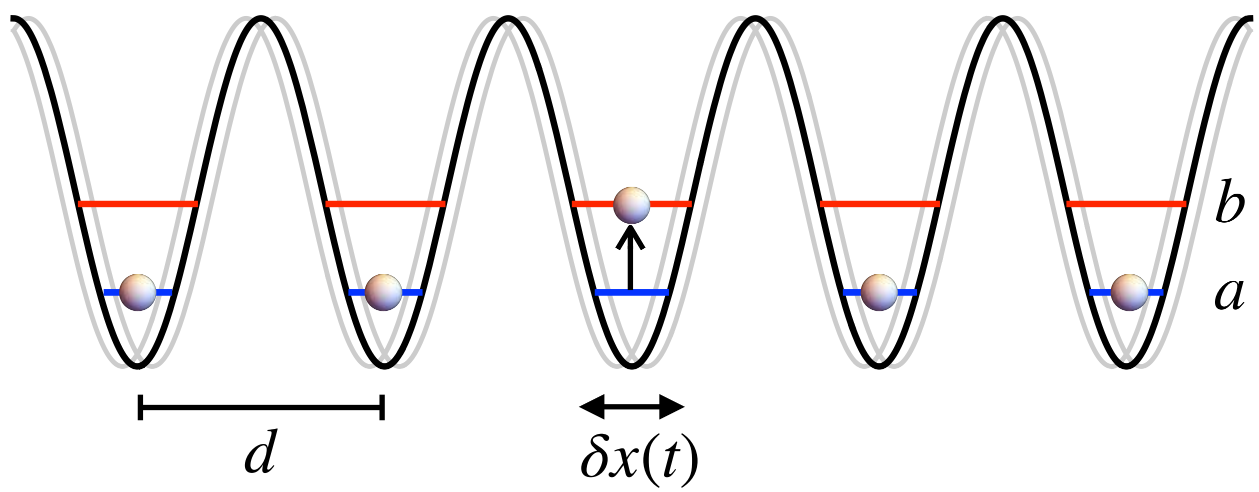
Shaken lattice

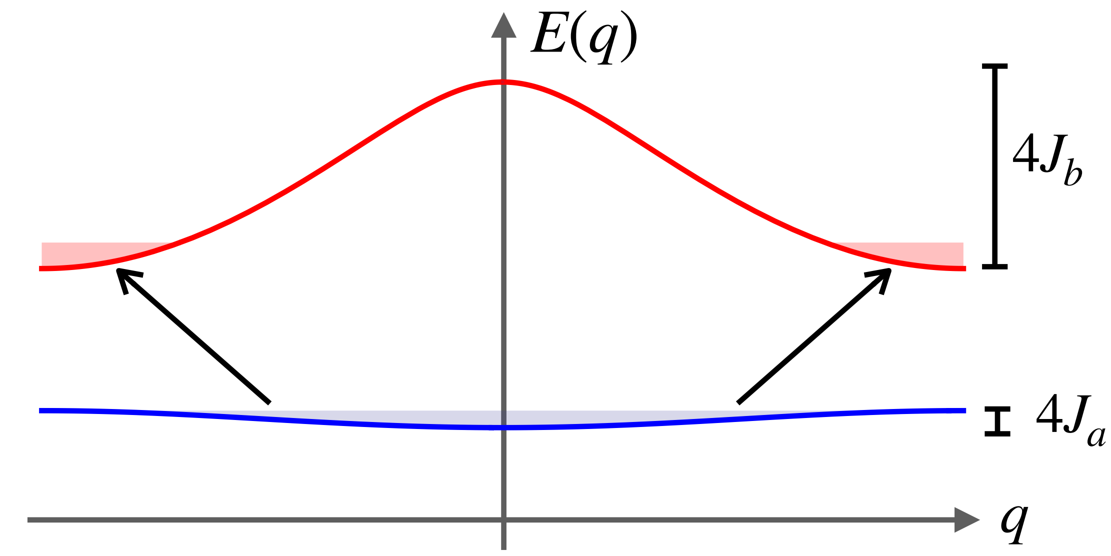




$J_a/U_a \approx 0.04$ (Mott), $J_b/U_b \approx 0.7$ (π-superfluid)

Shaken lattice





$J_a/U_a \approx 0.04$ (Mott), $J_b/U_b \approx 0.7$ (π-superfluid)

Higher bands off-resonant (more later)

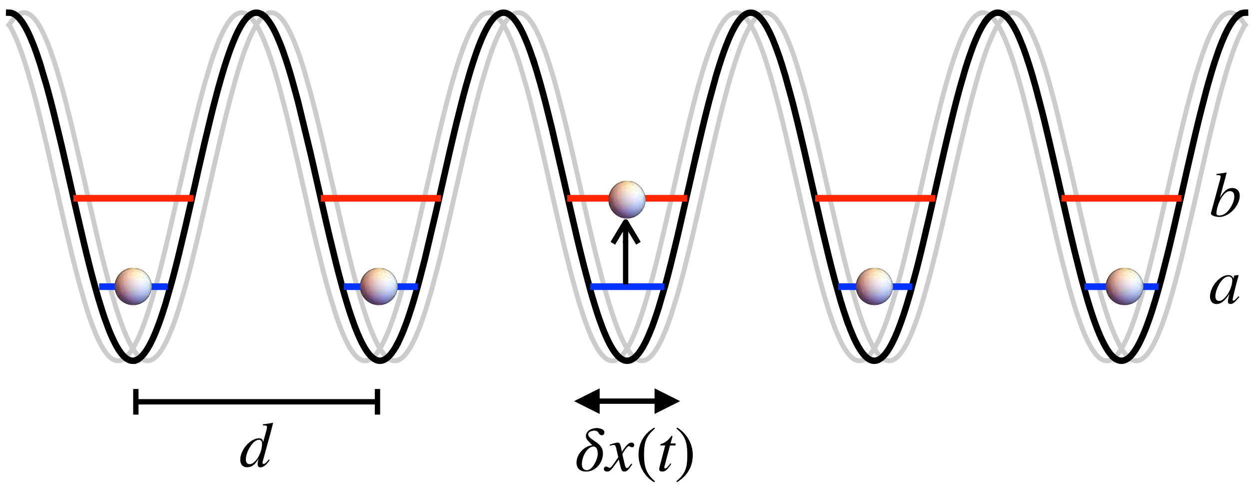
$$\hat{H}(t) = \int dx\ \hat{\psi}^\dagger(x)\ \left[-\partial_x^2 + V_{lat}\big(x - \delta x(t)\big)\right]\ \hat{\psi}(x) + g\ \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\psi}(x)$$ Lab frame

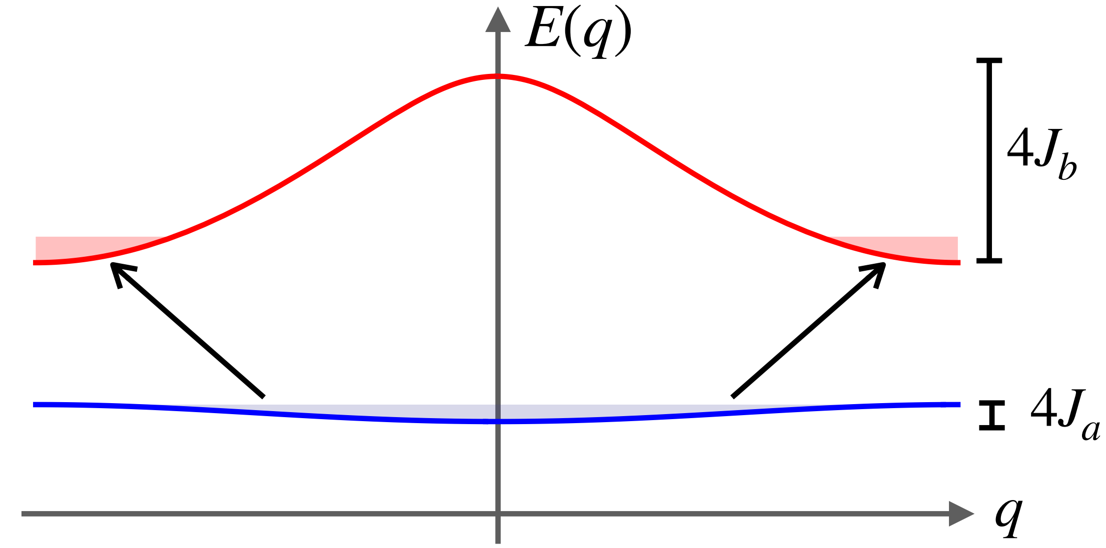
$\downarrow$ Shaking $\rightarrow$ periodic force $F(t) = m\ \delta\ddot{x}(t)$

$$\hat{H}(t) = \int dx\ \hat{\psi}^\dagger(x)\ \left[-\partial_x^2 + V_{lat}(x) + F(t)\ x\right]\ \hat{\psi}(x) + g\ \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\psi}(x)$$ Lattice frame

$\hat{\psi}(x) = \sum_j w_a(x - x_j)\ \hat{a}_j + w_b(x - x_j)\ \hat{b}_j \implies$ Two-band Hubbard model

Two-band Hamiltonian





$$\hat{H}(t) = \hat{T} + \hat{U} + \hat{S}(t)$$

$$\hat{T} = \bar{E}_a \sum_j \hat{a}_j^\dagger \hat{a}_j + \bar{E}_b \sum_j \hat{b}_j^\dagger \hat{b}_j - J_a \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + J_b \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j$$ Kinetic

$$\hat{U} = \sum_j \tfrac{1}{2} U_a \, \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + \tfrac{1}{2} U_b \, \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j + U_{ab} \, \hat{a}_j^\dagger \hat{a}_j \hat{b}_j^\dagger \hat{b}_j + \tfrac{1}{4} U_{ab} \left(\hat{a}_j^\dagger \hat{a}_j^\dagger \hat{b}_j \hat{b}_j + \textbf{H.c.} \right)$$ Interaction

$$\hat{S}(t) = F(t) d \sum_j \underbrace{\boxed{\alpha \left(\hat{a}_j^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{a}_j \right)}}_{\text{Rabi coupling}} + \underbrace{\boxed{j \left(\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j^\dagger \hat{b}_j \right)}}_{\text{Force}}$$ Drive

Energy scales: $J_a = 0.1$ kHz, $J_b = 1.3$ kHz

$U_a = 2.9$ kHz, $U_b = 1.7$ kHz, $U_{ab} = 2.4$ kHz

$E_b(\pi) - E_a(\pi) = 17$ kHz, Time $\sim$ ms

Time-averaged dynamics

Constant periodic shaking $F(t) := m\ \delta\ddot{x}(t) = m\omega^2 A \cos\omega t$

Rotating-frame Hamiltonian $\hat{H}_{\text{rot}}(t) = \hat{H}_* + \boxed{\left(\hat{H}_1 e^{i\omega t} + \hat{H}_2 e^{i2\omega t} + \text{H.c.}\right)}$ Fast oscillations (micromotion)

$[\hat{b}_j \to e^{-i\omega t}\hat{b}_j]$

$$\hat{H}_* = -J_a \sum\nolimits_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + J_b \sum\nolimits_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \sum\nolimits_j \Delta\ \hat{b}_j^\dagger \hat{b}_j + \Omega\ \left(\hat{a}_j^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{a}_j\right)$$
$$+ \sum\nolimits_j \tfrac{1}{2} U_a\ \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + \tfrac{1}{2} U_b\ \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j + U_{ab}\ \hat{a}_j^\dagger \hat{a}_j \hat{b}_j^\dagger \hat{b}_j$$

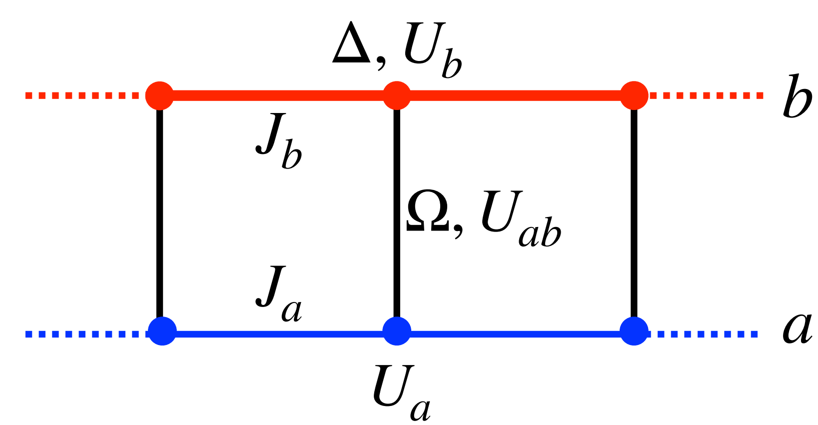
$\Delta := \bar{E}_b - \bar{E}_a - \hbar\omega$

$\Omega := m\omega^2 A d\alpha/2$

$$\hat{H}_1 = (F_0 d/2) \sum\nolimits_j j(\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j^\dagger \hat{b}_j)$$

$$\hat{H}_2 = \sum\nolimits_j \Omega\ \hat{b}_j^\dagger \hat{a}_j + (Uab/4)\ \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j$$

$\hat{H}_* \equiv$



Time-averaged dynamics generated by Floquet Hamiltonian

$$\hat{H}_{eff} = \hat{H}_* \boxed{+ \frac{1}{\hbar\omega} \sum\nolimits_{l=1}^{2} \frac{1}{l} [\hat{H}_l, \hat{H}_l^\dagger] + \frac{1}{2(\hbar\omega)^2} \sum\nolimits_{l=1}^{2} \dots}$$

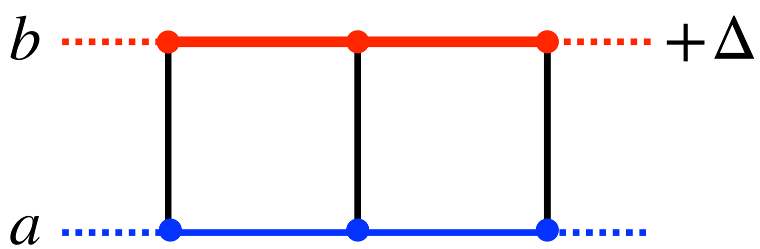
(Magnus expansion)

Renormalisation insignificant

Goldman & Dalibard, PRX 4 031027 (2014)

Inducing phase transition

$\hat{H}_{eff} =$

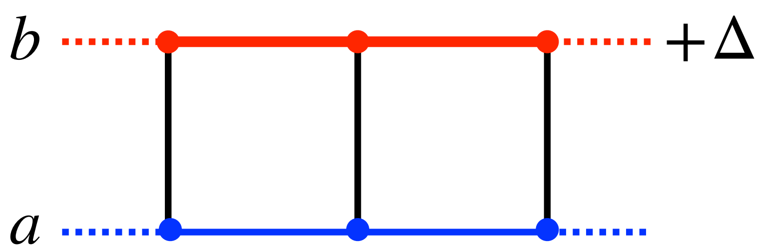


Low-freq ($\Delta \gg 0$) ground state: Mott (a)

High-freq ($\Delta \ll 0$) ground state: π-superfluid (b)

Strategy: sweep freq from low to high and follow ground state

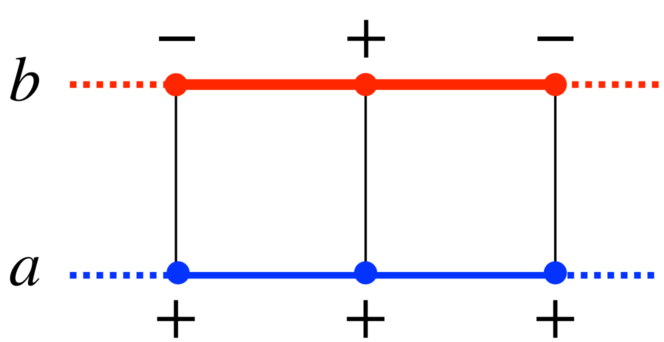
Discontinuous phase transition

$\hat{H}_{eff} =$ 

Low-freq ($\Delta \gg 0$) ground state: Mott (a)
High-freq ($\Delta \ll 0$) ground state: π-superfluid (b)

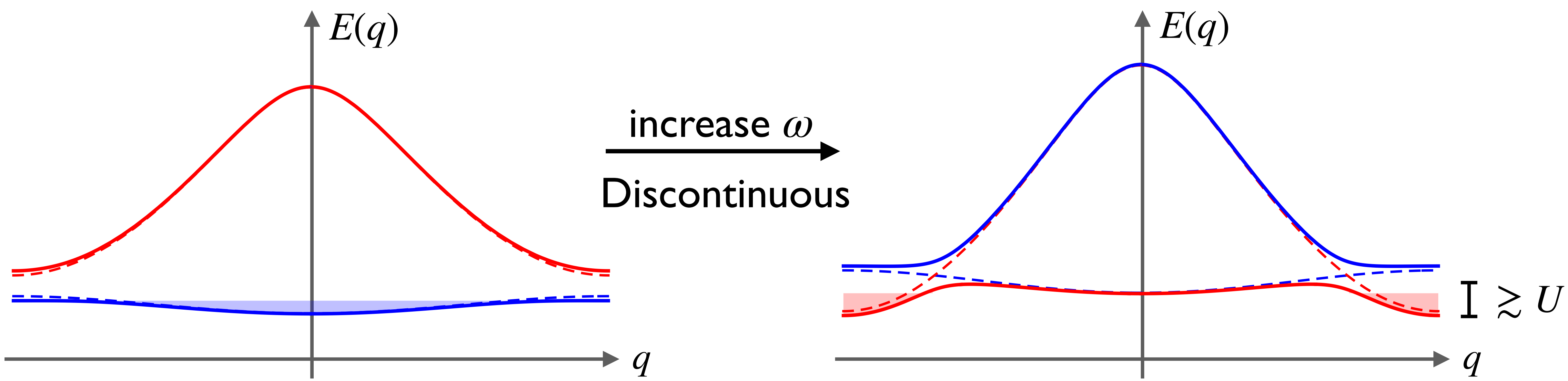
Strategy: sweep freq from low to high and follow ground state — *does not happen stuck in Mott (a)*

Case I. Weak coupling



Frustration between staggered & non-staggered phases

$\Longrightarrow$ Ground state jumps between two minima

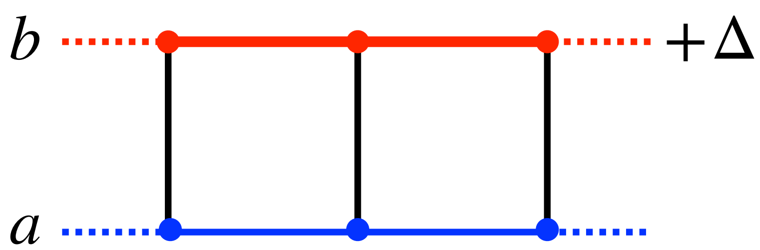


Non-staggered Mott

(Staggered) π-superfluid

Continuous phase transition

$\hat{H}_{eff} =$

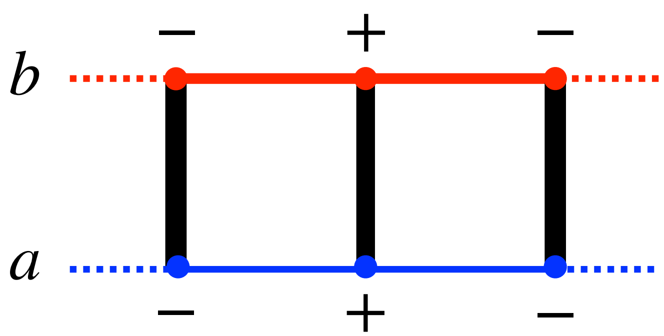


Low-freq ($\Delta \gg 0$) ground state: Mott (a)

High-freq ($\Delta \ll 0$) ground state: π-superfluid (b)

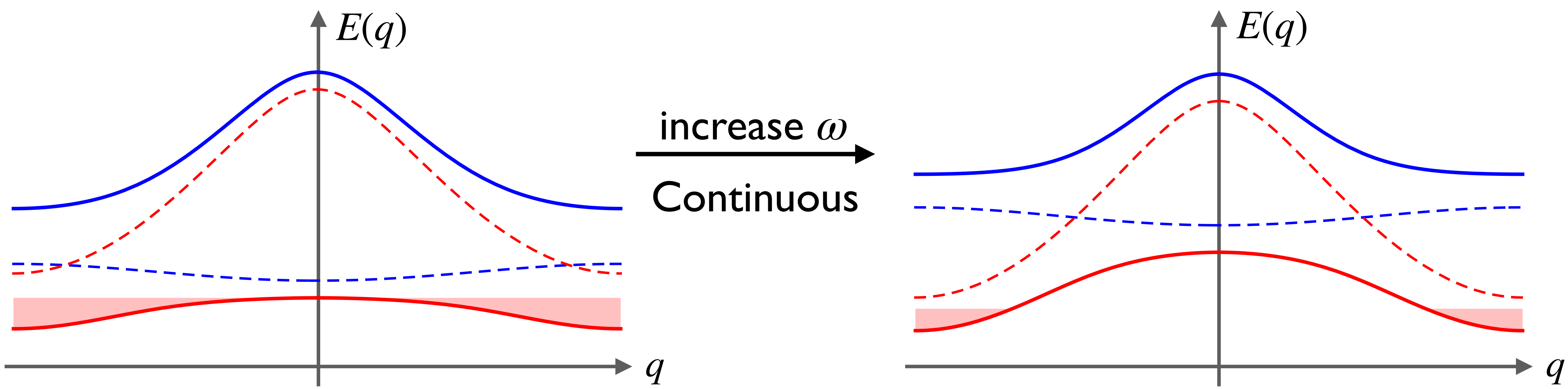
Strategy: sweep freq from low to high and follow ground state — *Kibble-Zurek*

Case II. Strong coupling



Bands strongly hybridised

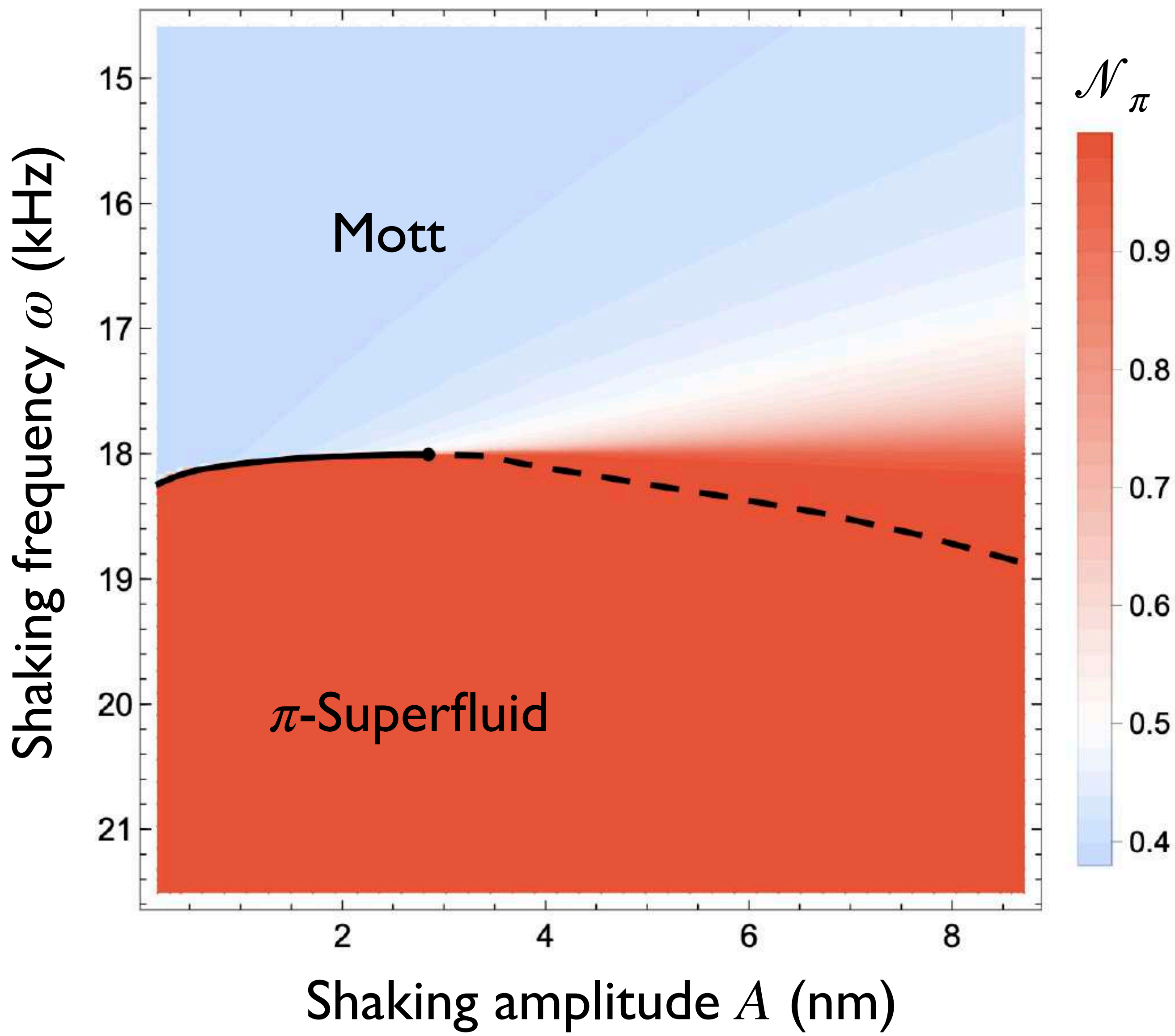
$\Longrightarrow$ Continuous transition in a single effective band



Staggered Mott

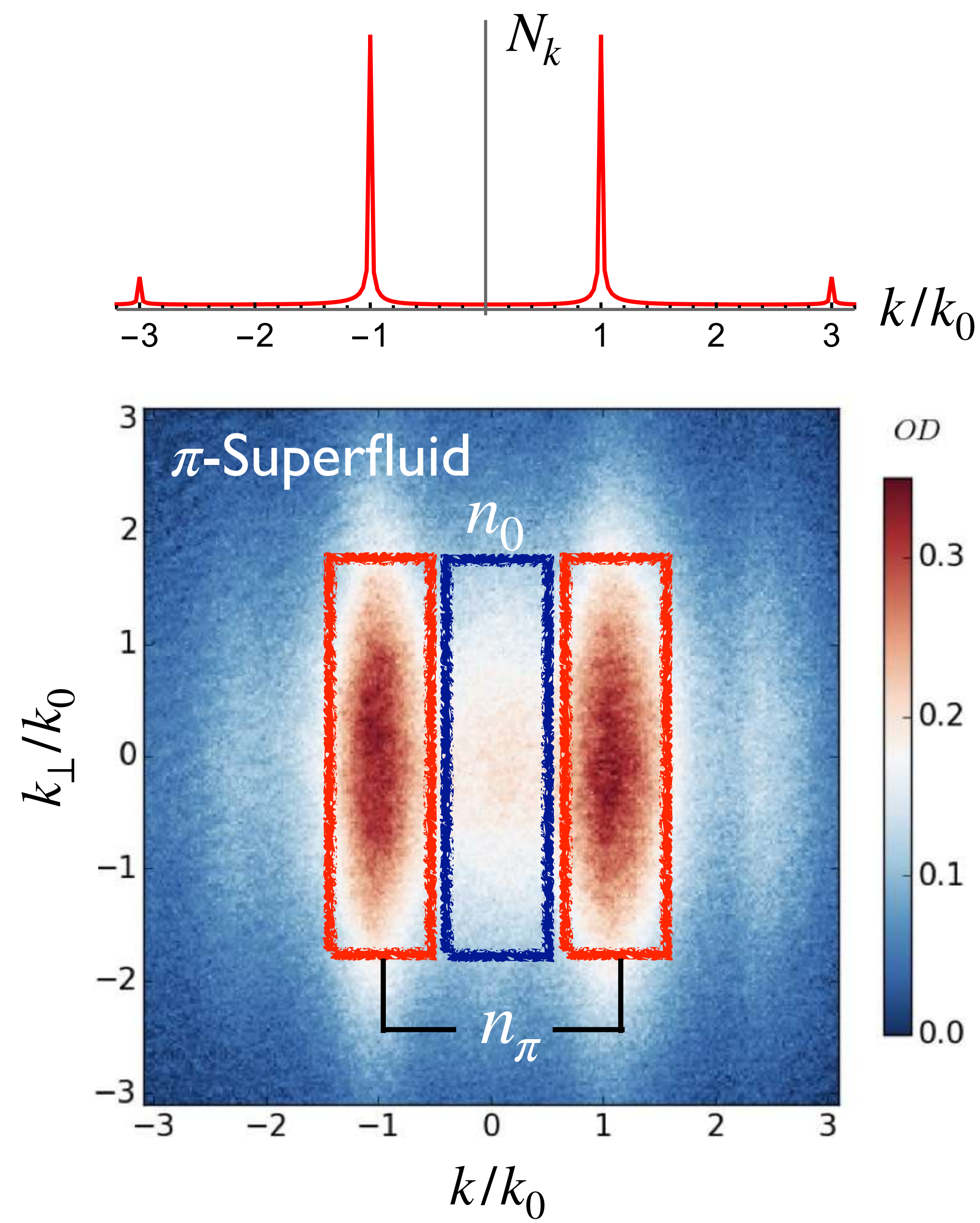
(Staggered) π-superfluid

Phase diagram

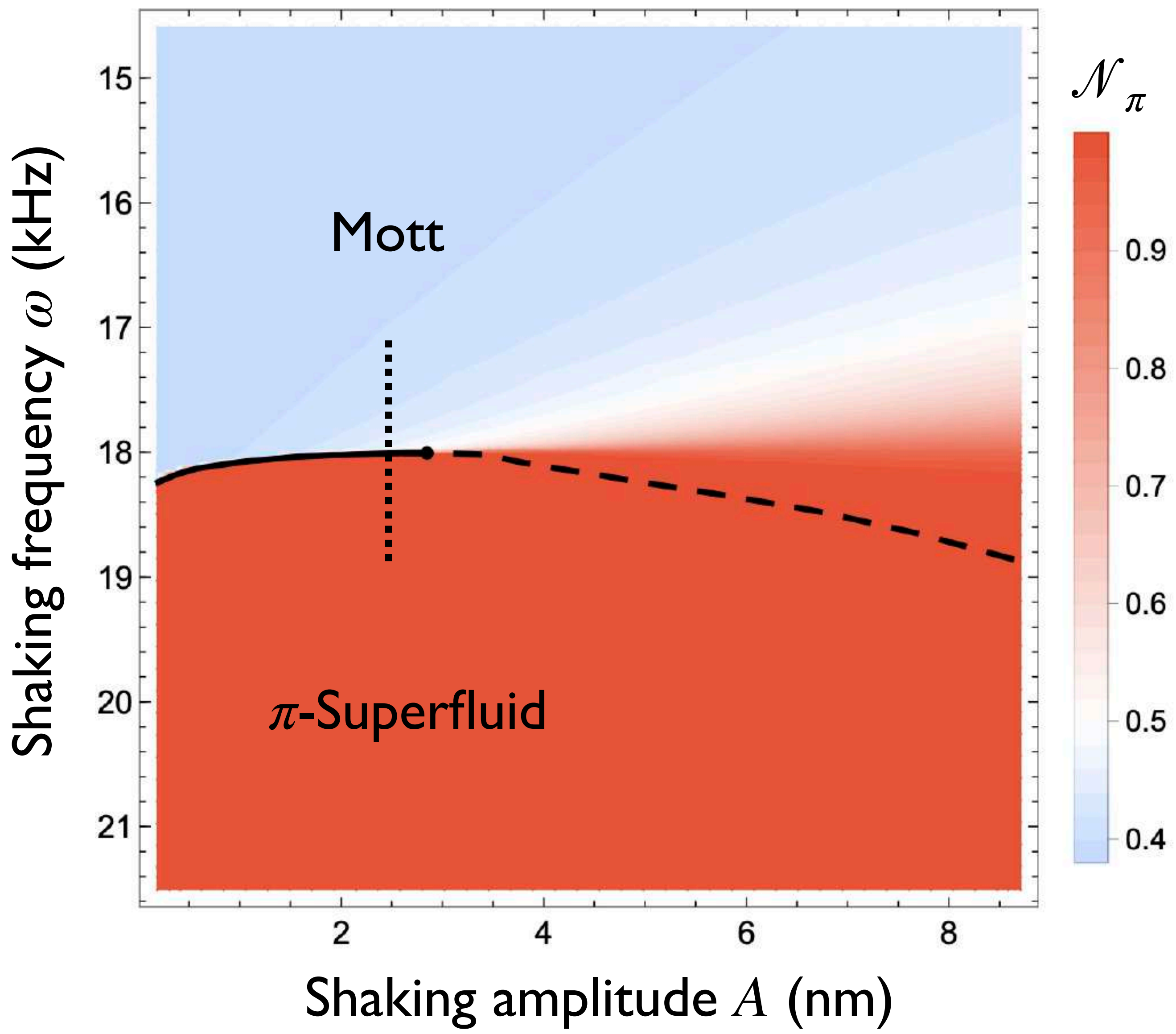


$$\mathcal{N}_\pi := n_\pi/(n_0 + n_\pi)$$

Experiment measures time-of-flight images — plane-wave occupations

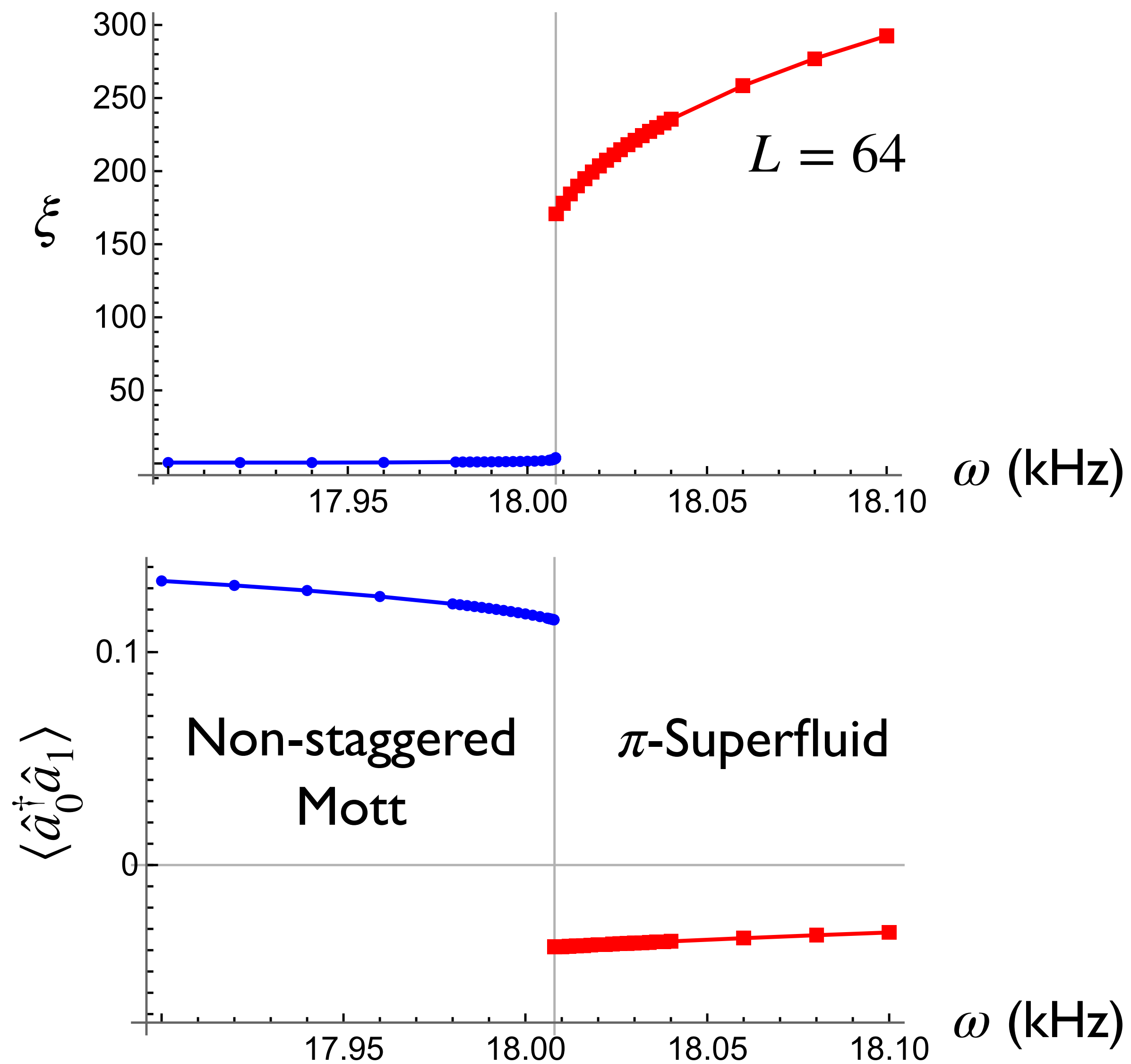


Phase diagram

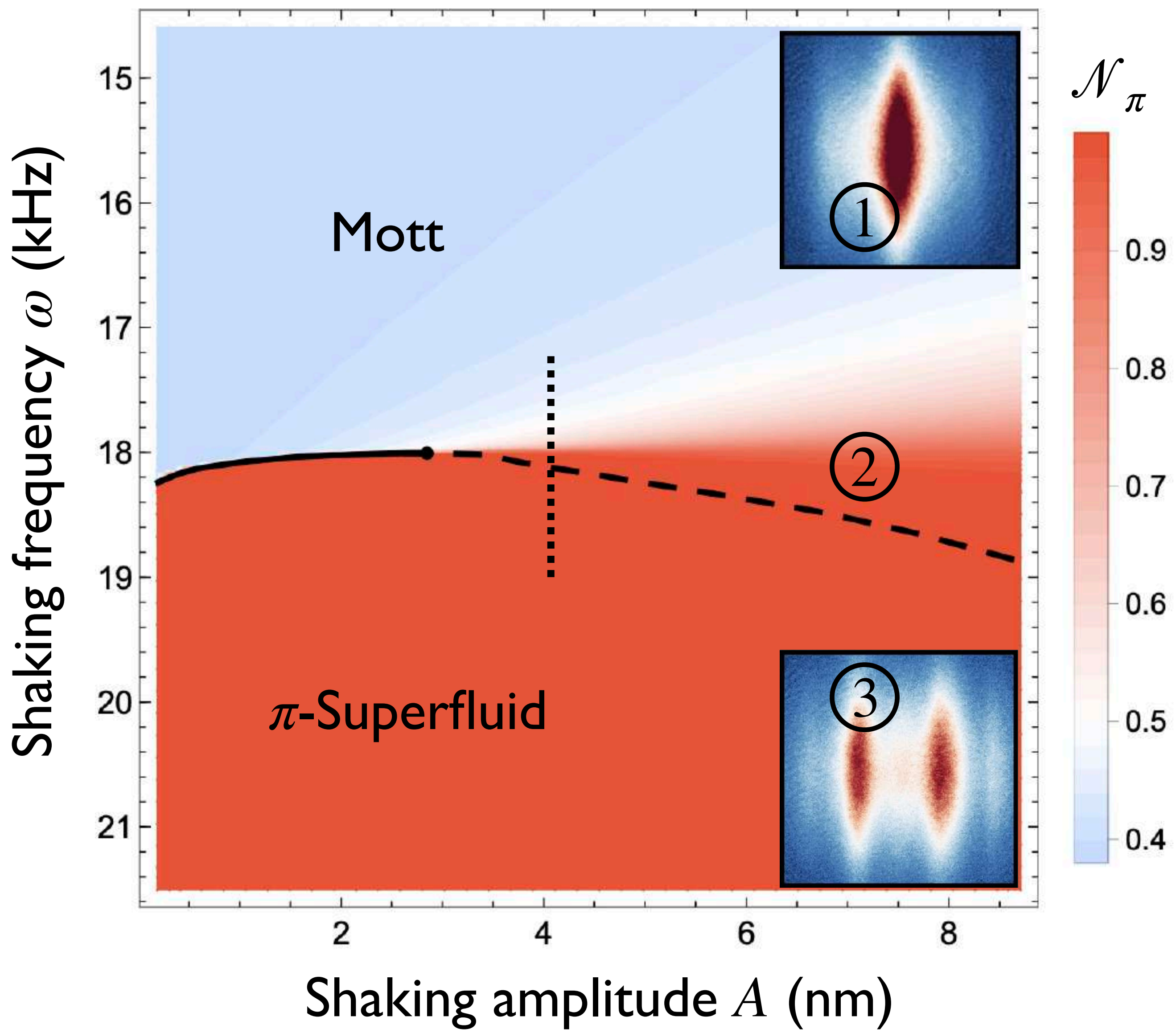


Small amplitude: discontinuous transition

$$|\langle \hat{a}_0^\dagger \hat{a}_r \rangle| \sim e^{-r/\xi}\, r^{-1/(2K)}$$

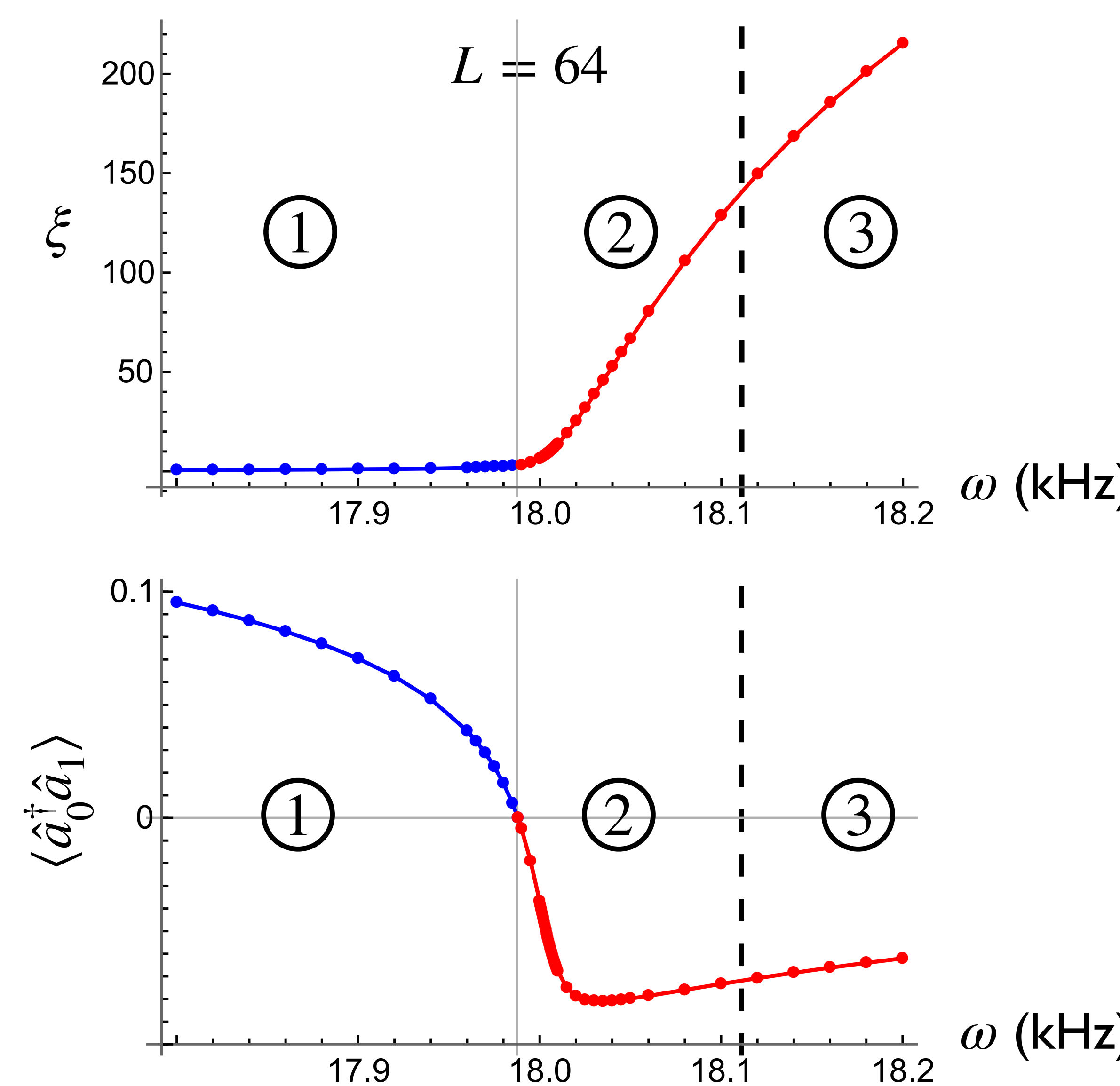


Phase diagram



Large amplitude: continuous transition

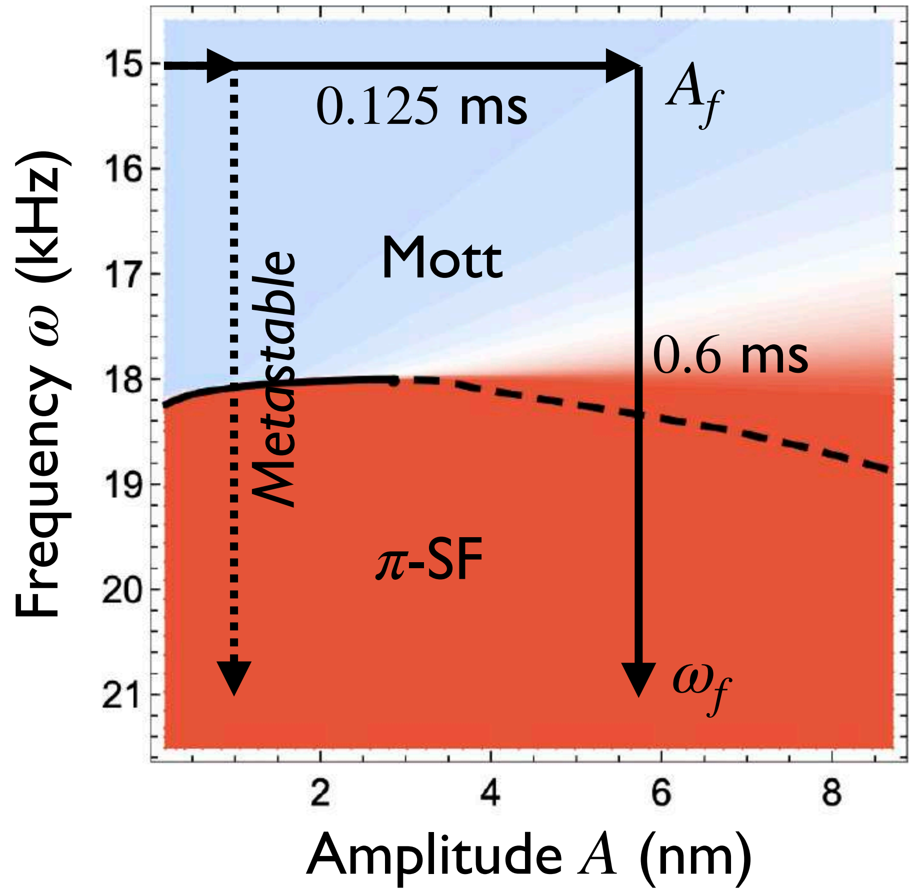
$$|\langle \hat{a}_0^\dagger \hat{a}_r \rangle| \sim e^{-r/\xi}\, r^{-1/(2K)}$$

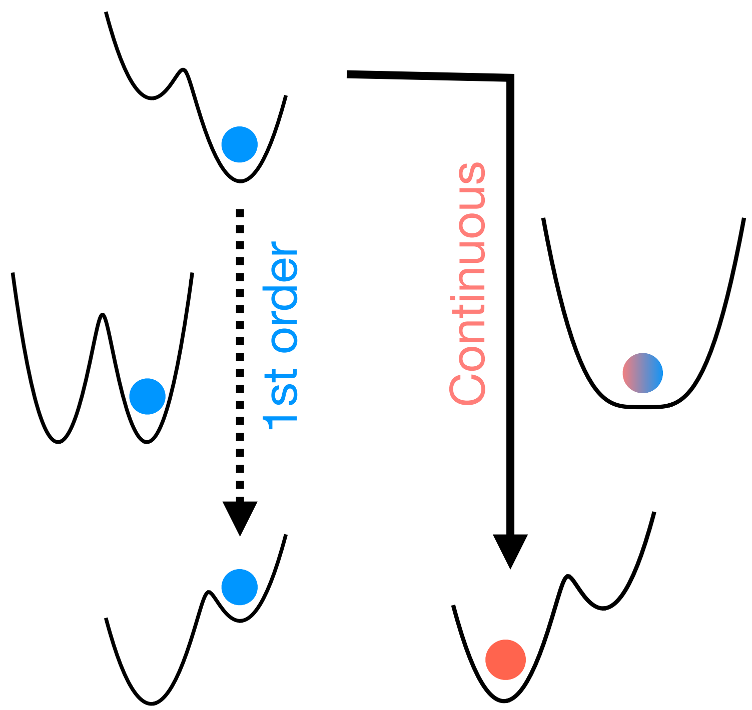


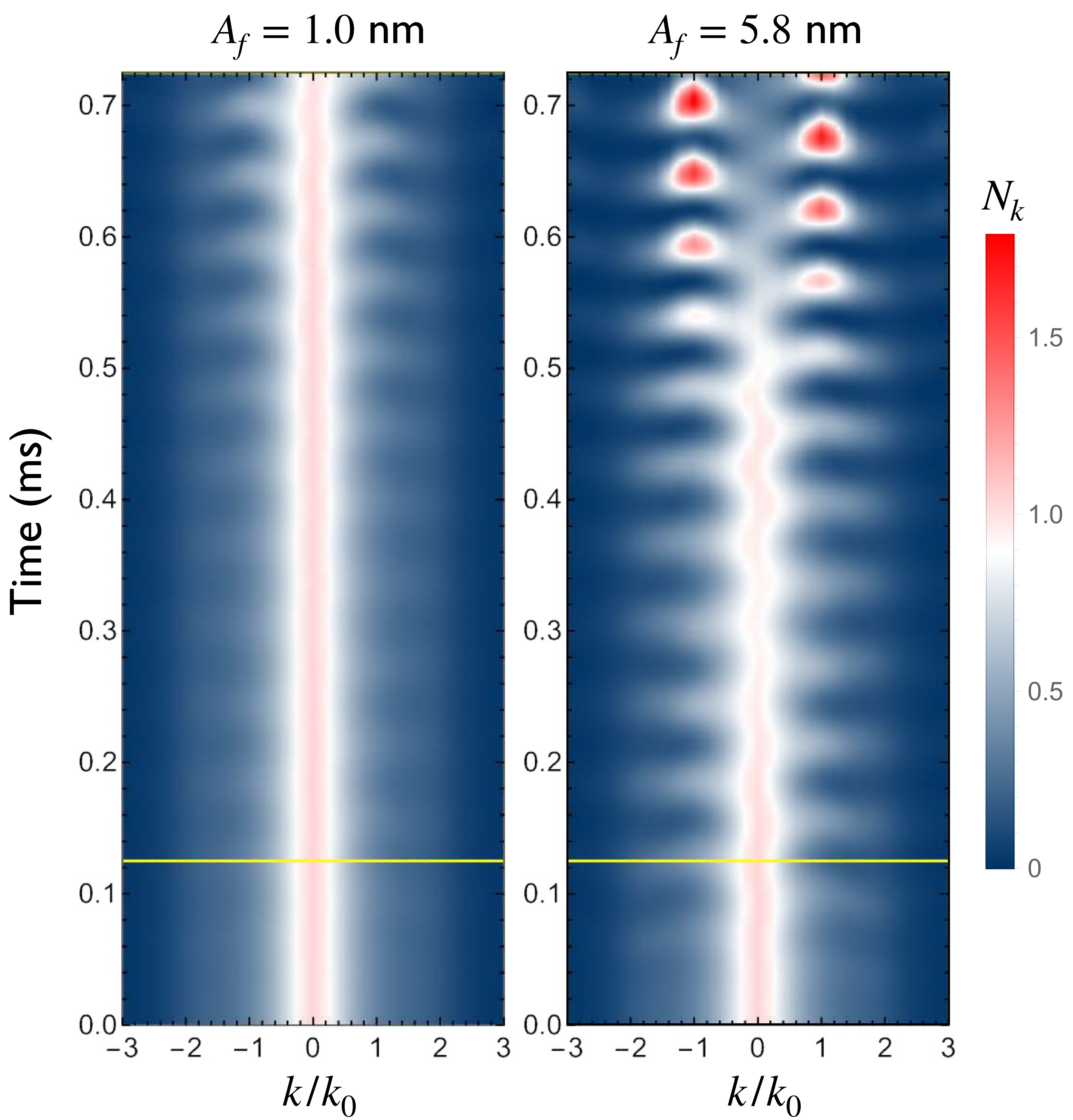
1 Non-staggered Mott

2 Staggered Mott

Metastability

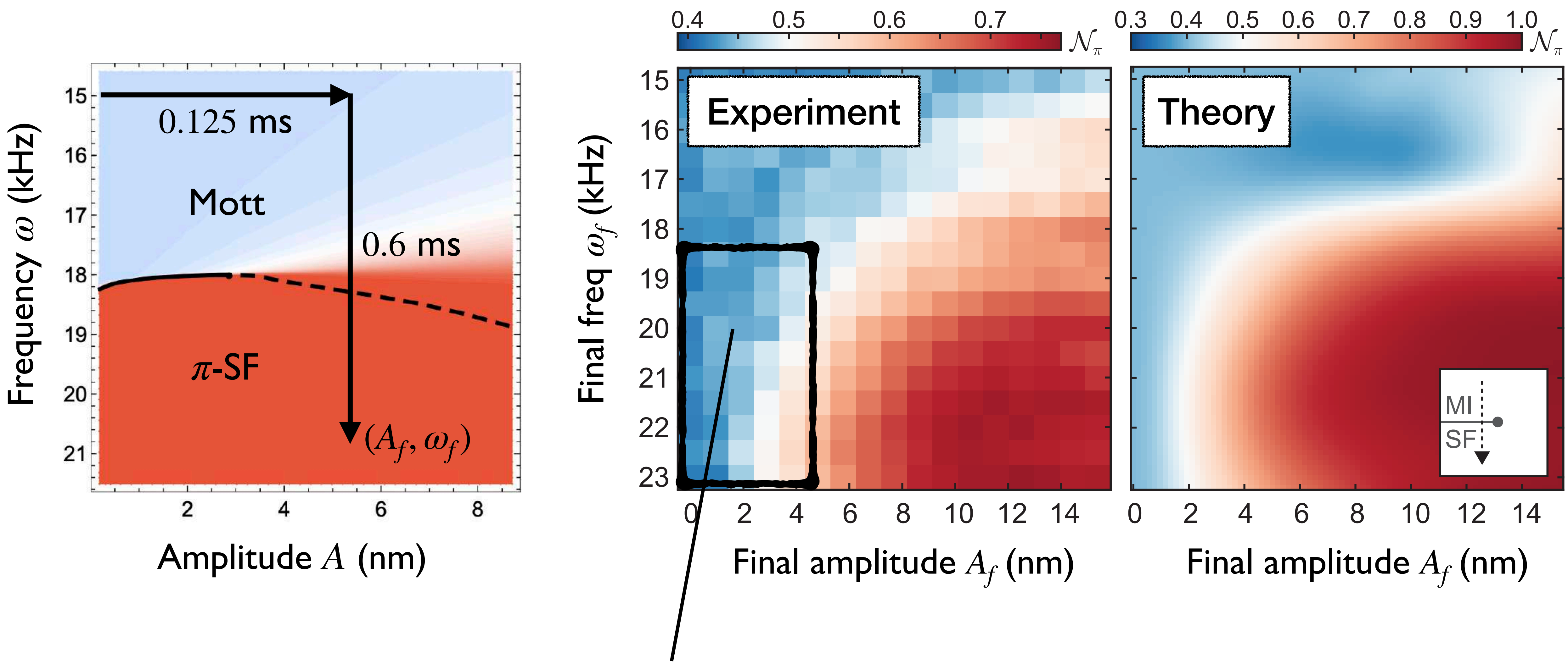






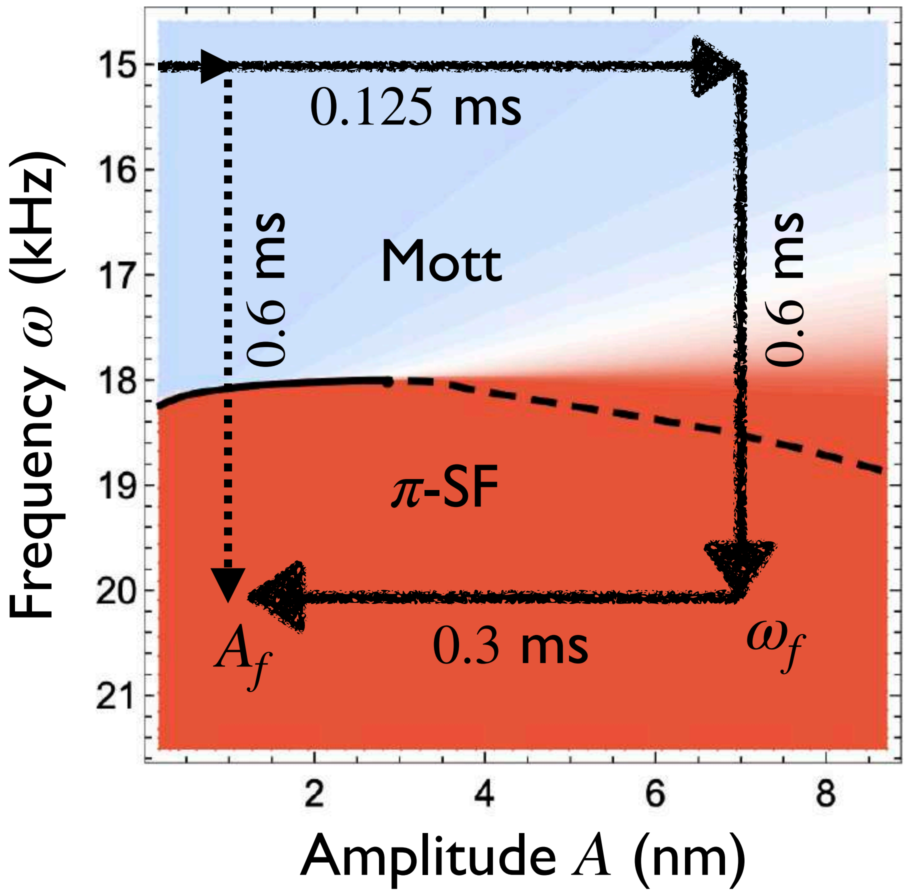
Metastability

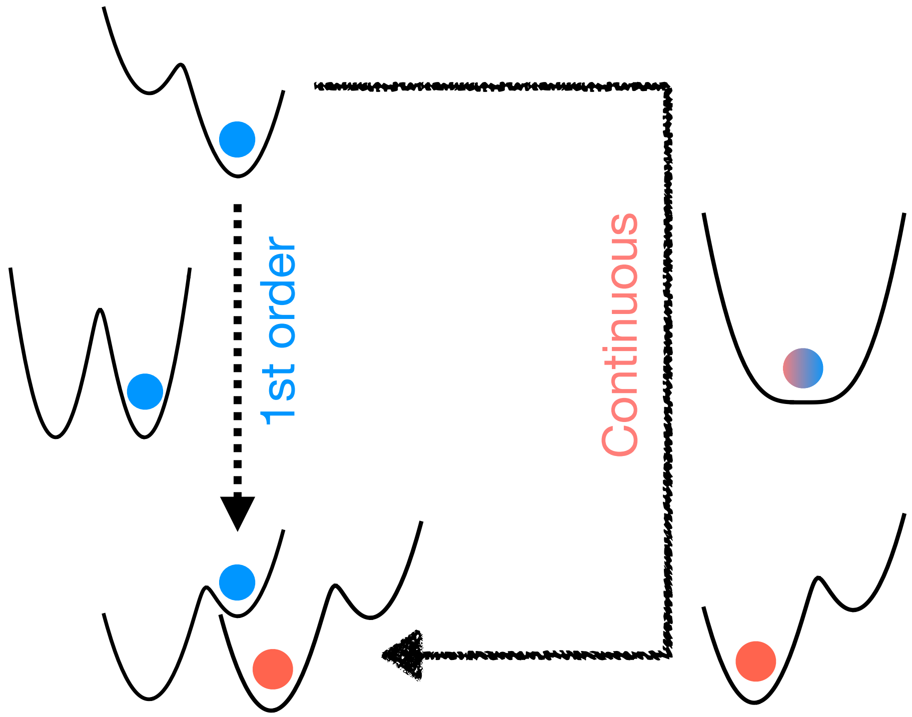
Final occupations vs Ground-state occupations

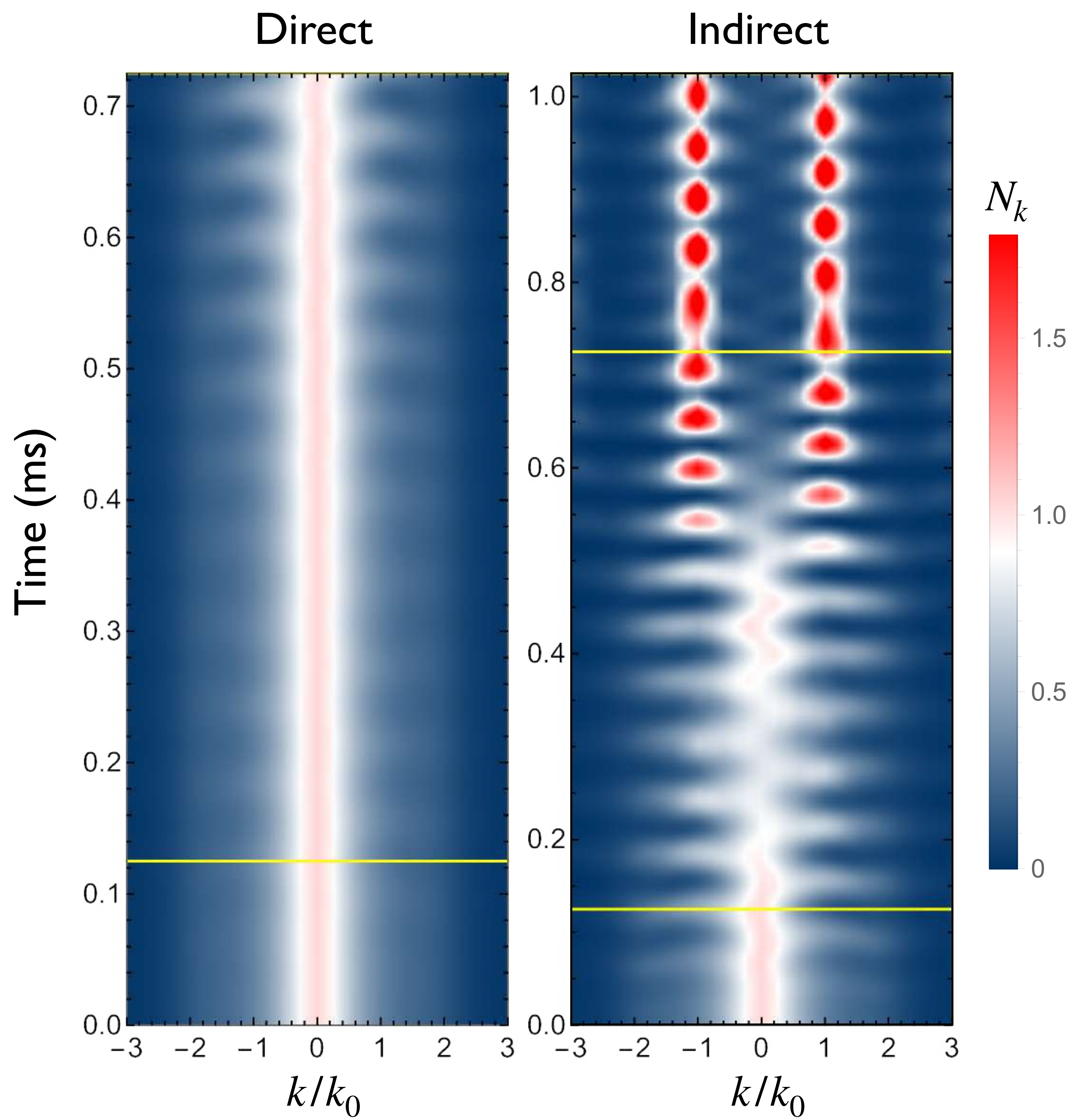


Stuck in metastable Mott phase

Hysteresis: direct vs indirect sweeps

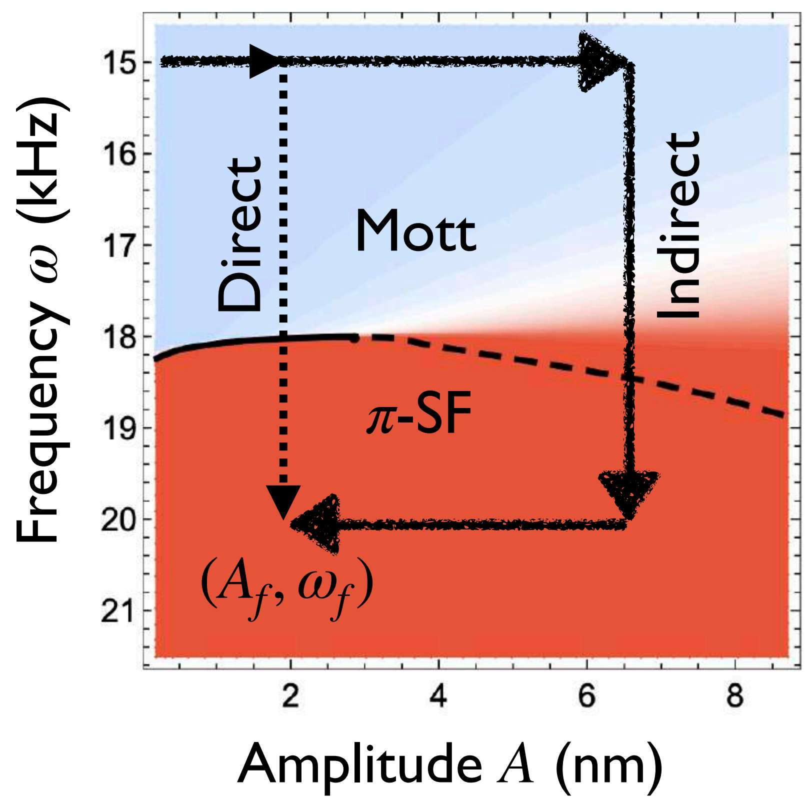


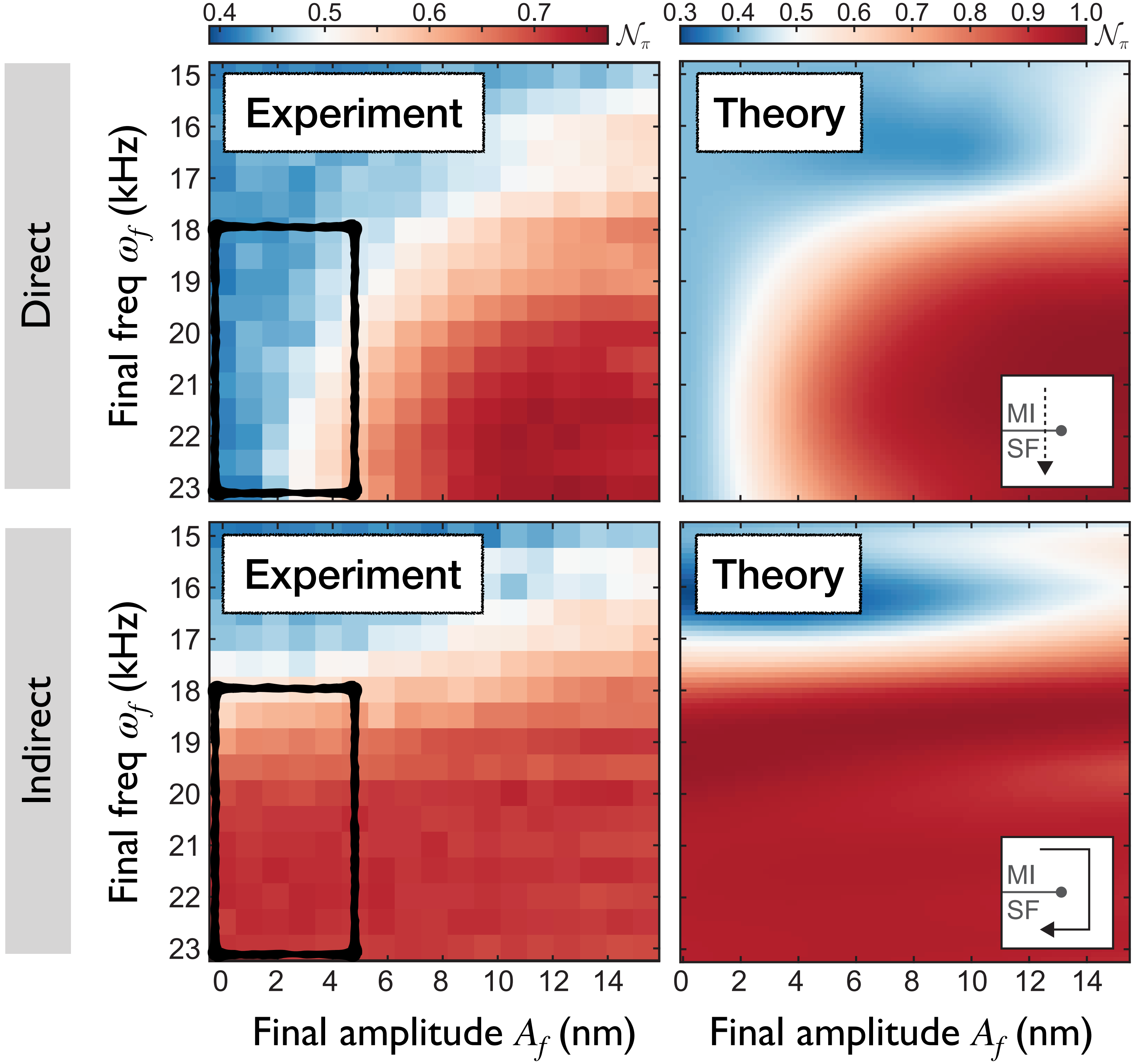




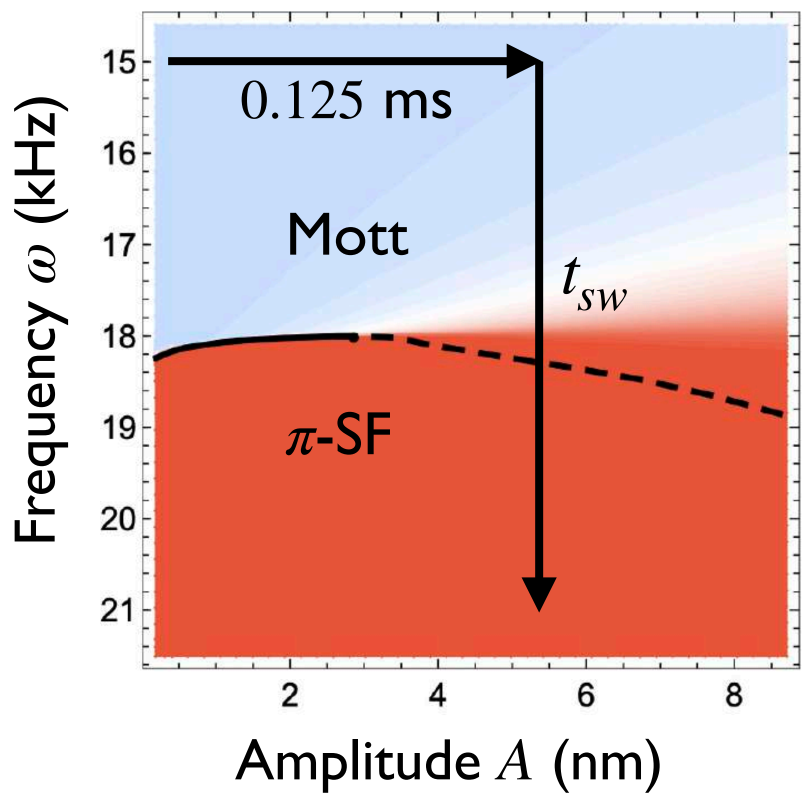
Hysteresis: direct vs indirect sweeps

Indirect sweeps follow ground states but direct sweeps do not

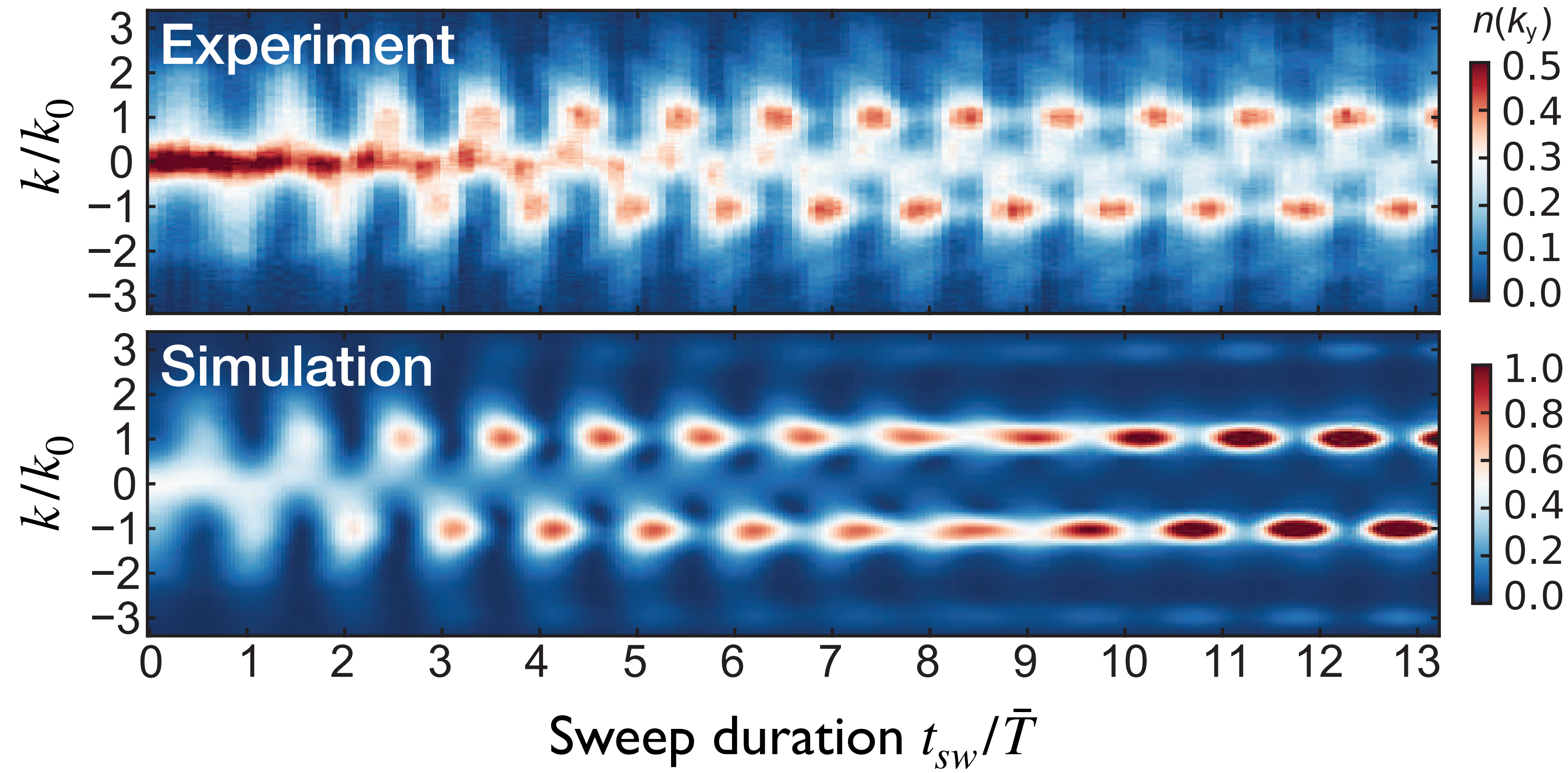


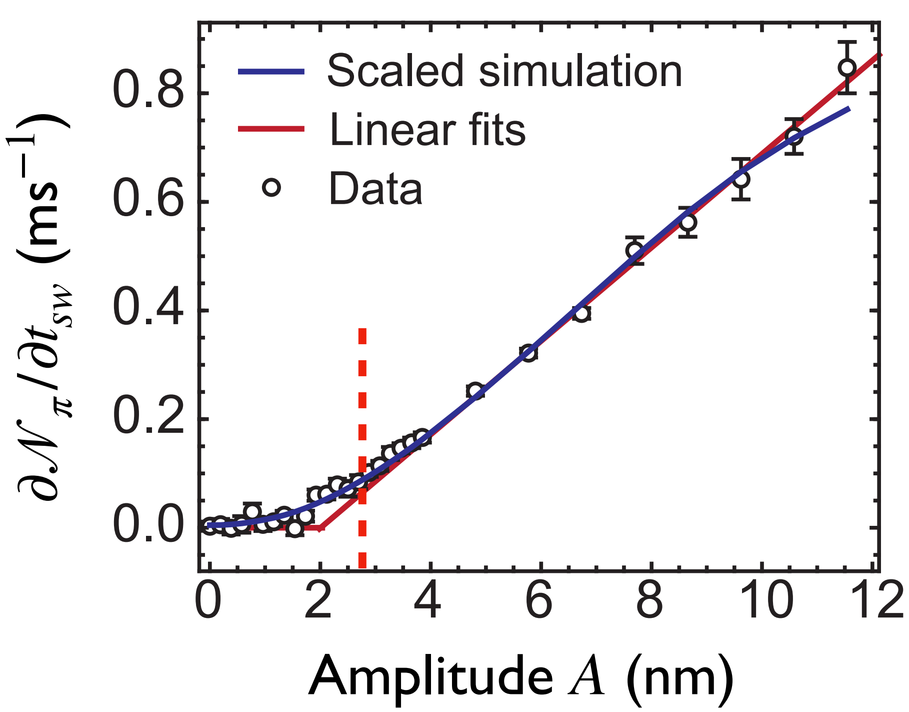


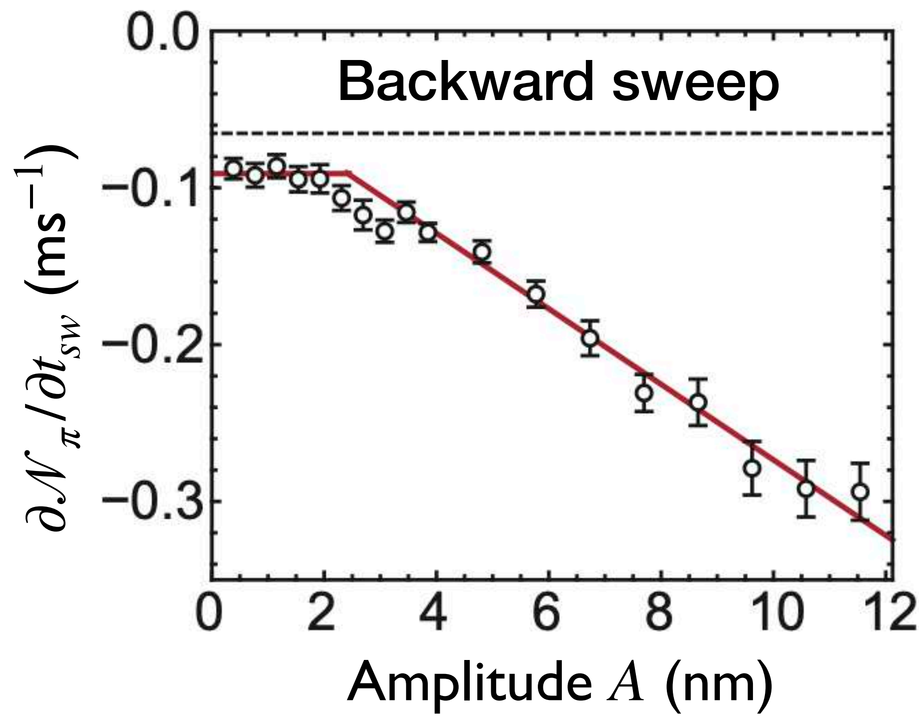
Variable excitation rates



Final occupations for $A = 11.5$ nm



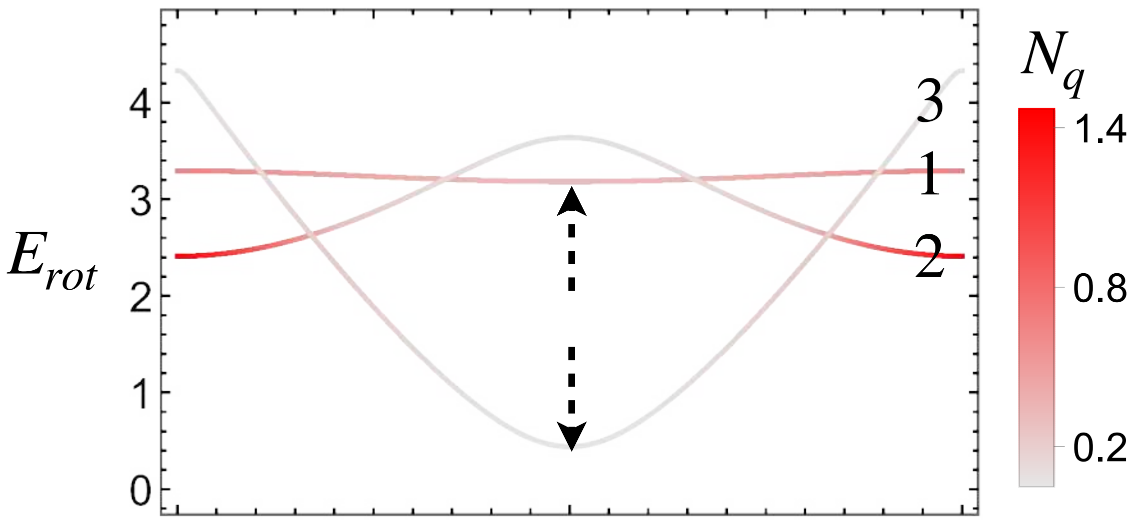




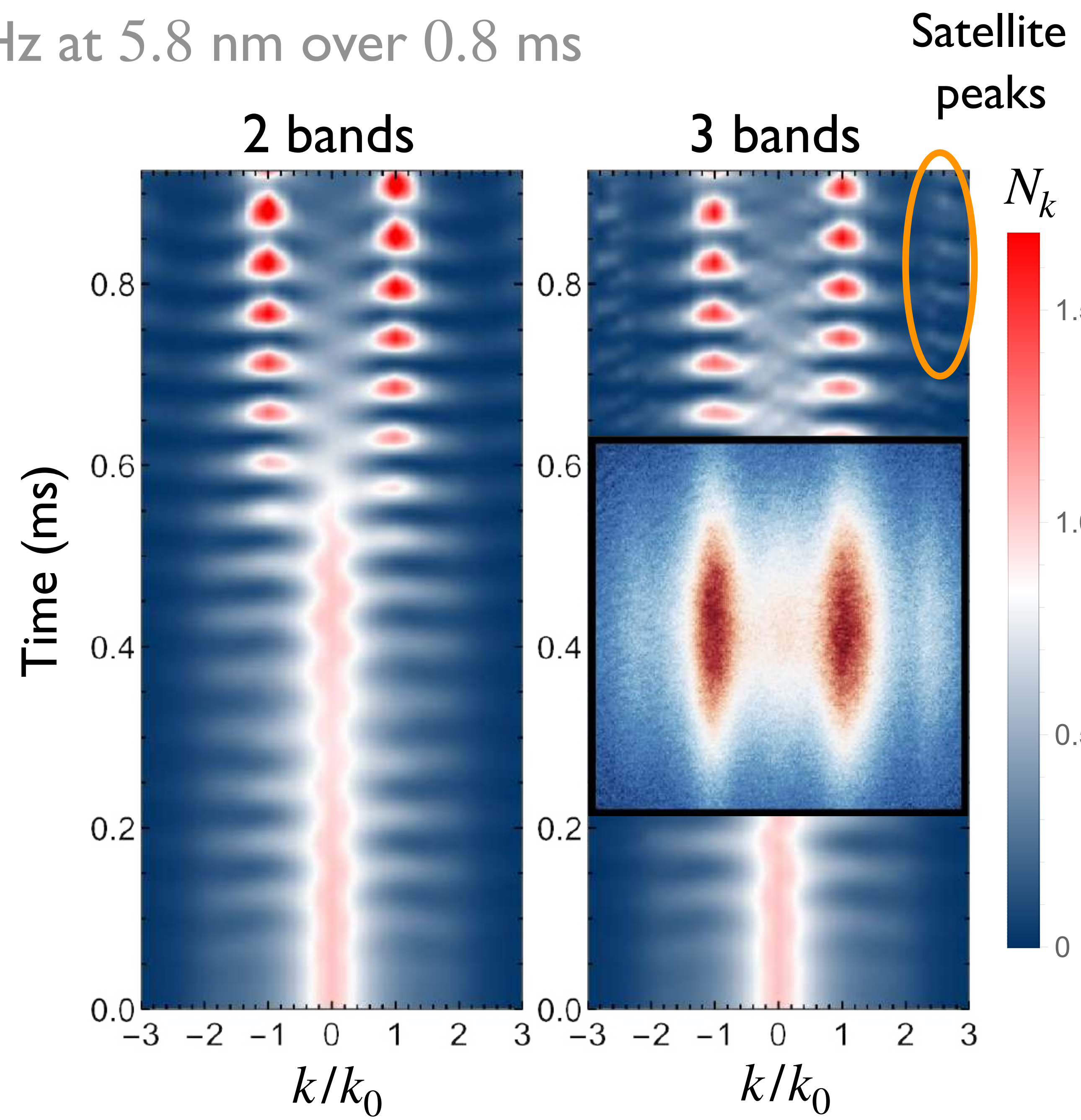
Effect of third / higher bands

- 1st and 3rd band have same parity — no direct coupling by shaking
- Particles populate 2nd band near π, which remains off-resonant with 3rd band as long as $\hbar\omega < E_3(\pi) - E_2(\pi) \approx 2[E_2(\pi) - E_1(\pi)]$

Example: direct sweep 15 kHz → 21 kHz at 5.8 nm over 0.8 ms

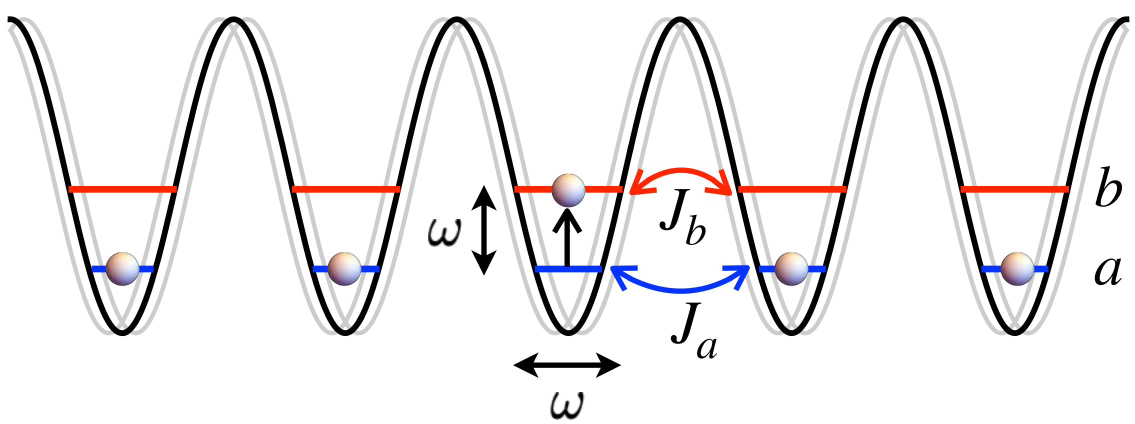


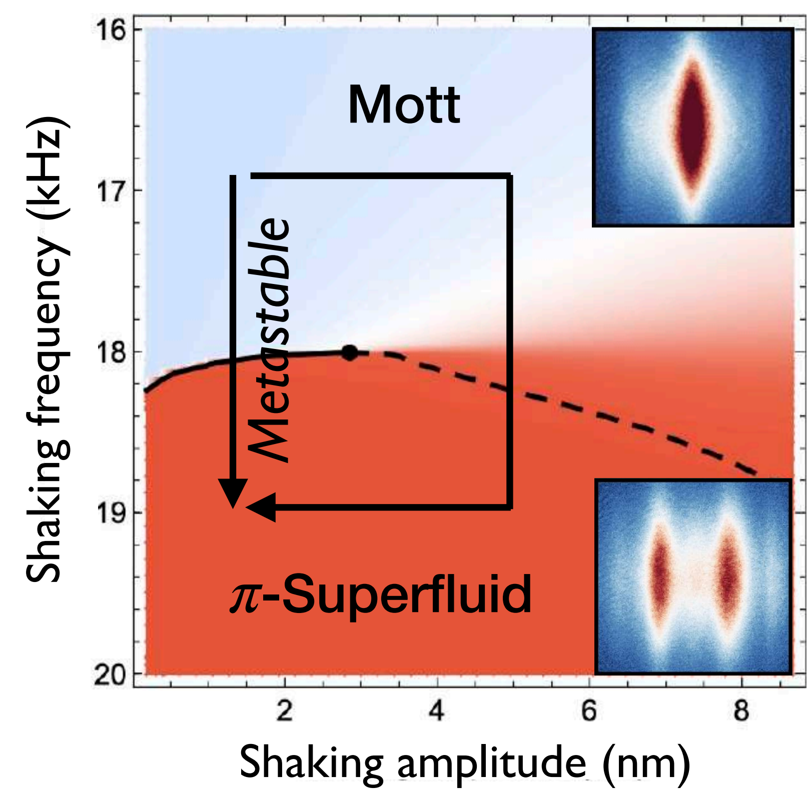
Small fraction excited to 3rd band at intermediate momenta



Summary

Tuneable quantum phase transitions with strikingly different dynamics in a driven strongly-correlated system, exhibiting metastability & hysteresis





Outlook:

- Investigate decay mechanism of metastable many-body states
- Structure formation from quantum fluctuations — false vacuum decay
- Universal properties — scaling laws

B. Song, SD, S. Bhave, J.-C. Yu, E. Carter, N. Cooper, & U. Schneider — arXiv:2105.12146 (Nature Physics)