

Controllable long-range entanglement in a lossy qubit array

Shovan Dutta
University of Cambridge
(now at MPI-PKS)

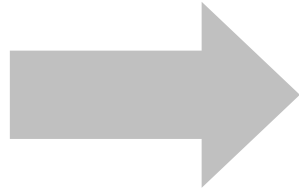
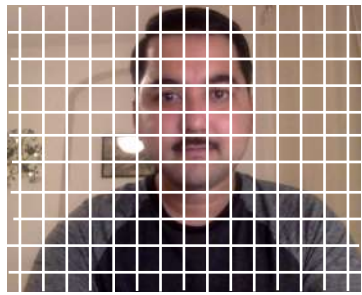
SD & Nigel Cooper, [PRL 125, 240404 \(2020\)](#)

[PRR 3, L012016 \(2021\)](#)

SD, Stefan Kuhr, & Nigel Cooper, coming soon

Qubits and Entanglement

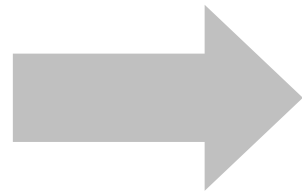
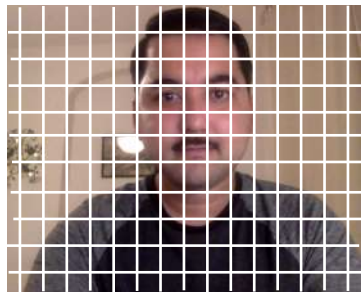
Classical bit — unit of information: 0 or 1



Data stream ...0010110101...

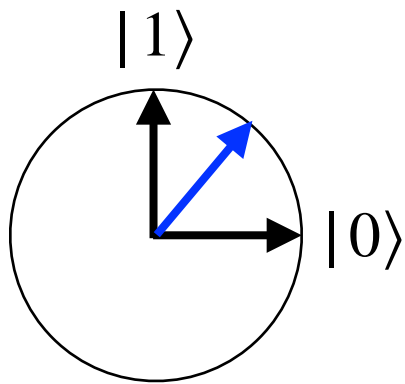
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Quantum bit (“qubit”) — can be in a **superposition** $\alpha|0\rangle + \beta|1\rangle$



Measurement gives:

$|0\rangle$: probability $|\alpha|^2$

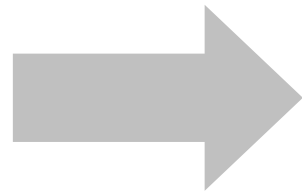
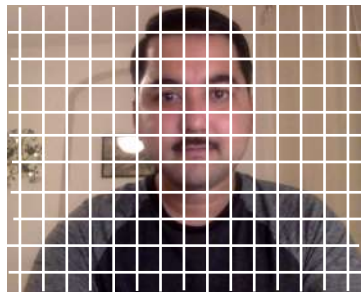
$|1\rangle$: probability $|\beta|^2$

Spin: $\alpha|\downarrow\rangle + \beta|\uparrow\rangle$

Lattice site: $\alpha|\circ\rangle + \beta|\bullet\rangle$

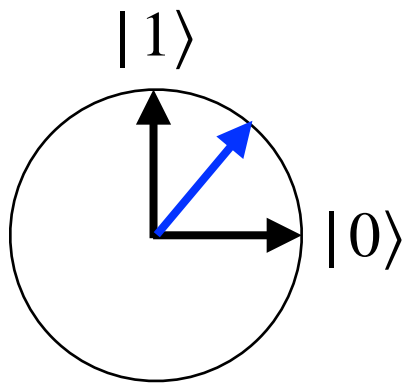
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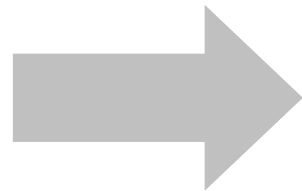
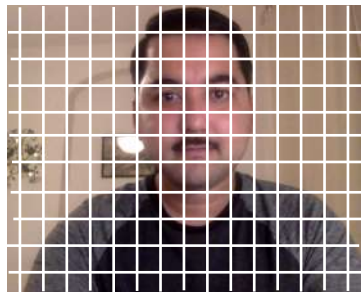
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Entangled pair of qubits $\alpha|01\rangle + \beta|10\rangle$

$|\alpha| = |\beta|$: “**Bell pair**”

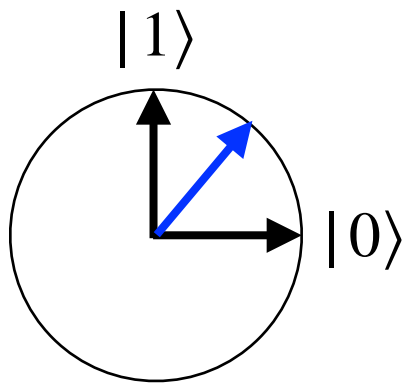
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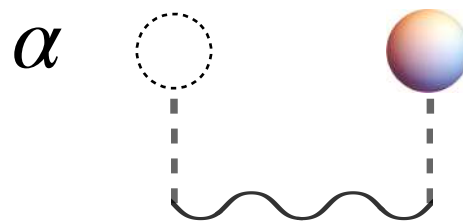
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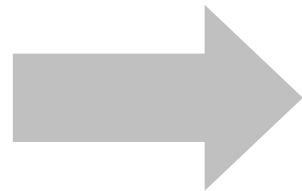
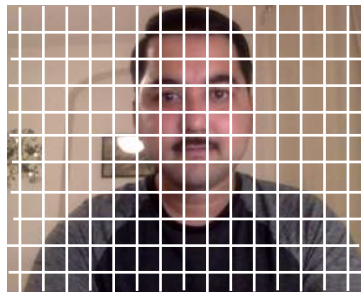
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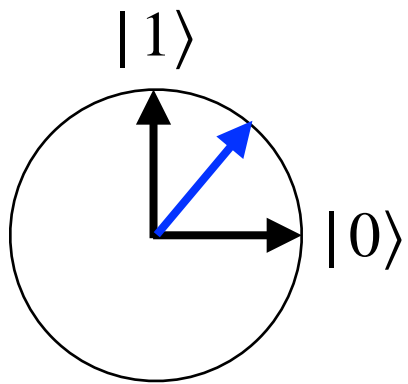
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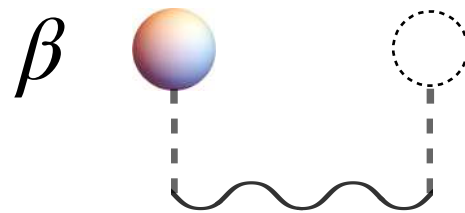
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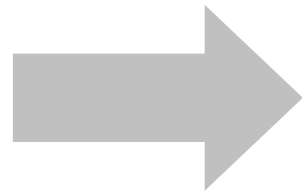
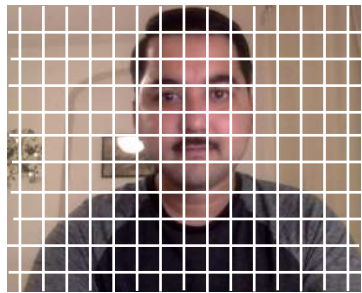
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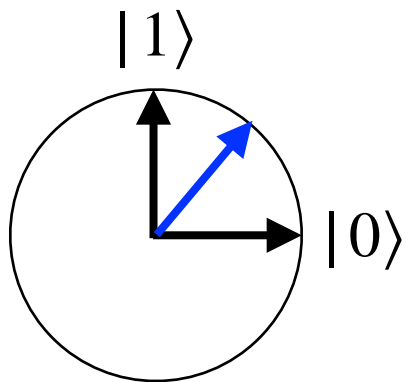
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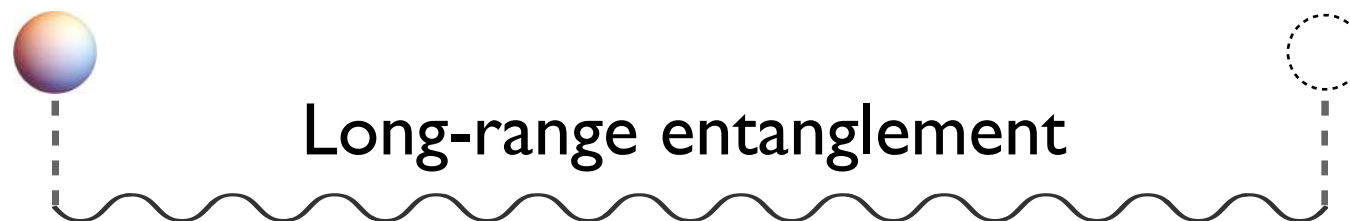
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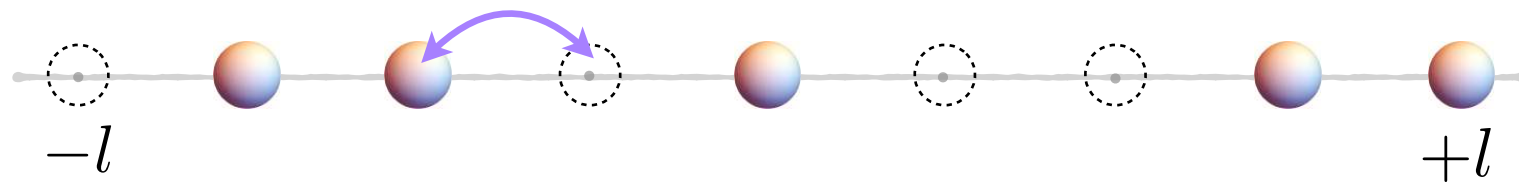
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At the heart of quantum communication/computing — **but hard to stabilise**

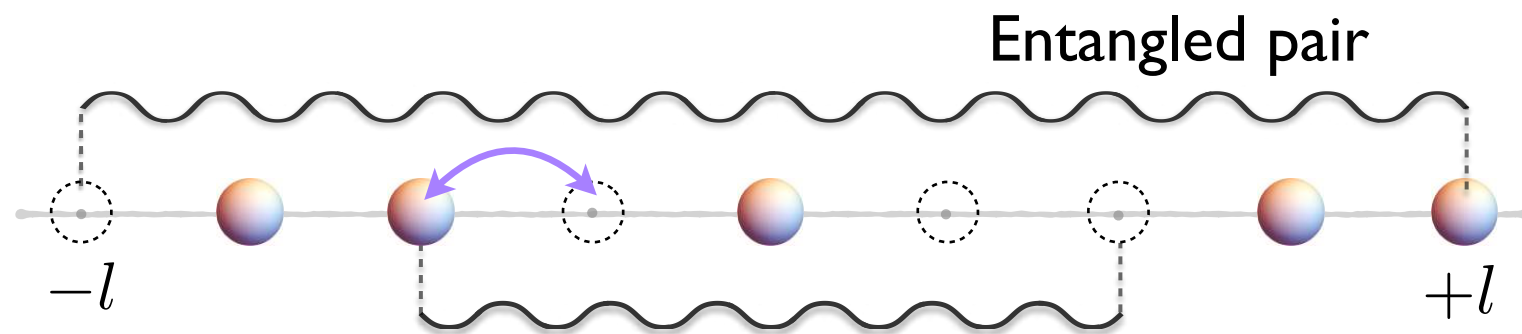
Main result

Array of coupled qubits:



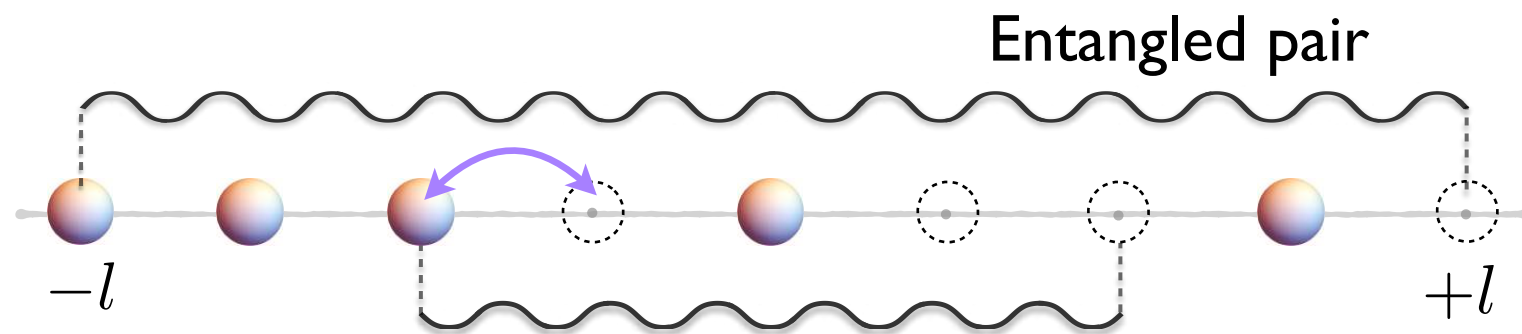
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Multiple Bell pairs:



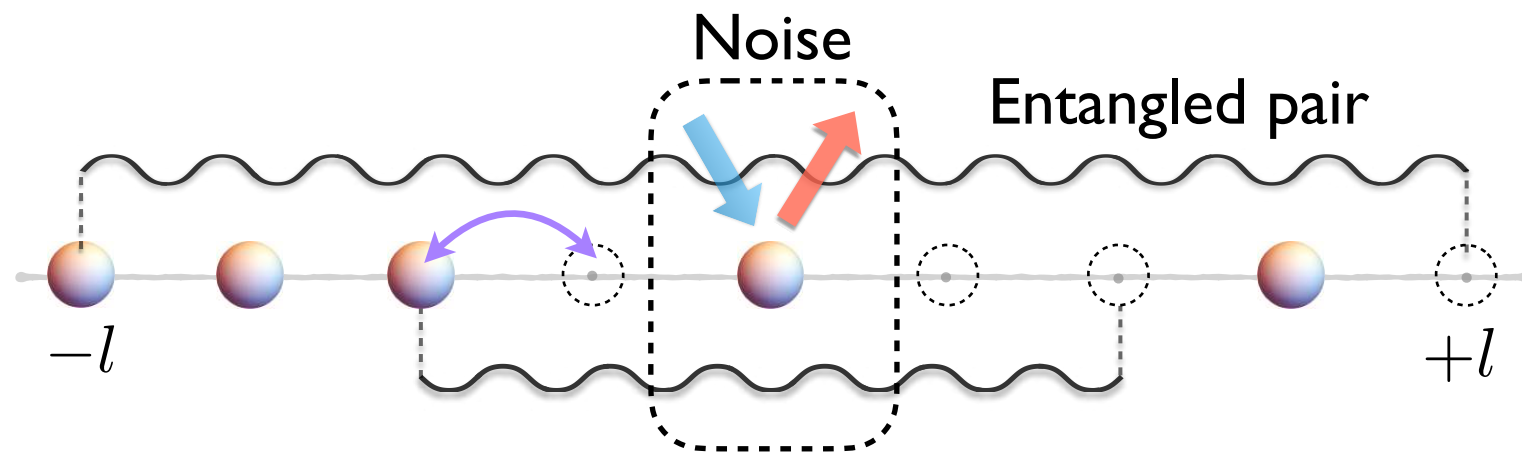
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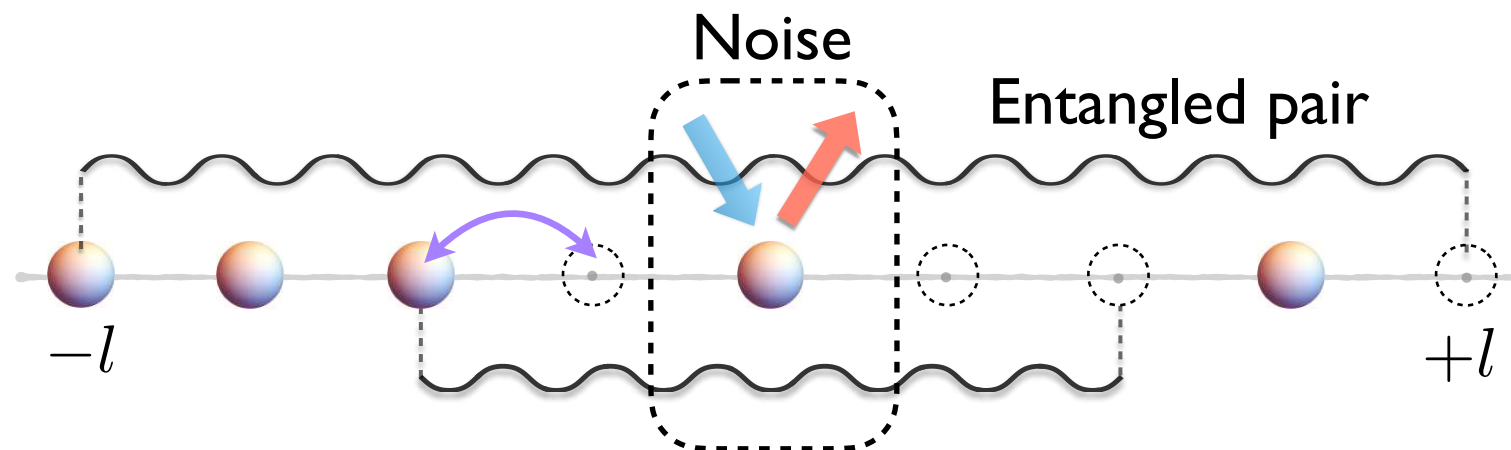
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Multiple Bell pairs **protected from noise by a symmetry:**



Main result

Multiple Bell pairs **protected from noise by a symmetry:**



Hidden symmetry could be key to more robust quantum systems,...

<https://www.cam.ac.uk/research/news/hidden-symmetry-could-be-key-to-more-robust-quantum-systems-researchers-find>

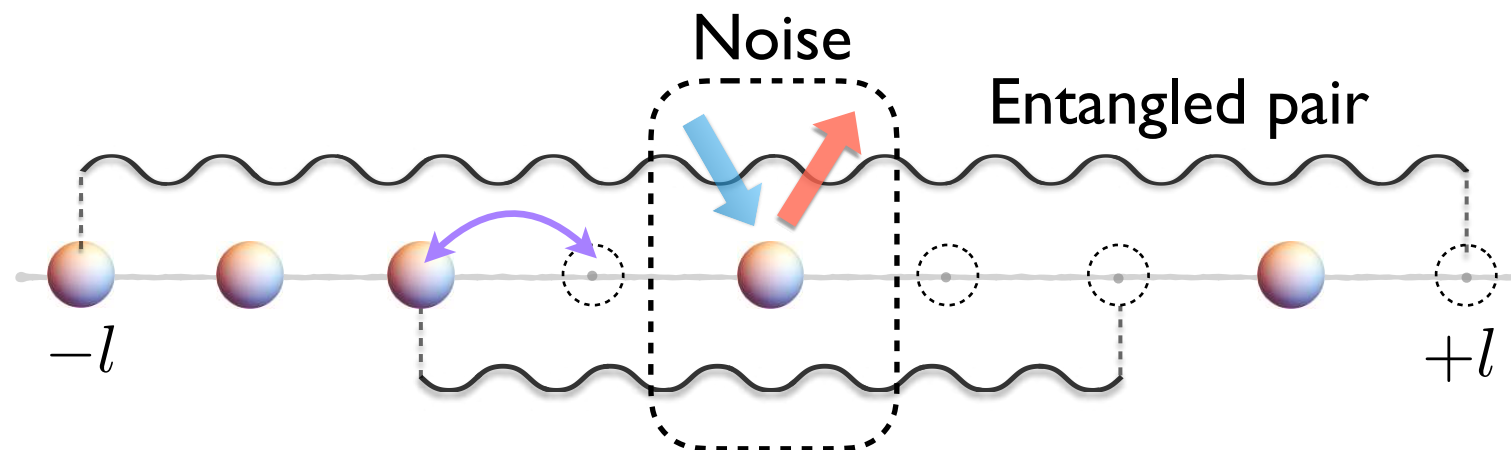
9 Dec 2020:

The researchers, from the University of Cambridge, have shown that microscopic particles can remain intrinsically linked, or entangled, over long distances even if there are random disruptions between them.

By: @cambridge_uni

Main result

Multiple Bell pairs **protected from noise by a symmetry**:



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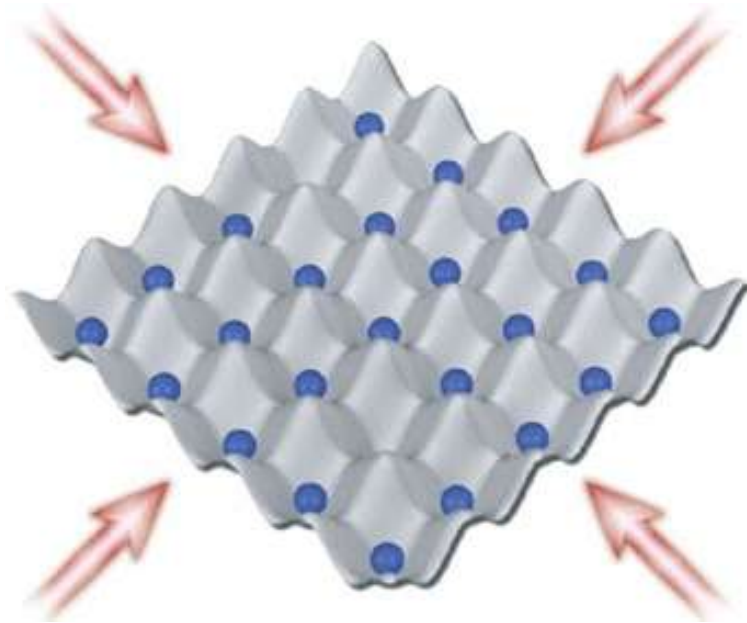
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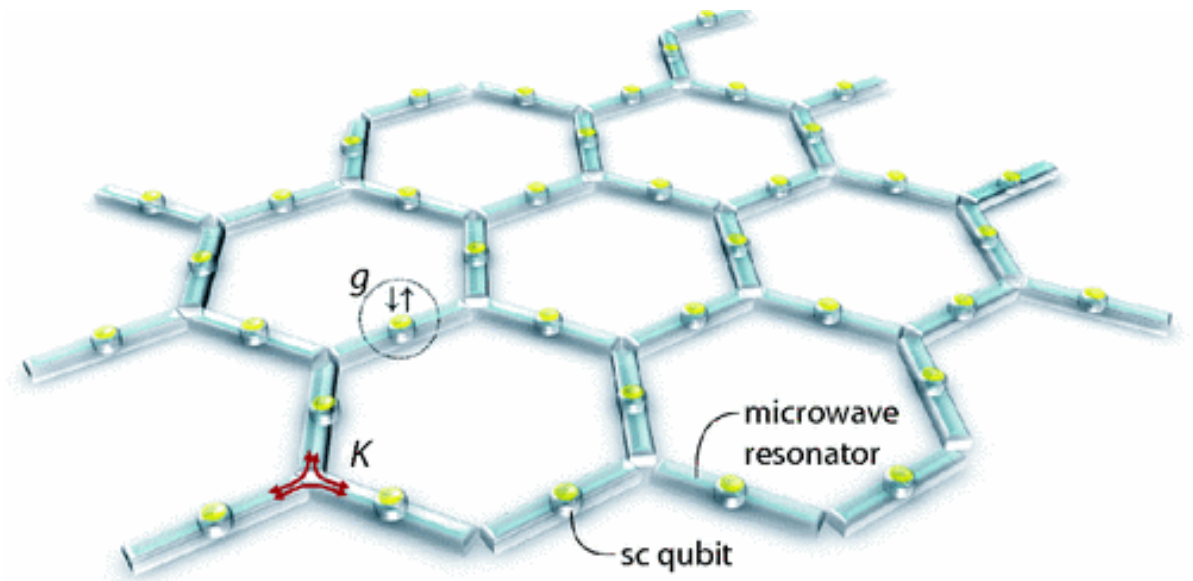
- ⇒
- Entanglement can be stable for certain types of noise
 - Symmetry can be a resource

Big picture — engineering quantum systems

Designer experimental platforms



cold atoms [Bloch, Nature '08]



superconducting circuits [Koch et al, PRA '10]

- Simulating idealised models
- Novel many-body quantum phenomena
- Quantum technologies

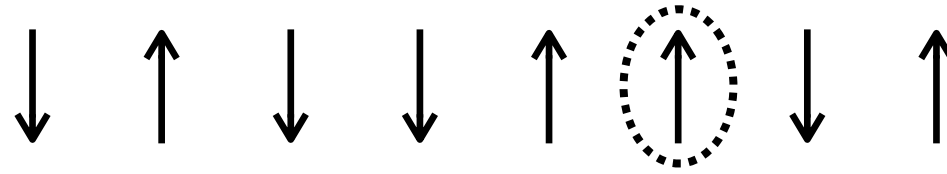
Lewenstein et al, Adv Phys '07
Müller et al, Adv At Mol Opt Phys '12
Georgescu et al, RMP '14

Long-range entanglement — resource for quantum information processing

Horodecki et al, RMP '09

Big picture — engineering open quantum systems

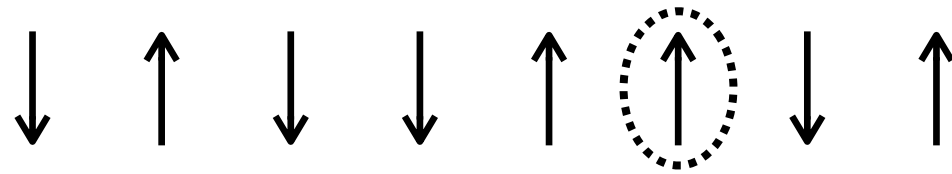
Environmental noise typically destroys quantum correlations



random spin flips $\implies$ infinite-temperature state

Big picture — engineering open quantum systems

Environmental noise typically destroys quantum correlations ...But

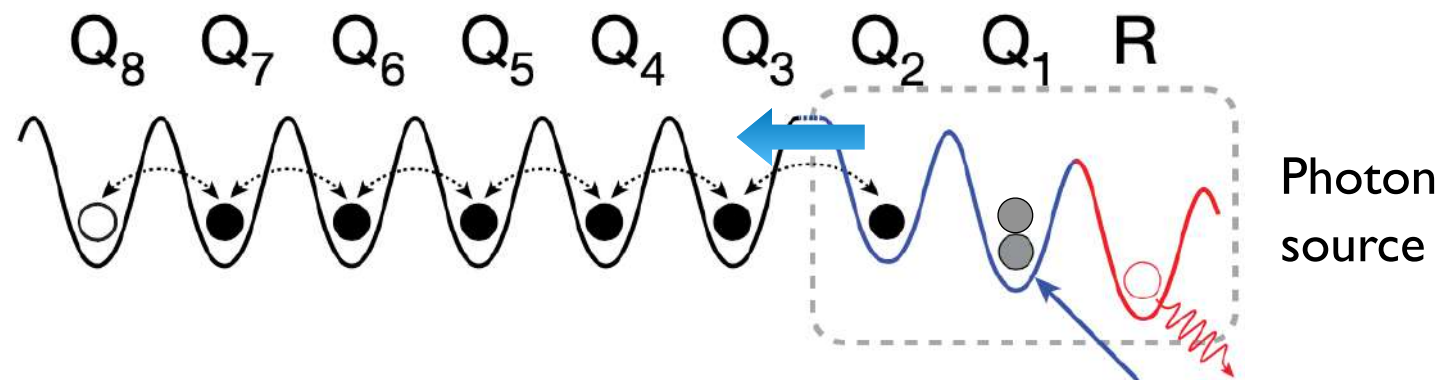


random spin flips $\Rightarrow$ infinite-temperature state

I) It is possible to engineer environment as resource

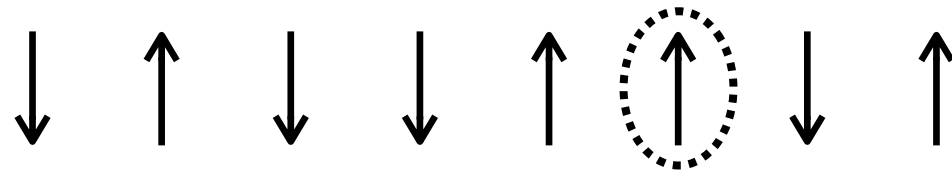
Sieberer *et al*, Rep. Prog. Phys. '16

Mott insulator
of photons
[Ma *et al*,
Nature '19]



Big picture — engineering open quantum systems

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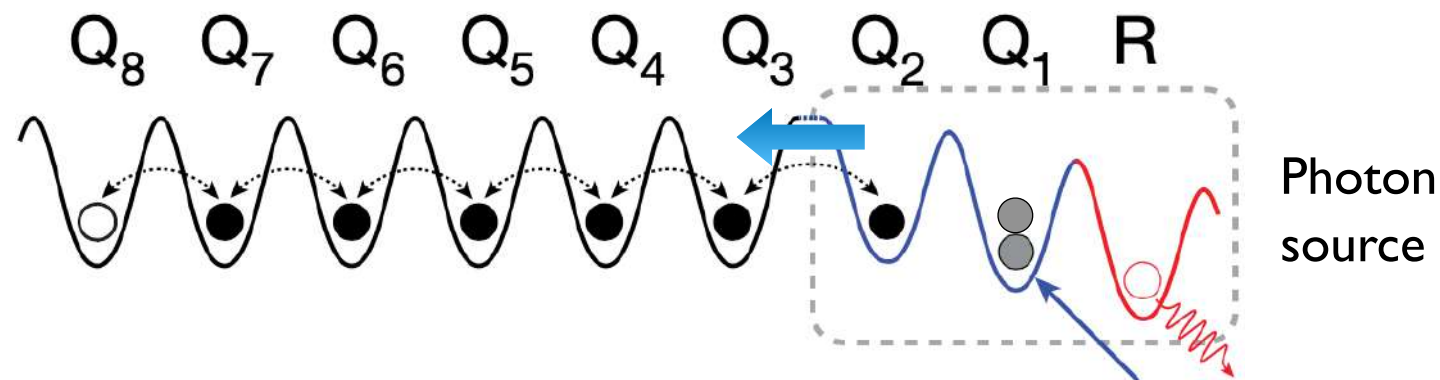


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1) It is possible to engineer environment as resource

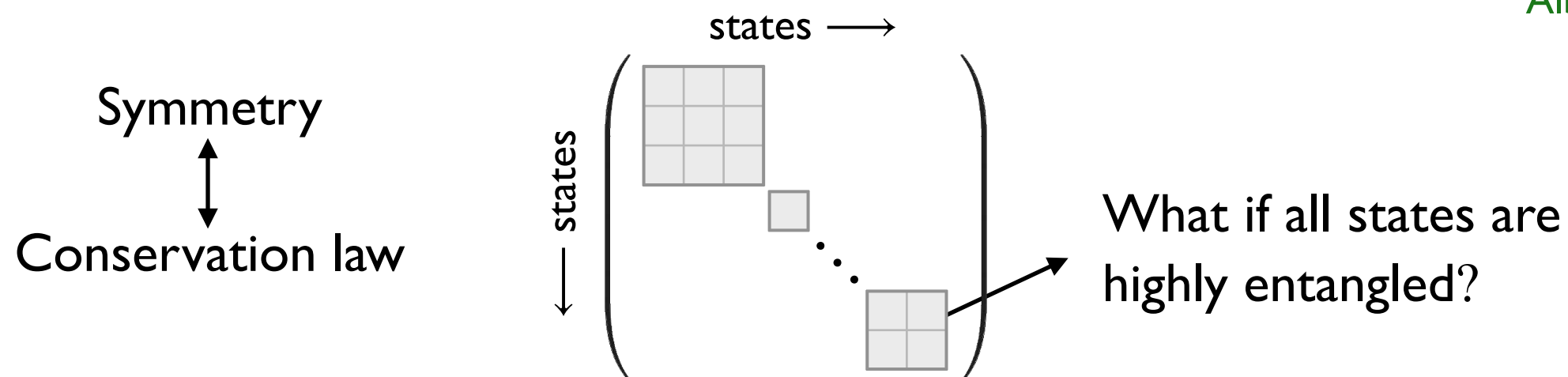
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Mott insulator
of photons
[Ma *et al*,
Nature '19]



2) **Symmetry** can lead to decoupled sectors

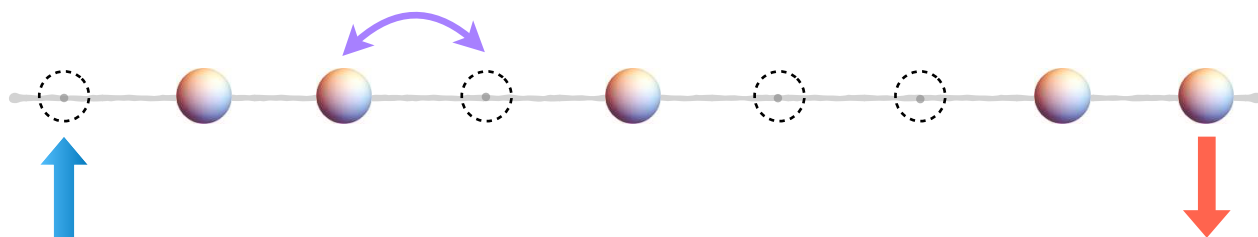
Buca, Prosen, NJP '12
Albert, Jiang, PRA '14



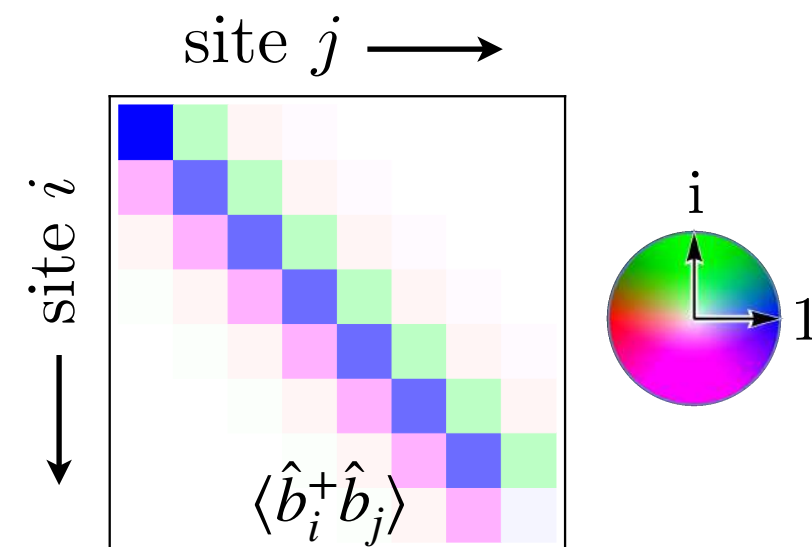
Context — steady state with pump & loss

Boundary-driven chain

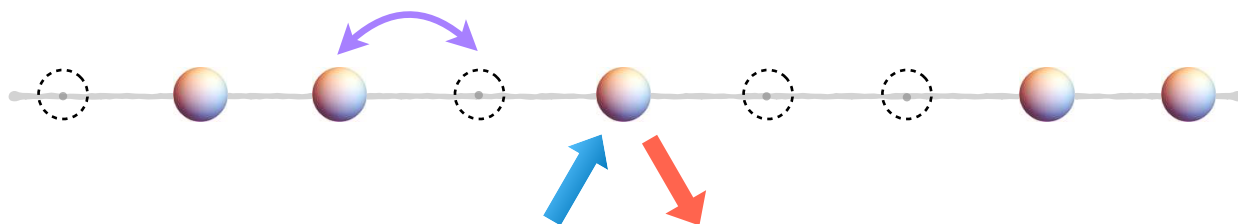
[Prosen, NJP '08; PRL '08; J Stat Mech '17]



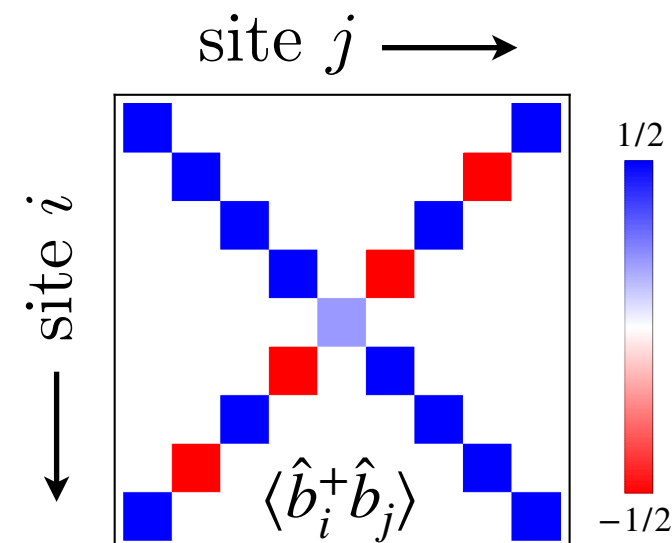
- Dynamics reduce to non-interacting fermions
- Unique steady state w/ short-range correlations



Here: Pump & loss not on the boundaries



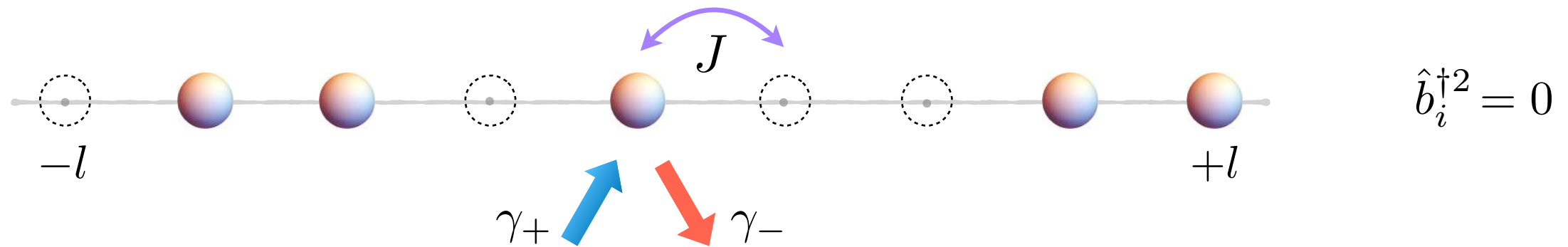
- Does not reduce to non-interacting system
 - Multiple steady states (hidden symmetry)
 - Stable long-range entanglement
- Rare!



Model

Model — centre-driven qubit array

“Hard-core bosons” with localised pump and loss



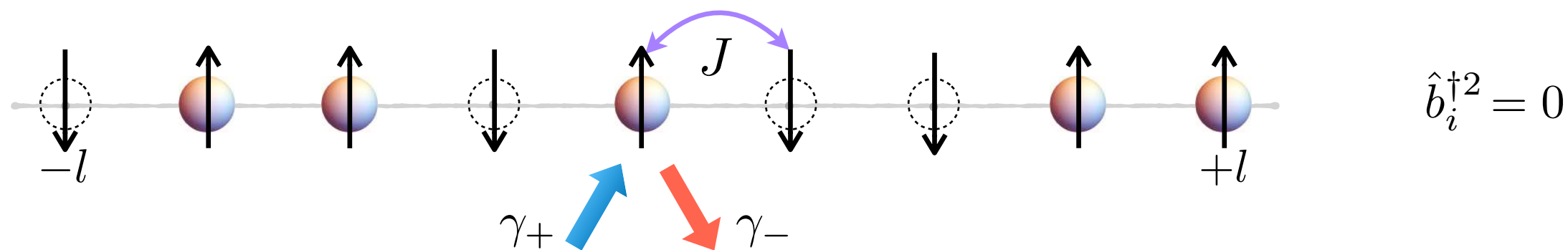
$$\hat{H} = -J \sum_{i=-l}^{l-1} (\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i)$$

Master equation for a Markovian environment:

$$\left. \begin{aligned} \frac{d\hat{\rho}}{dt} &= -i [\hat{H}, \hat{\rho}] + \sum_{\alpha} \hat{L}_{\alpha} \hat{\rho} \hat{L}_{\alpha}^{\dagger} - \{\hat{L}_{\alpha}^{\dagger} \hat{L}_{\alpha}, \hat{\rho}\} / 2 \\ \hat{L}_{+} &= \sqrt{\gamma_{+}} \hat{b}_0^{\dagger}, \quad \hat{L}_{-} = \sqrt{\gamma_{-}} \hat{b}_0 \quad \text{(Lindblad)} \end{aligned} \right] \begin{array}{l} \text{Not} \\ \text{essential} \end{array}$$

Model — centre-driven qubit array

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$$\hat{H} = -J \sum_{i=-l}^{l-1} (\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i) \quad \longleftrightarrow \quad -J \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

Master equation for a Markovian environment:

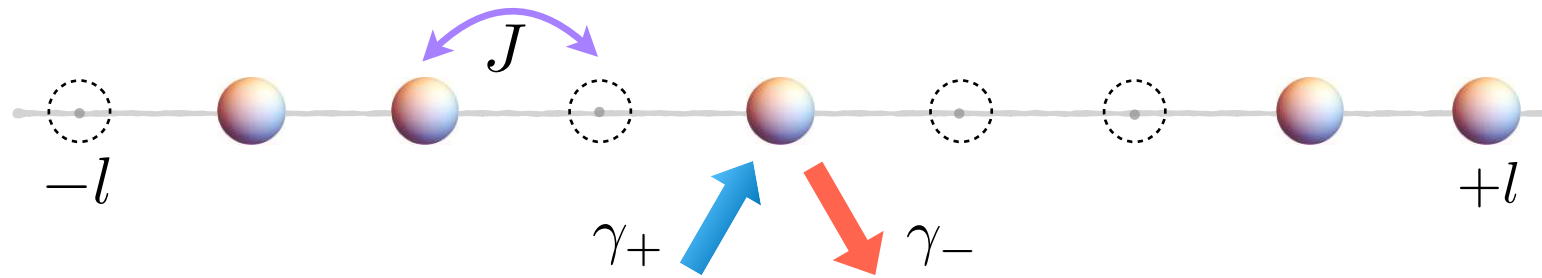
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$$\hat{L}_+ = \sqrt{\gamma_+} \hat{b}_0^{\dagger}, \quad \hat{L}_- = \sqrt{\gamma_-} \hat{b}_0 \quad (\text{Lindblad})$$

Equivalent to XY spin-1/2 chain with spin-flip “errors”

- Two-component bosons in a deep lattice [Duan, Demler, Lukin PRL '03]
- Spin-orbit coupled fermions [Mamaev, Kimchi, Nandkishore, Rey '20]
- Rydberg arrays [Browaeys, Lahaye Nat. Phys. '20]

Distinction from non-interacting fermions



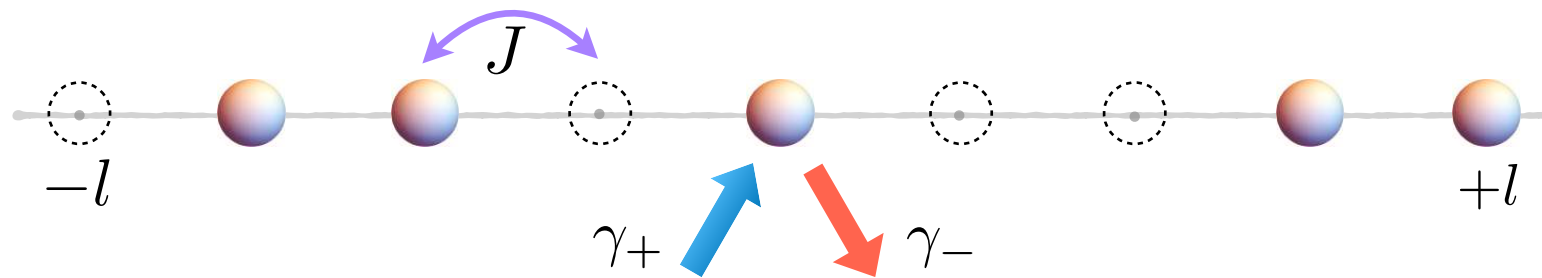
$$\hat{H} = -J \sum_i \hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.} \quad \hat{L}_+ = \sqrt{\gamma_+} \hat{b}_0^\dagger, \quad \hat{L}_- = \sqrt{\gamma_-} \hat{b}_0$$

Jordan-Wigner transformation:

$$\hat{f}_j = (-1)^{\sum_{i < j} \hat{n}_i} \hat{b}_j \quad [\hat{n}_i := \hat{b}_i^\dagger \hat{b}_i = \hat{f}_i^\dagger \hat{f}_i]$$

$\Rightarrow$ Hamiltonian maps to free fermions $\hat{H} = -J \sum_i \hat{f}_i^\dagger \hat{f}_{i+1} + \text{H.c.}$

Distinction from non-interacting fermions



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⇒ Hamiltonian maps to free fermions $\hat{H} = -J \sum_i \hat{f}_i^\dagger \hat{f}_{i+1} + \text{H.c.}$

But pump/loss mediates interactions

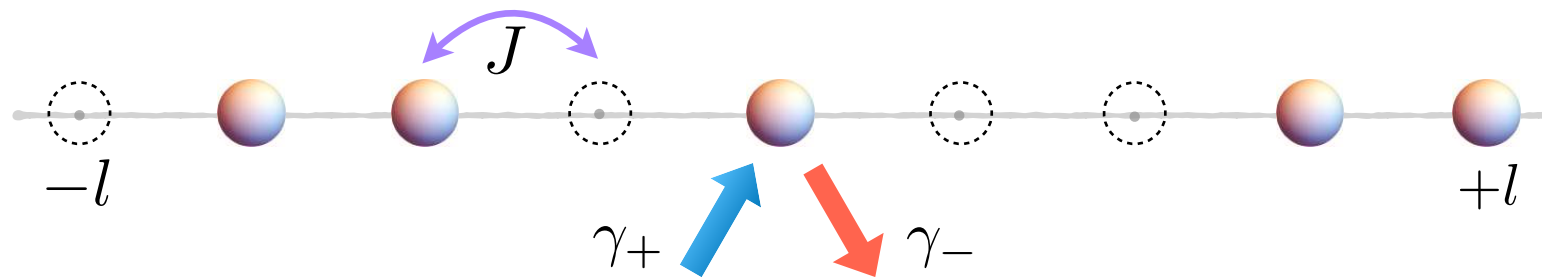
$$\hat{L}_+ := \sqrt{\gamma_+} \hat{b}_0^\dagger = \sqrt{\gamma_+} (-1)^{\sum_{i<0} \hat{n}_i} \hat{f}_0^\dagger \quad (\neq \hat{f}_0^\dagger)$$

$$\hat{L}_- := \sqrt{\gamma_-} \hat{b}_0 = \sqrt{\gamma_-} (-1)^{\sum_{i<0} \hat{n}_i} \hat{f}_0 \quad (\neq \hat{f}_0)$$

⇒ Dynamics strongly nonlinear

Symmetry

Hidden symmetry



$$\hat{L}_+ = \sqrt{\gamma_+} \hat{b}_0^\dagger$$

$$\hat{L}_- = \sqrt{\gamma_-} \hat{b}_0$$

Symmetry operator

$$\hat{C} := -1/2 + \sum_{i=-l}^l \hat{f}_i^\dagger \hat{f}_{-i}$$

$\hat{C}^2$ is a “strong” symmetry:

Strong/weak symmetries:
Buca, Prosen, NJP '12

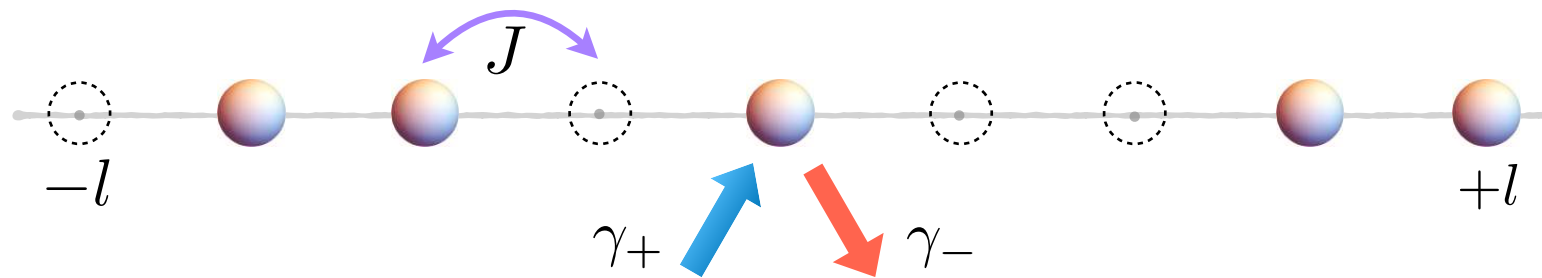
- $[\hat{H}, \hat{C}] = 0 \implies$ hopping J does not affect $\hat{C}$ (or $\hat{C}^2$)

- $\hat{C} = (-1)^{\hat{n}_0} \text{func}(\{\hat{b}_j, \hat{b}_j^\dagger : j \neq 0\}) \implies [\hat{b}_0^{(\dagger)}, \hat{C}^2] = 0$

$$\left[\hat{f}_j = (-1)^{\sum_{i<j} \hat{n}_i} \hat{b}_j \right]$$

$\implies \hat{C}^2$ is independent of the centre site

Hidden symmetry



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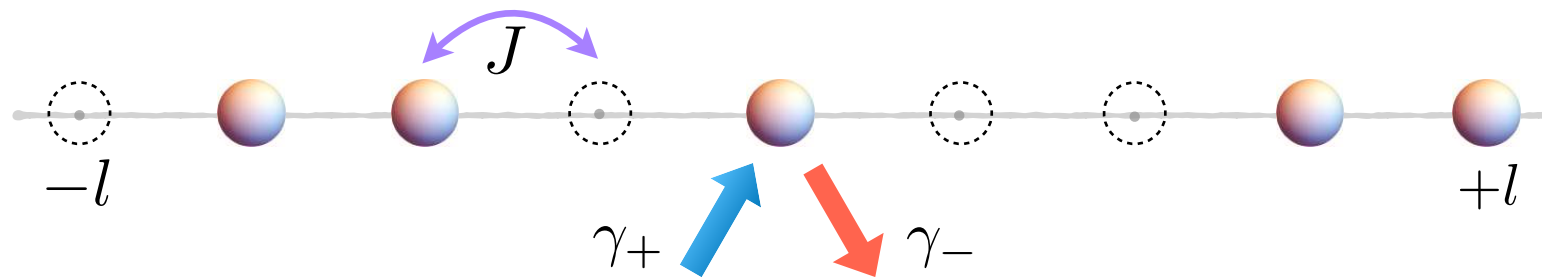
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$\implies \hat{C}^2$ is independent of the centre site

$\therefore$ Any coupling to centre site conserves $\hat{C}^2$

Dynamics split into different sectors of $\hat{C}^2 \Rightarrow$ multiple steady states

Hidden symmetry



$$\hat{L}_+ = \sqrt{\gamma_+} \hat{b}_0^\dagger$$

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 $\implies \hat{C}^2$ is independent of the centre site

$\therefore$ Any coupling to centre site conserves $\hat{C}^2 \leftrightarrow$ number of Bell pairs

Dynamics split into different sectors of $\hat{C}^2 \Rightarrow$ multiple steady states

Interpretation of symmetry — Bell pairs

$$\hat{C} = -1/2 + \hat{n}_0 + \sum_{i=1}^l \hat{a}_{i,+}^\dagger \hat{a}_{i,+} - \hat{a}_{i,-}^\dagger \hat{a}_{i,-} \qquad \hat{a}_{i,\pm} := \frac{1}{\sqrt{2}}(\hat{f}_i \pm \hat{f}_{-i})$$

$$\hat{a}_{i,\pm}^\dagger |0 \dots 0\rangle \sim |0_i 1_{-i}\rangle \pm |1_i 0_{-i}\rangle \text{ — Bell pair at sites } i \text{ and } -i$$

Interpretation of symmetry — Bell pairs

$$\hat{C} = -1/2 + \hat{n}_0 + \underbrace{\sum_{i=1}^l \hat{a}_{i,+}^\dagger \hat{a}_{i,+} - \hat{a}_{i,-}^\dagger \hat{a}_{i,-}}_{\text{Total "charge" } \nu = 0, \pm 1, \dots, \pm l} \quad \hat{a}_{i,\pm} := \frac{1}{\sqrt{2}}(\hat{f}_i \pm \hat{f}_{-i})$$

Total “charge” $\nu = 0, \pm 1, \dots, \pm l$

$\hat{a}_{i,\pm}^\dagger |0 \dots 0\rangle \sim |0_i 1_{-i}\rangle \pm |1_i 0_{-i}\rangle$ — Bell pair at sites i and $-i$ with “charge” ± 1

Interpretation of symmetry — Bell pairs

$$\hat{C} = -1/2 + \hat{n}_0 + \underbrace{\sum_{i=1}^l \hat{a}_{i,+}^\dagger \hat{a}_{i,+} - \hat{a}_{i,-}^\dagger \hat{a}_{i,-}}_{\text{Total “charge” } \nu = 0, \pm 1, \dots, \pm l} \qquad \hat{a}_{i,\pm} := \frac{1}{\sqrt{2}}(\hat{f}_i \pm \hat{f}_{-i})$$

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$\hat{C}^2$ has eigenvalues $\Lambda = (\nu + n_0 - 1/2)^2$

$\implies l + 1$ distinct sectors $\Lambda \in \{(\eta + 1/2)^2 : \eta = 0, \dots, l\}$

$\eta = 0$: minimally entangled

$\eta = l$: maximally entangled

Steady states

Steady states

Unique steady state in each sector

$$\hat{\rho}_\eta = \text{const.} (\gamma_+/\gamma_-)^{\hat{N}} \hat{P}_\eta$$

$\left[\begin{array}{l} \hat{N}: \text{total particle number} \\ \hat{P}_\eta: \text{projector onto sector } \eta \end{array} \right.$

— Fully mixed + chemical potential $\mu = \ln(\gamma_+/\gamma_-)$ **within a given sector!**

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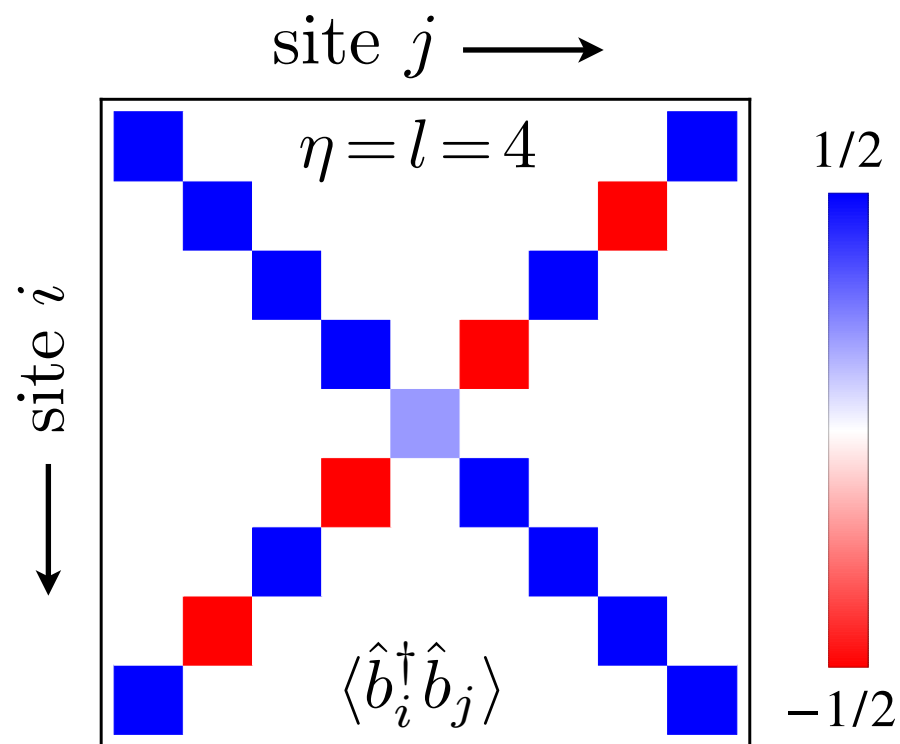
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example: $\eta = l$



Maximally entangled sector

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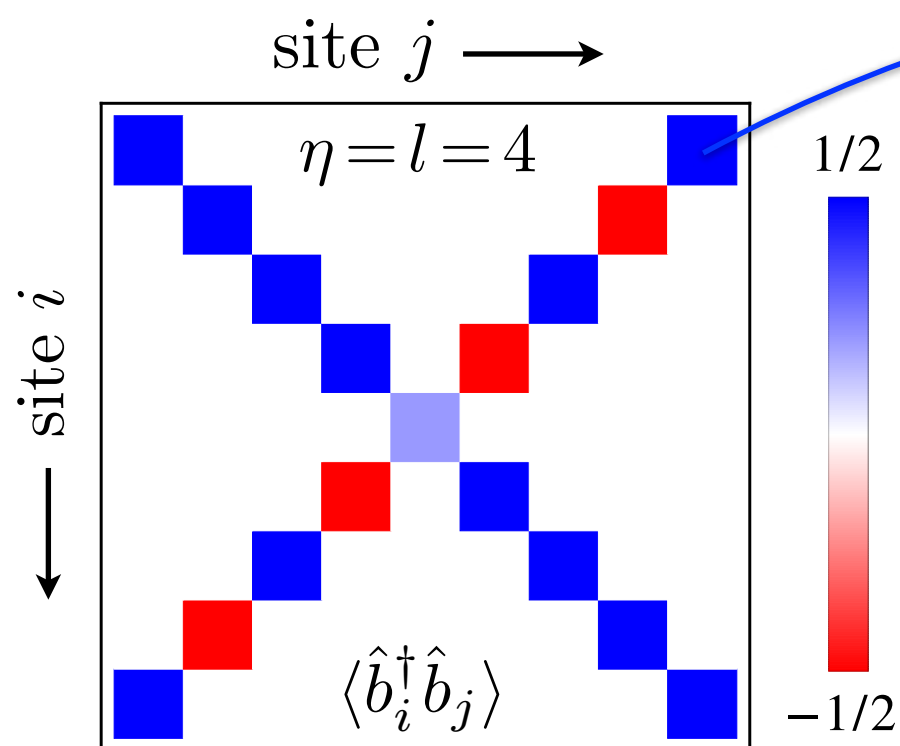
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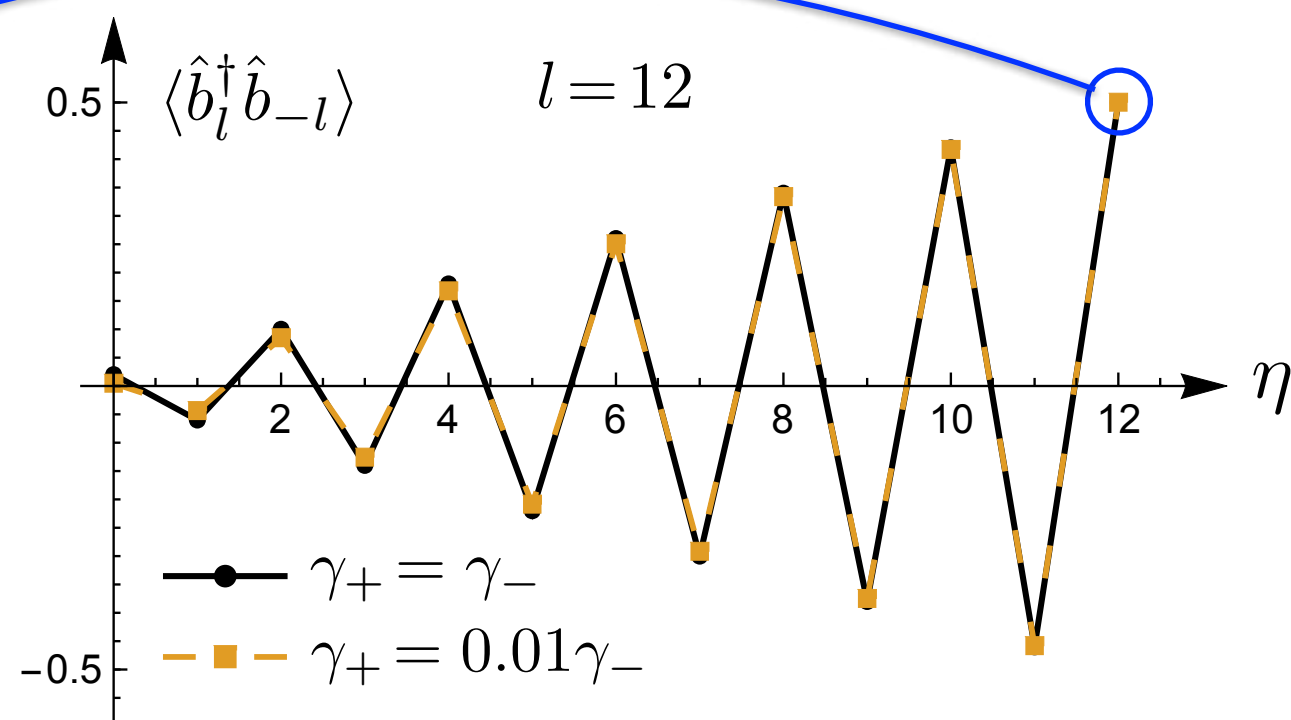
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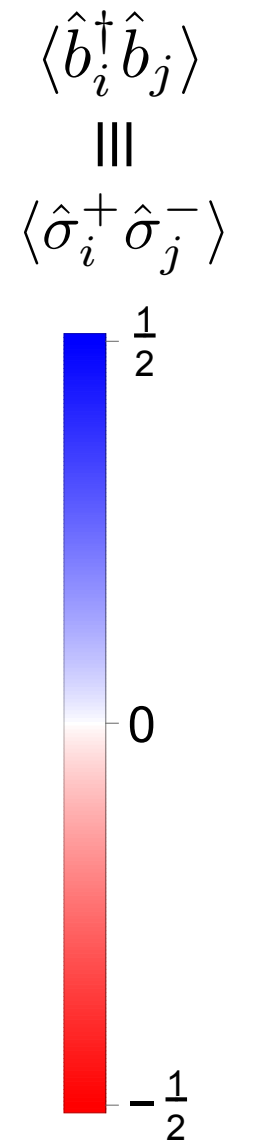
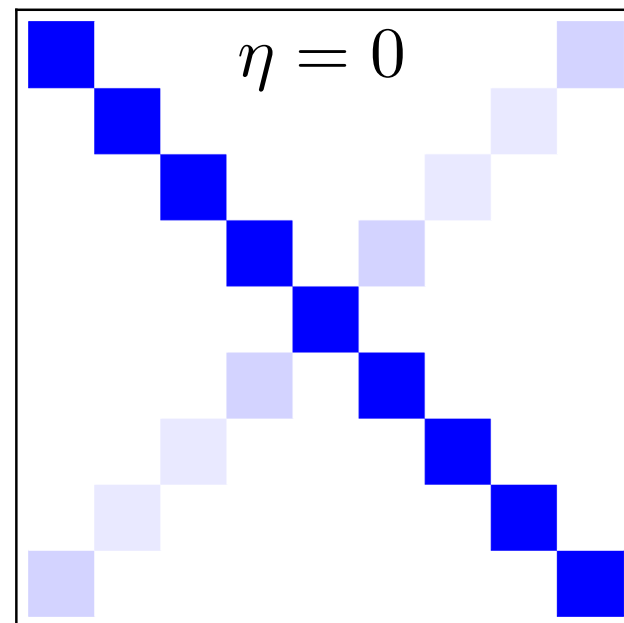
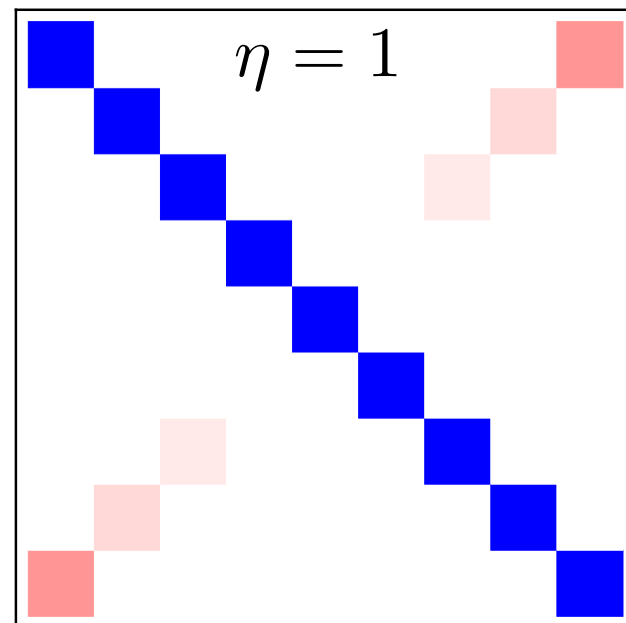
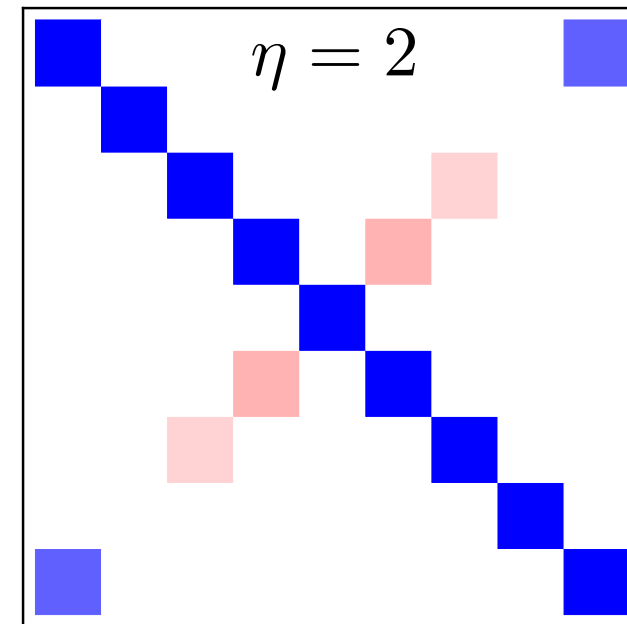
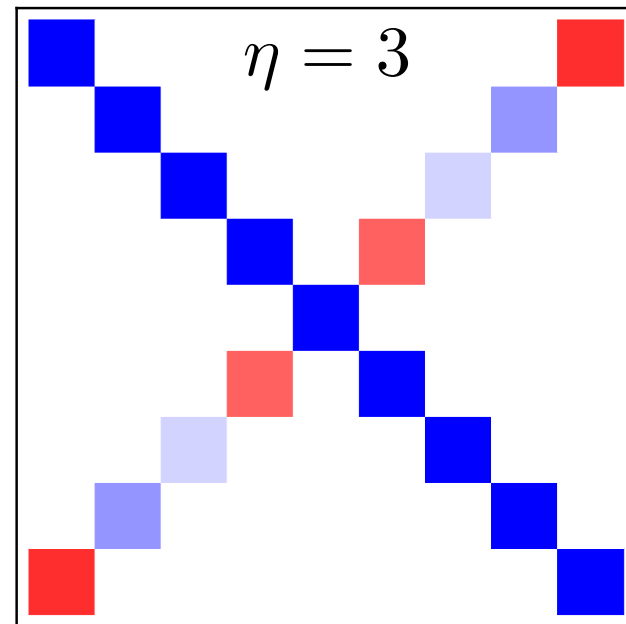
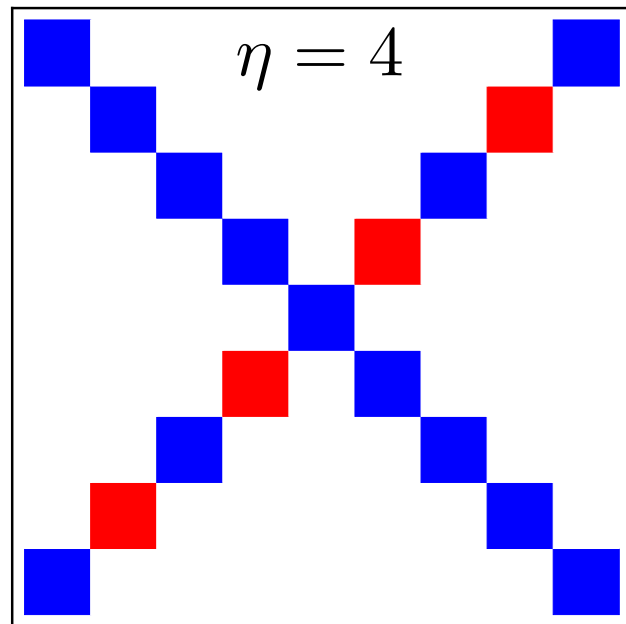
Maximally entangled sector



End-to-end correlation in different sectors

Steady states

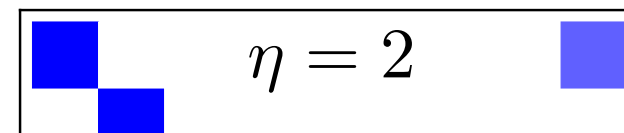
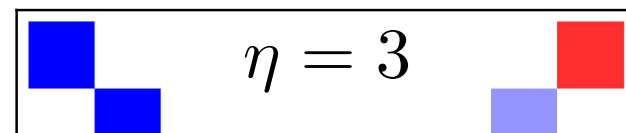
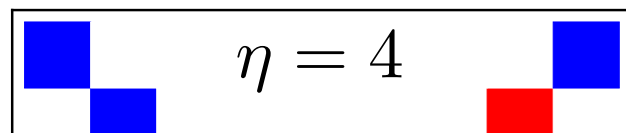
$$l = 4, \gamma_+ = \gamma_-$$



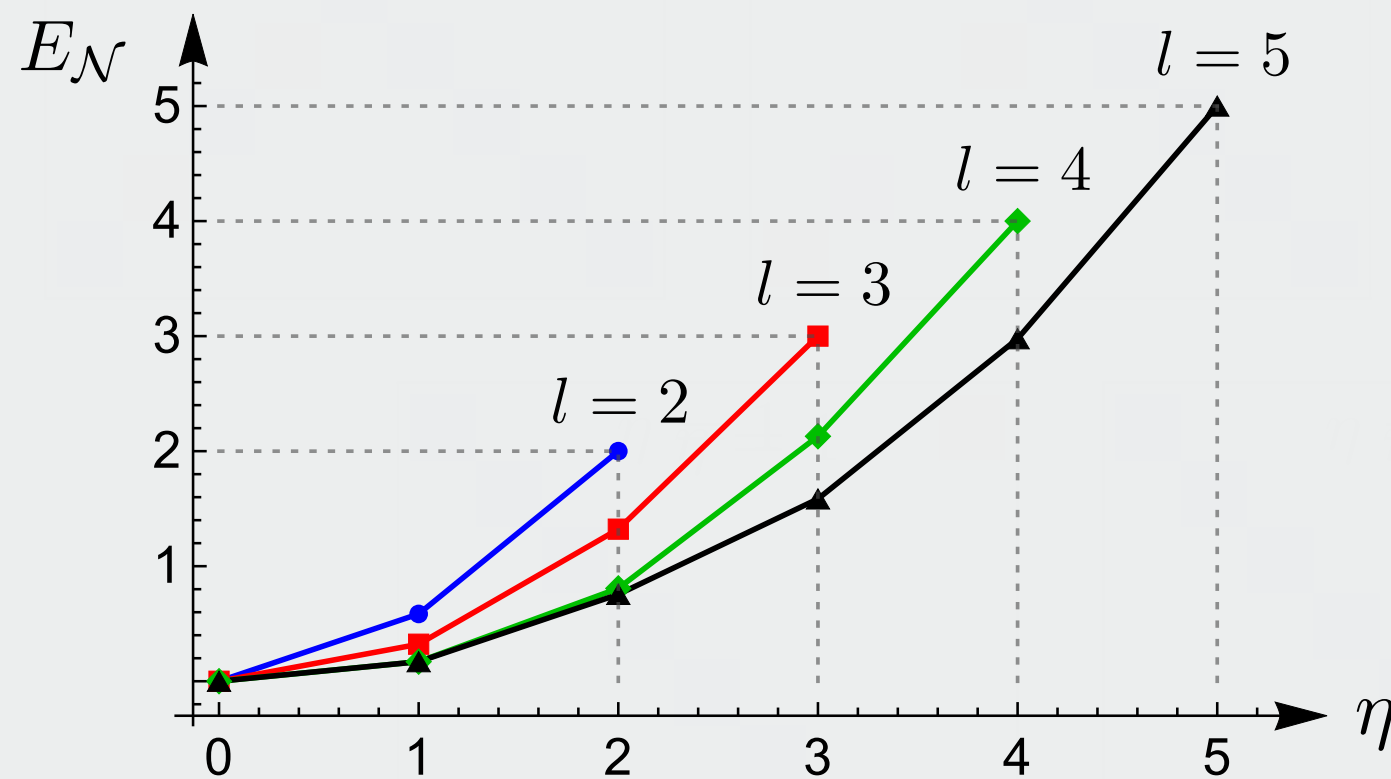
Similar signatures in density-density correlations $\equiv \langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle$

Steady states

$$l = 4, \gamma_+ = \gamma_-$$



Entanglement



Log negativity E_N :
entanglement between left
and right halves (max number
of distillable Bell pairs)



Similar signatures in density-density correlations $\equiv \langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle$

Selective Preparation

How to prepare

If initial state has sector weights w_η , final state $\hat{\rho}_\infty = \sum_\eta w_\eta \hat{\rho}_\eta$

— how to prepare a given sector?

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Exact solution using engineered loss: [PRL 125, 240404 \(2020\)](#)

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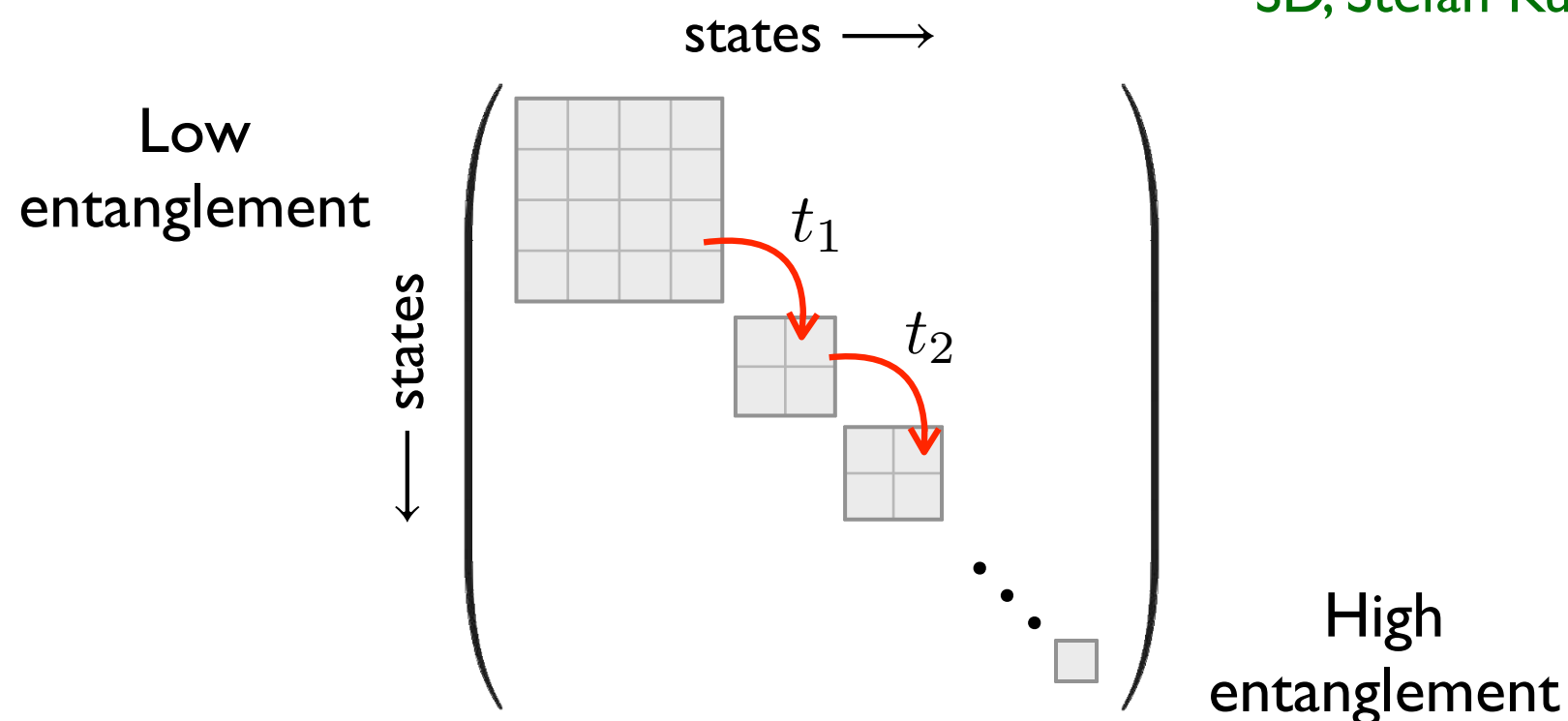
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Here, approximate solution — pump C using timed pulses

[SD, Stefan Kuhr, & Nigel Cooper, in prep](#)



Generalises to other symmetries: e.g., η pairing Hubbard model

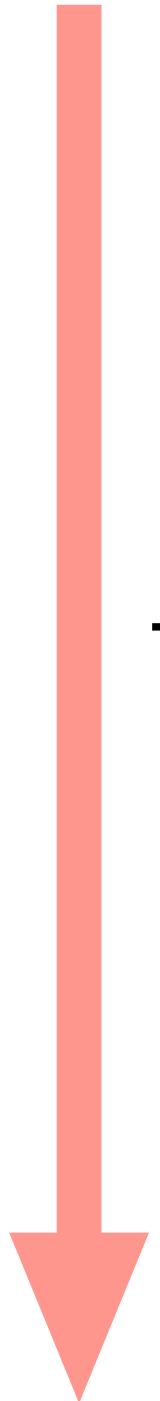
How to prepare — schematic

$$\hat{C} = \hat{\sigma}_0^z/2 + \sum_{i \neq 0} \hat{f}_i^\dagger \hat{f}_{-i}$$

$$\text{●} \equiv \uparrow \quad \text{○} \equiv \downarrow$$

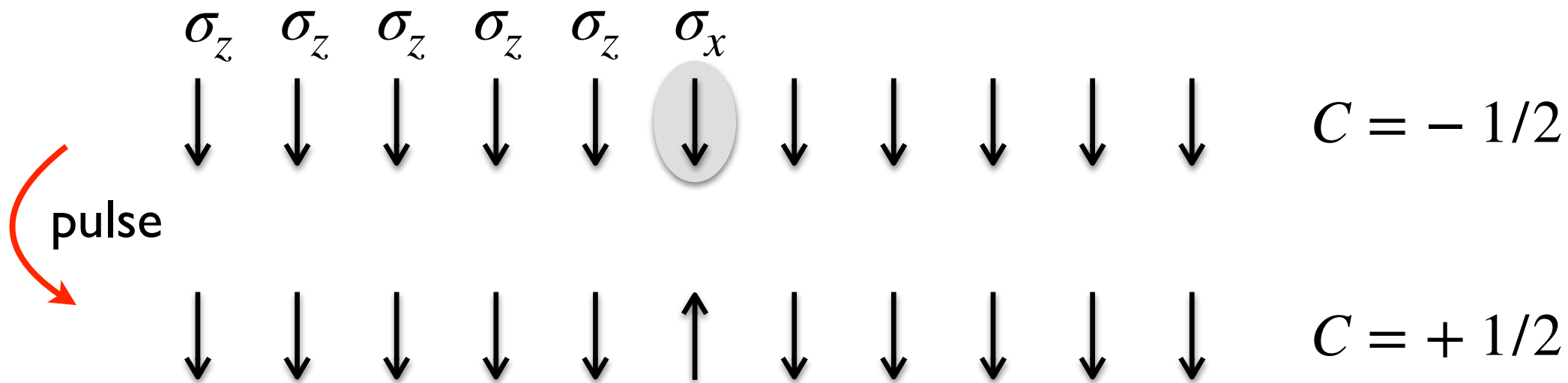
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ $C = -1/2$

Time



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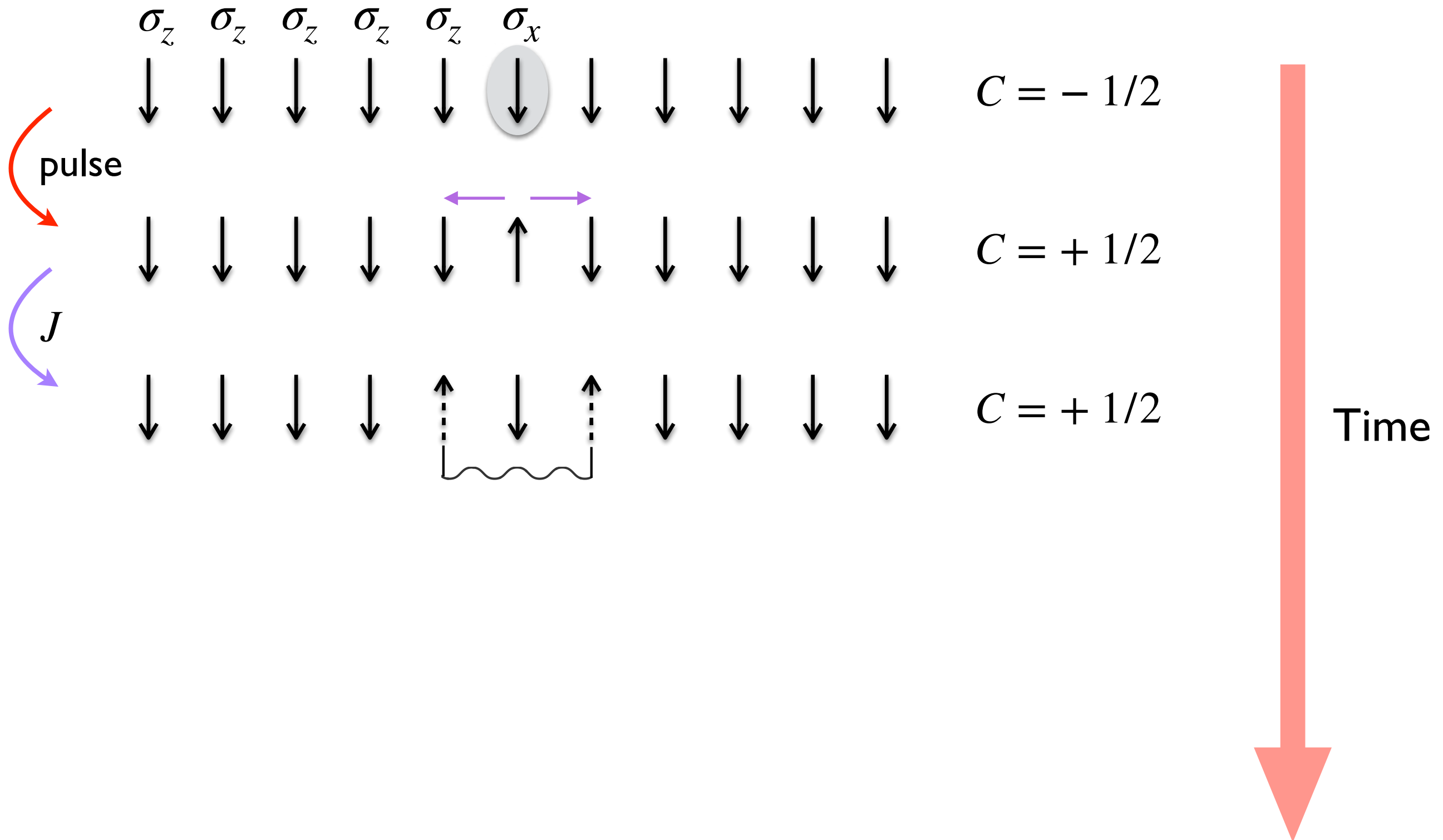


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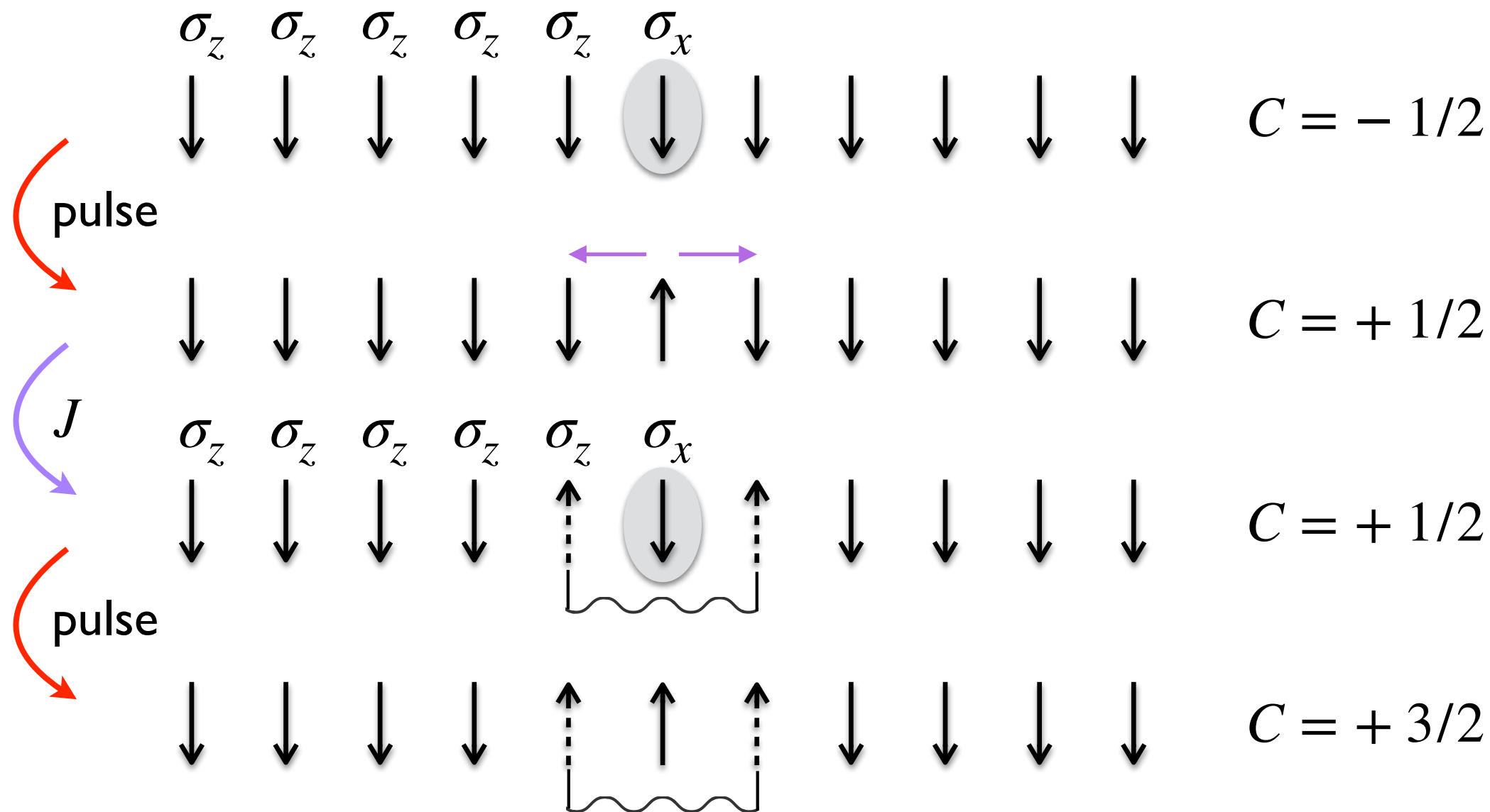
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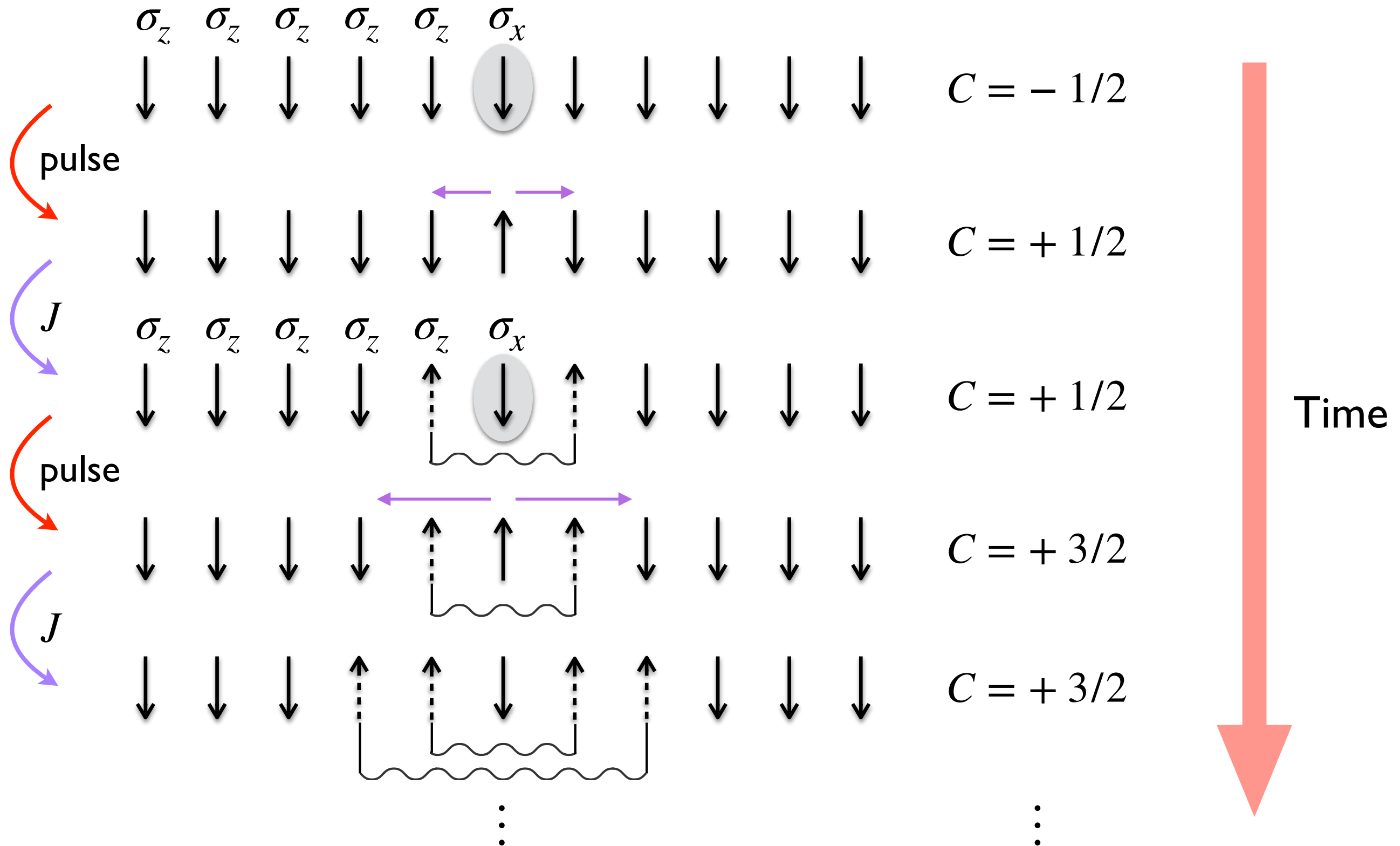
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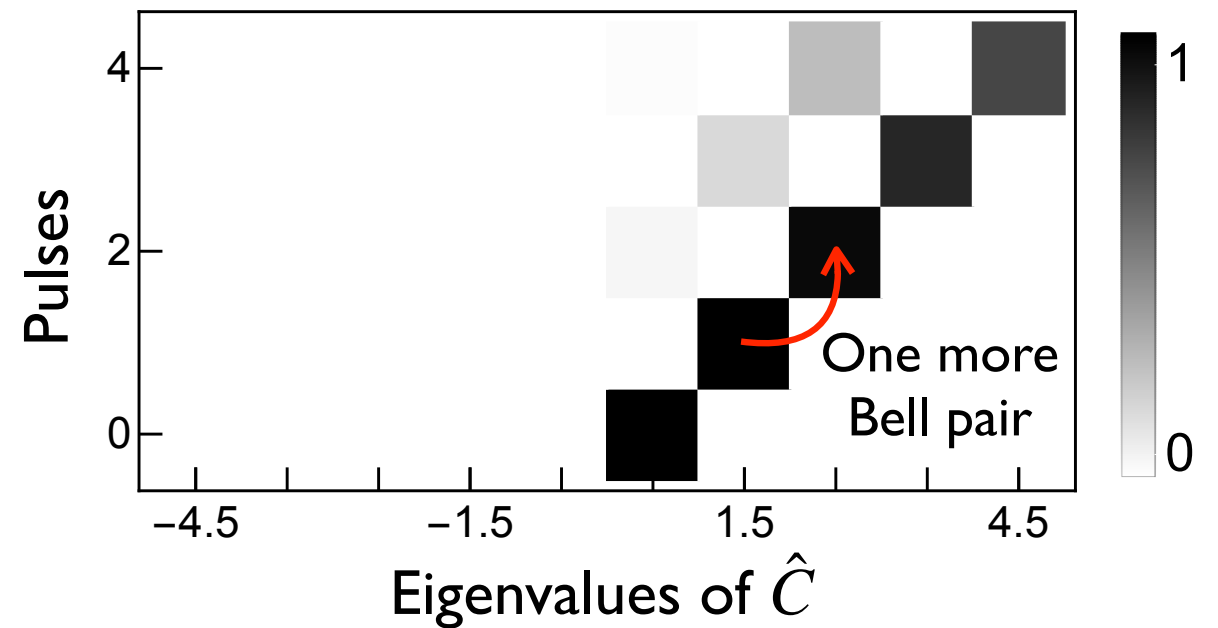
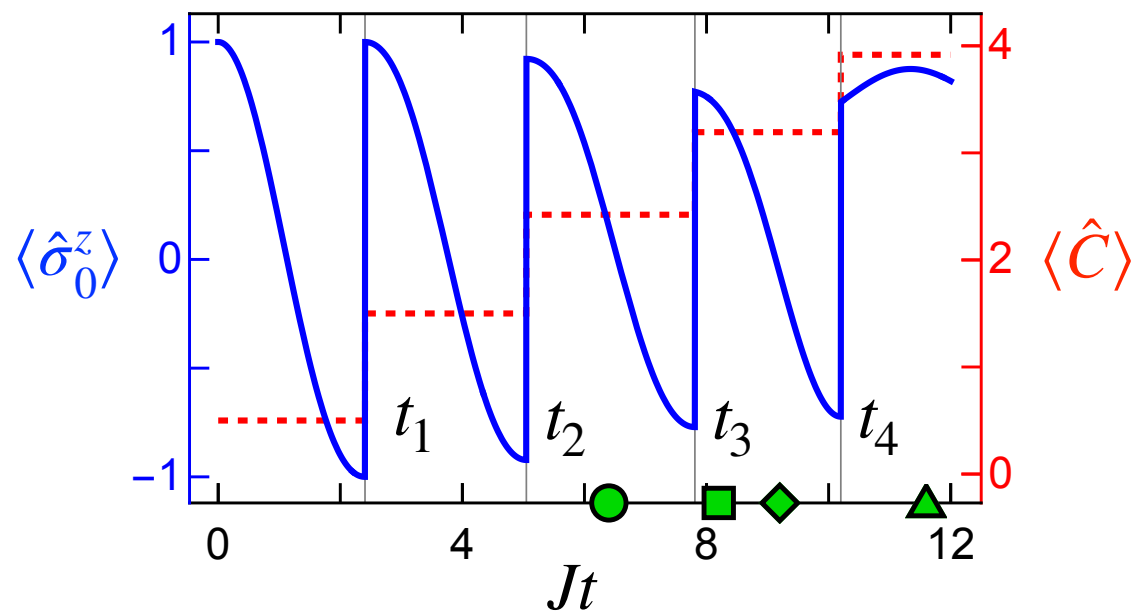
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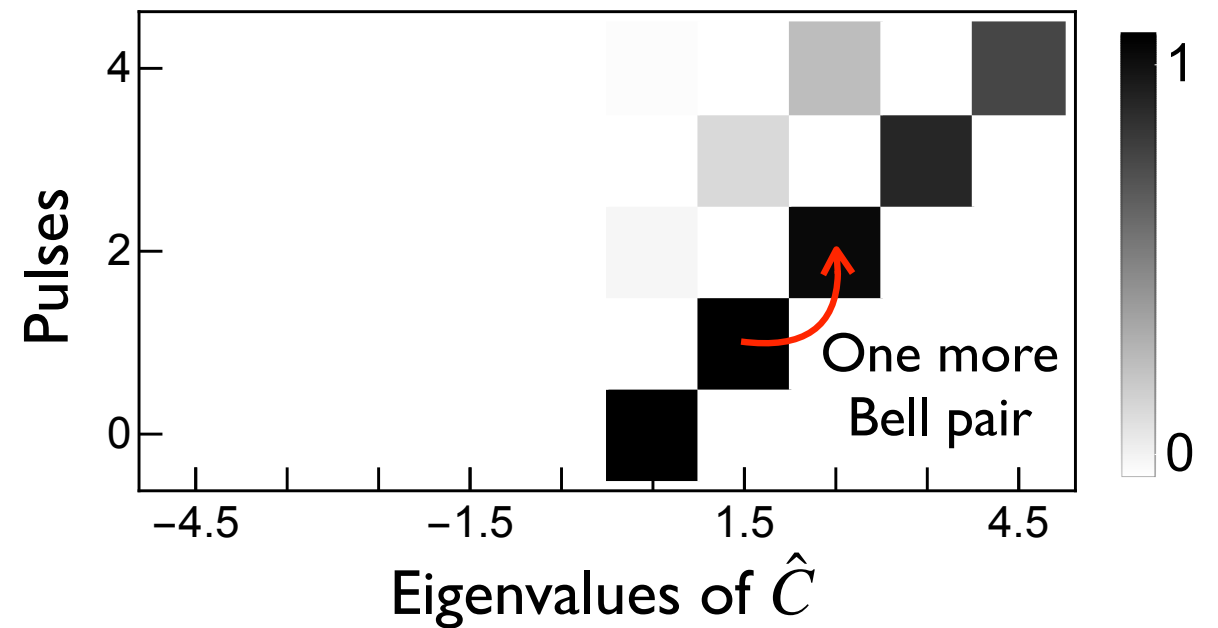
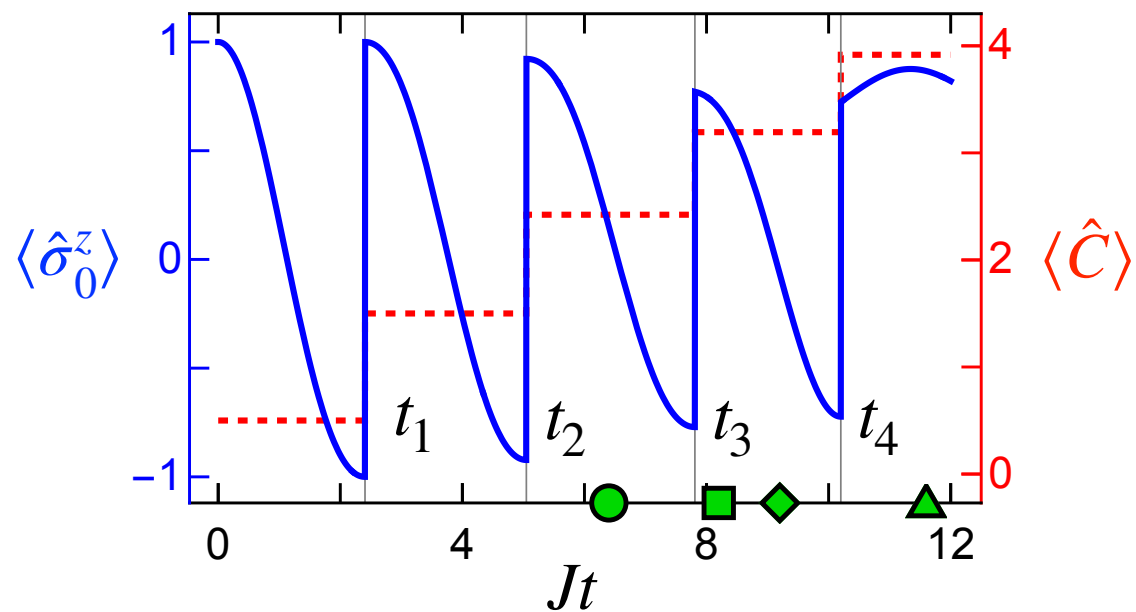
How to prepare — exact numerics

Pulse whenever centre site is mostly $\downarrow$, up to l times

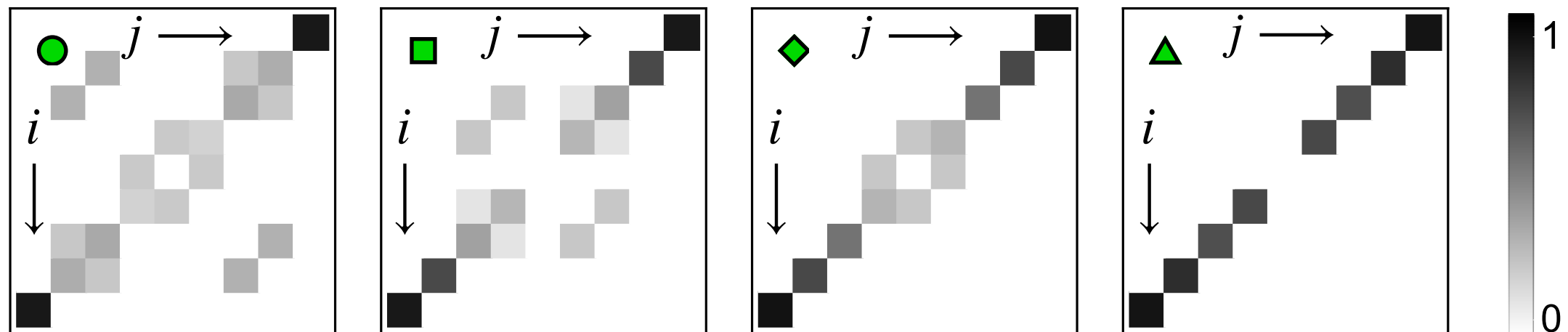


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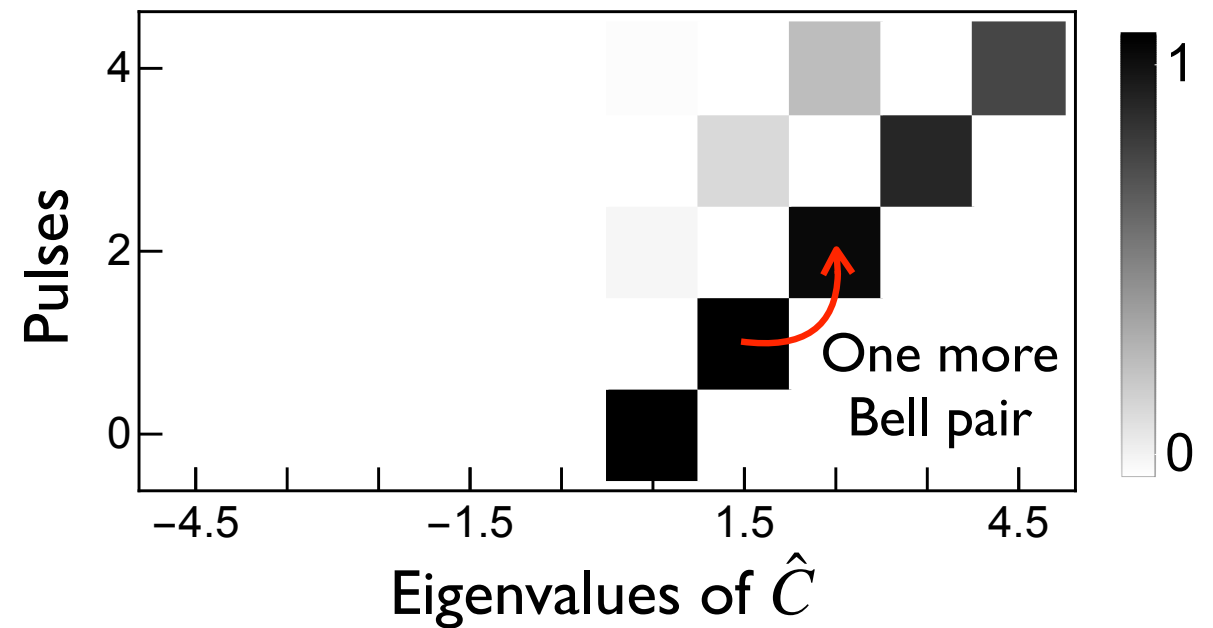
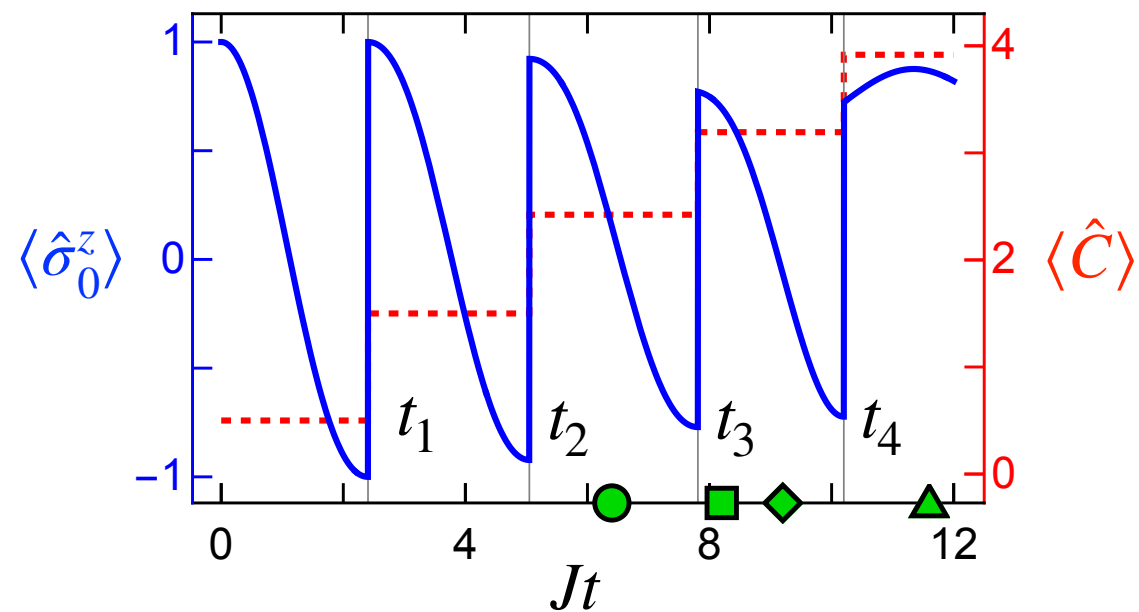


Concurrences $\mathcal{C}_{i,j}$ show Bell pairs are successively stacked inward
(same reflected in two-site correlations)

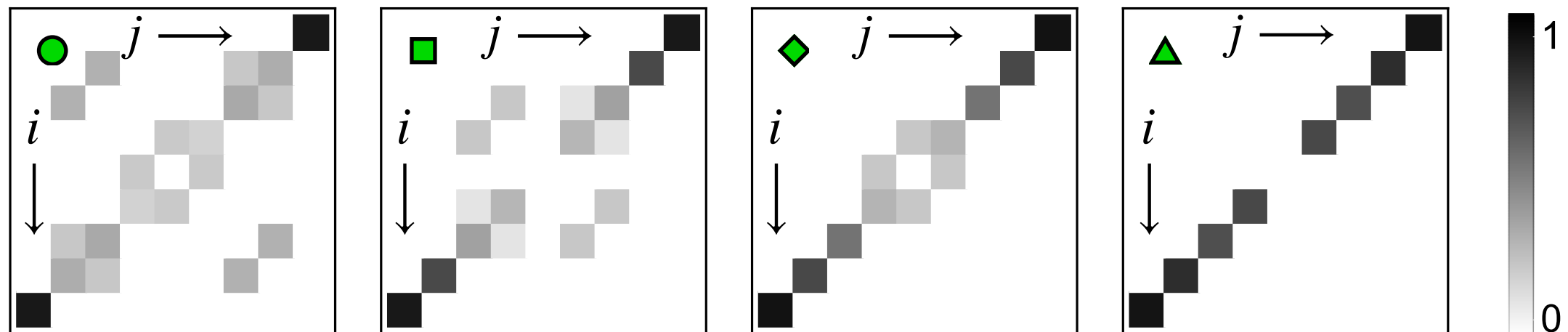


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Number of Bell pairs and preparation time both linear in l

Robustness

$\hat{C}^2$ remains a symmetry for

- Hamiltonians quadratic in the JW fermions and reflection symmetric
 - symmetric traps, XY spin-1/2 chain
 - transverse-field Ising model $\leftrightarrow$ Kitaev chain
- Periodic boundary conditions
- Any coupling at the centre site
- Any function of “pair occupations” $\hat{n}_k + \hat{n}_{-k}$
- . . .

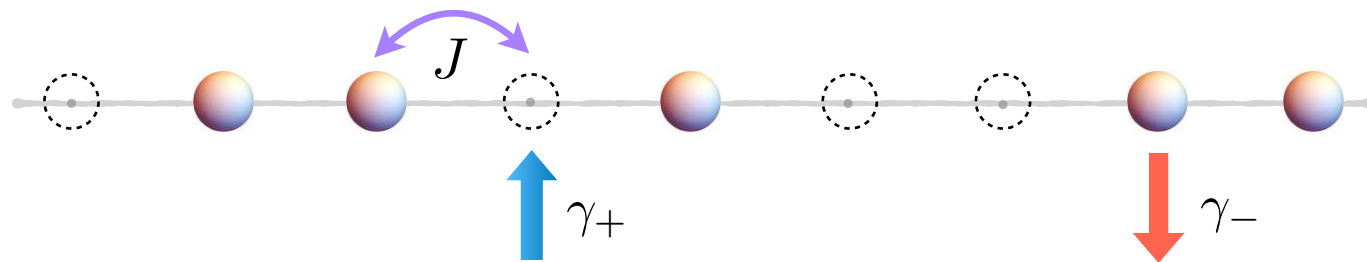
$\hat{C}^2$ symmetry is broken for

- Dissipation away from centre $\leftarrow$ uniform dephasing
 - Nearest-neighbour interactions $\leftarrow$ XXZ chain
 - Long-range tunnelling $\leftarrow$ Rydberg arrays
-] decay rate $\sim \varepsilon$
] decay rate $\sim \varepsilon^2$

Other pump-loss configurations

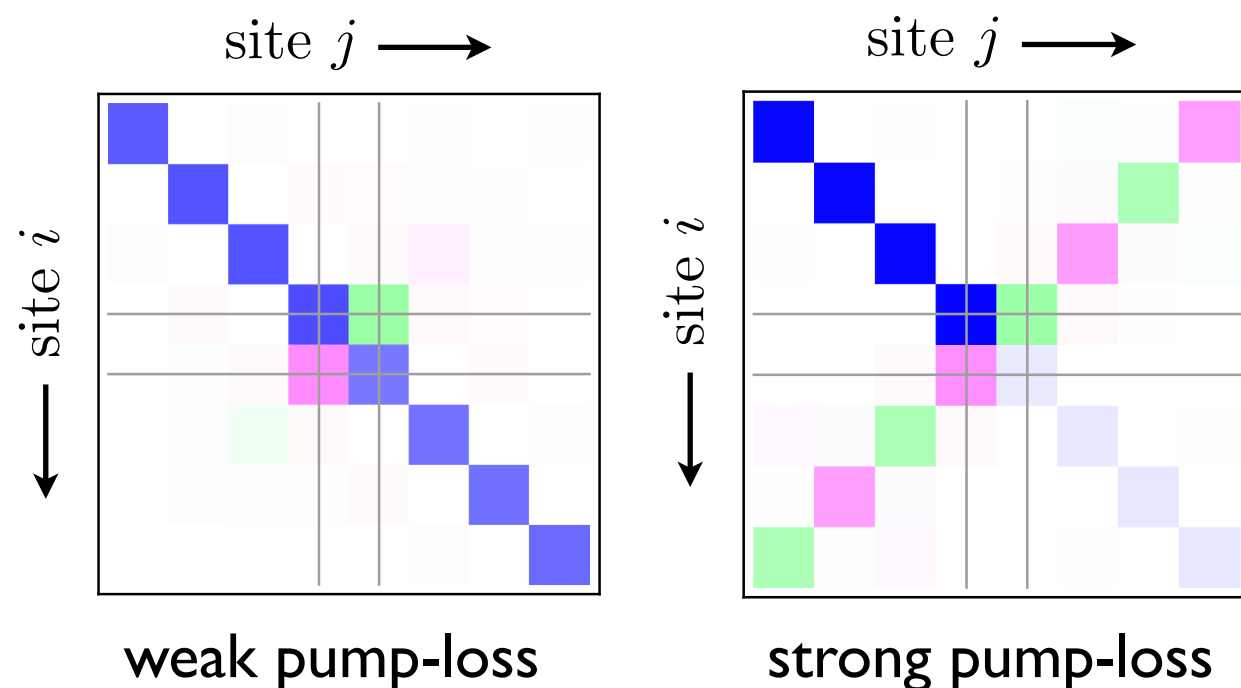
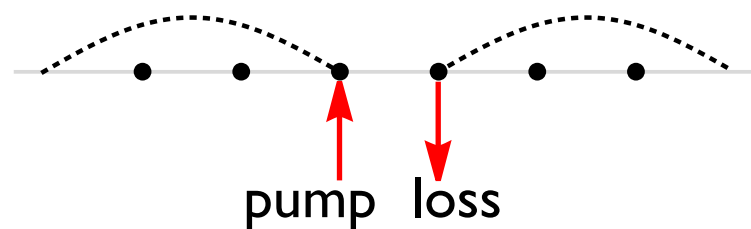
Long-range order in other configurations

PRR 3, L012016 (2021)

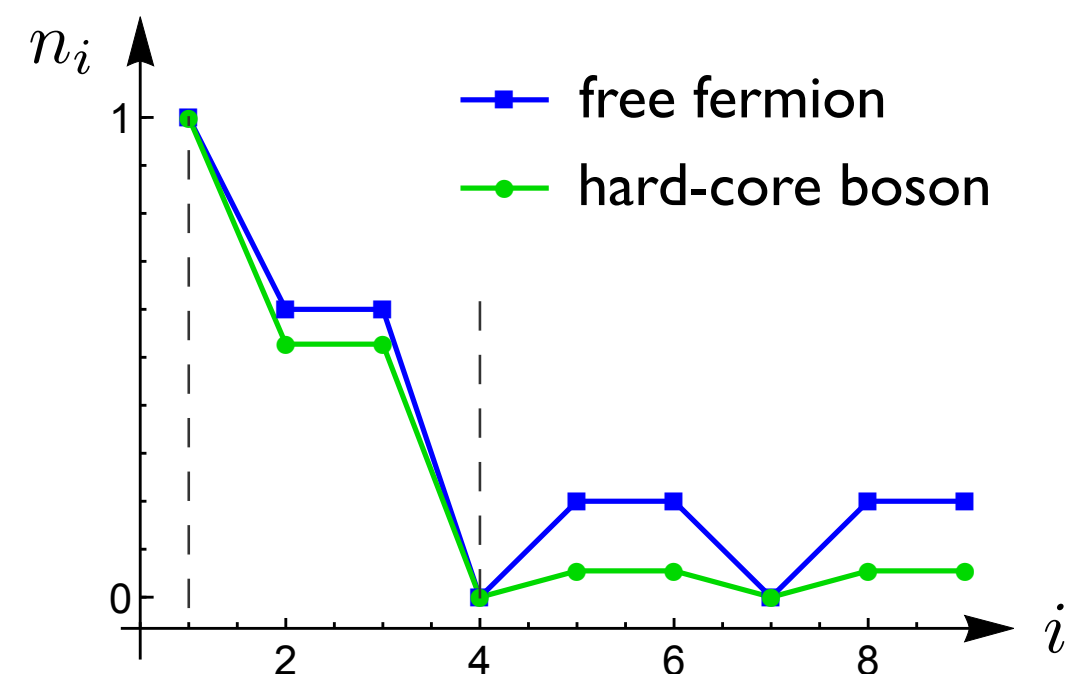
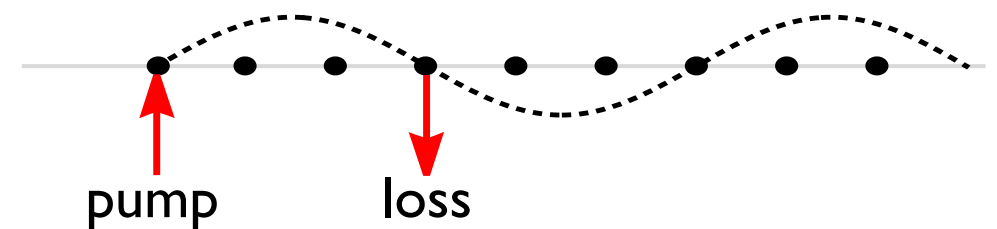


No symmetry $\Rightarrow$ Unique steady state

Pump/loss-induced long-range correlation

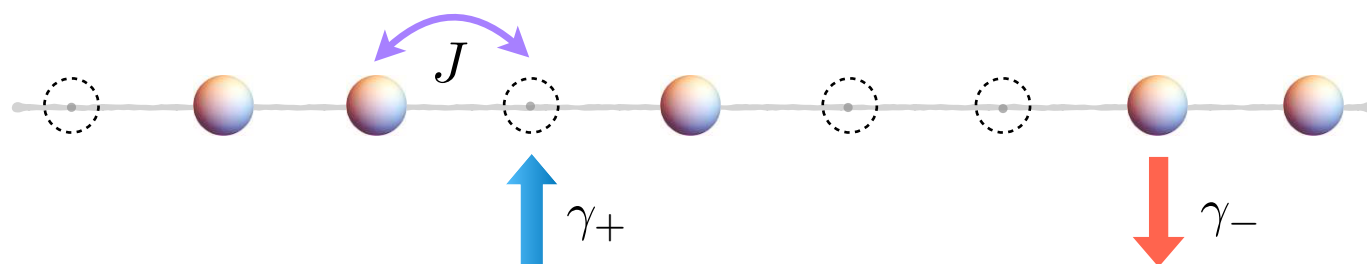


Density wave in "resonant" geometries



Long-range order in other configurations

PRR 3, L012016 (2021)

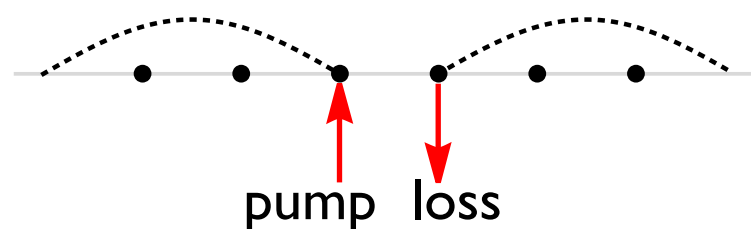


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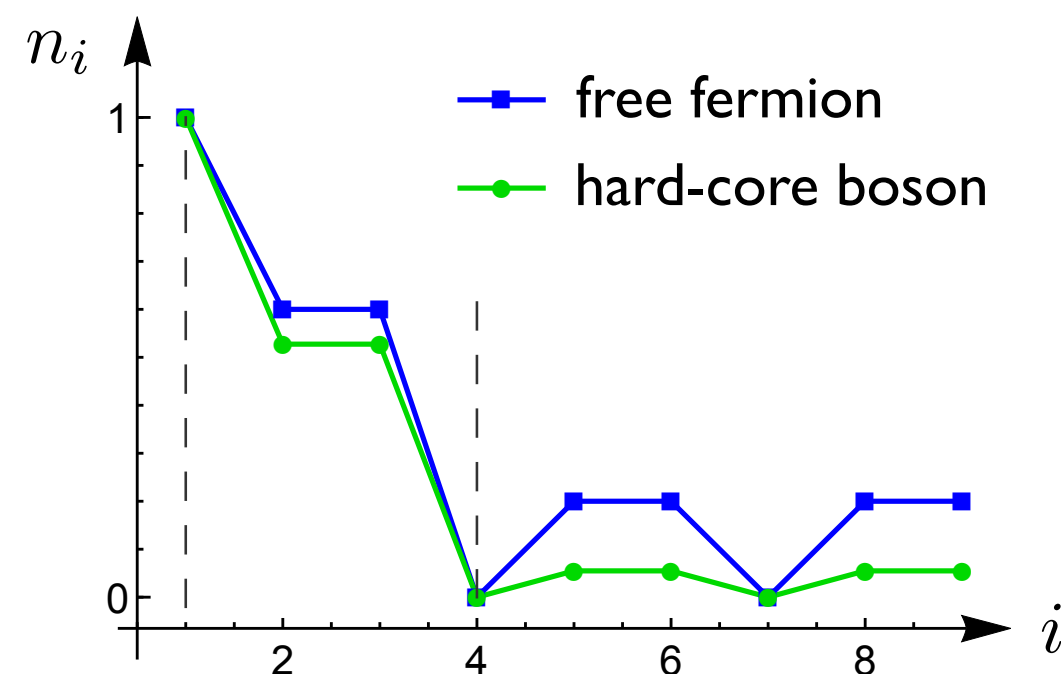
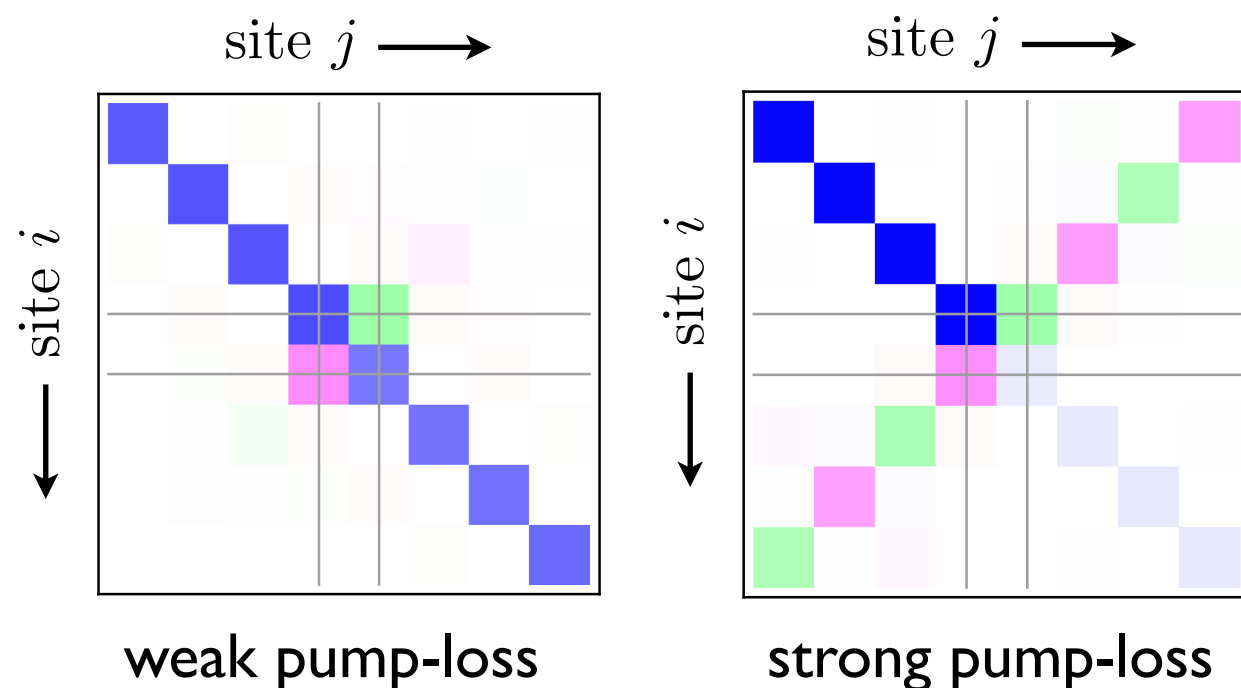
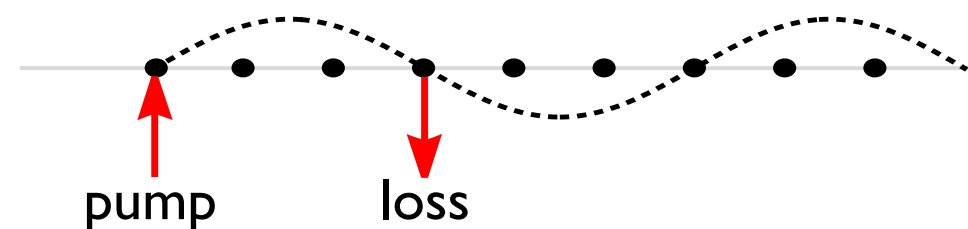
Techniques:

- DMRG + quantum trajectories
- Effective rate eqns

Pump/loss-induced long-range correlation

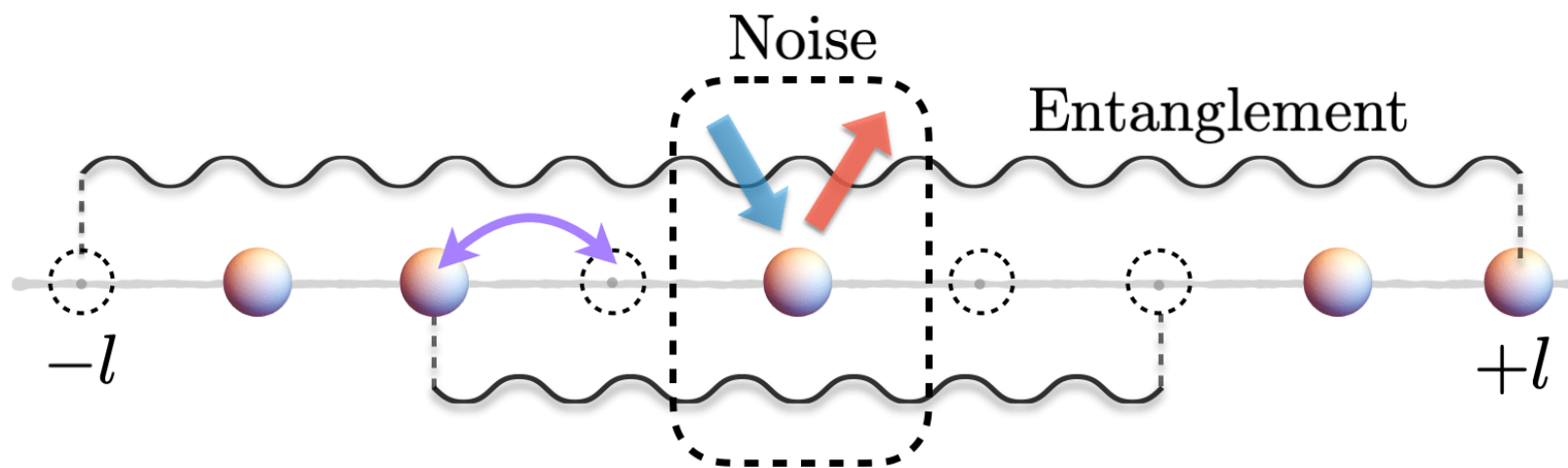


Density wave in “resonant” geometries



Summary

- Multiple (exactly solvable) steady states with long-range entanglement in a lossy qubit array — highly unusual
- Symmetry in open system protecting Bell pairs — applies to wider class of models
- Can be selectively prepared using timed pulses in cold atoms, superconducting circuits,... — symmetry harnessing



Shovan Dutta & Nigel Cooper, [PRL 125, 240404 \(2020\)](#)

SD, Stefan Kuhr, & NC, coming soon!

- Counterintuitive features in other geometries — [PRR 3, L012016 \(2021\)](#)