

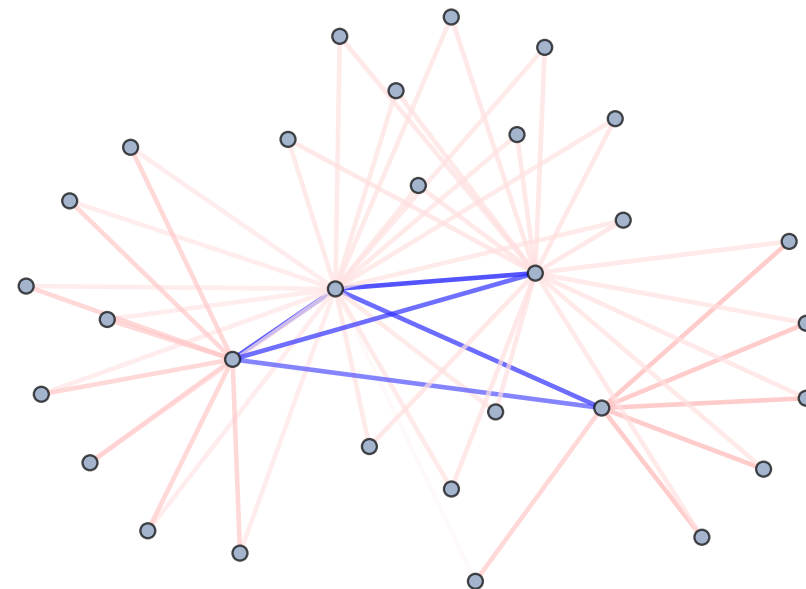
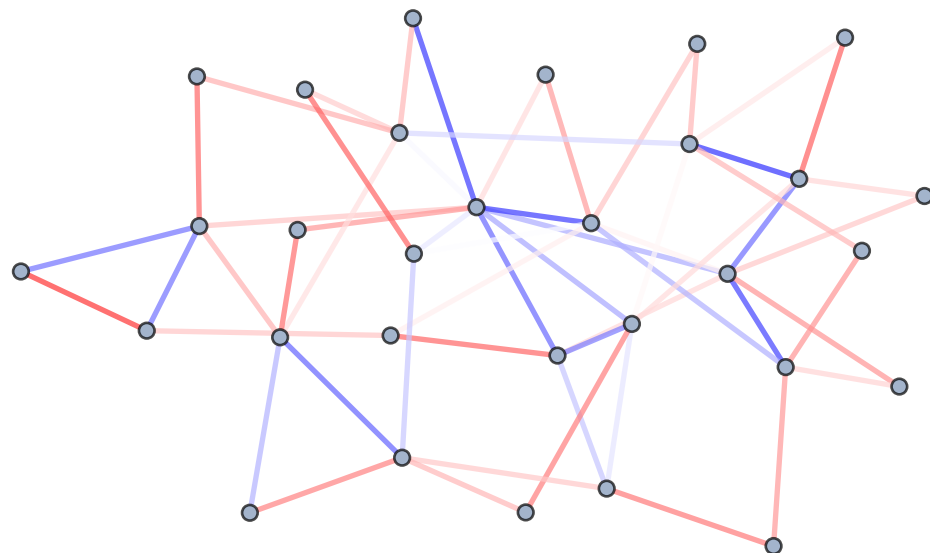
Frustrated magnetism on complex networks: What sets the total spin

Shovan Dutta

Raman Research Institute



Preethi G
IISER TVM



विज्ञान एवं प्रौद्योगिकी विभाग
DEPARTMENT OF
SCIENCE & TECHNOLOGY

[arXiv:2403.09116](https://arxiv.org/abs/2403.09116)



Why networks?

- Why limit to regular lattices?

Networks can host new many-body physics

- Novel classical phenomena, e.g., explosive synchronization

Network topology controls disease spreading

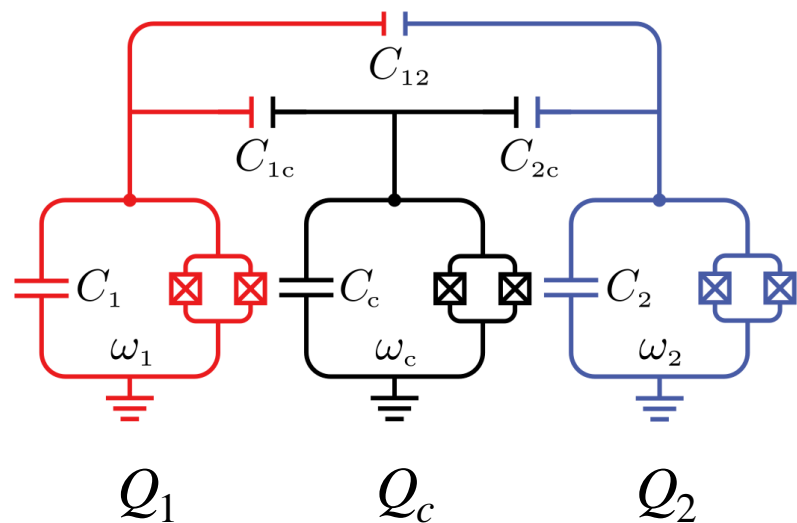
Small-world property, community structure

Strogatz, Nature 410, 268 (2001)

D'Souza *et al.*, Adv. Phys. 68, 123 (2019)

Sousa da Mata, Braz. J. Phys. 50, 658 (2020)

- Possible to synthesize arbitrary network of quantum spins



Superconducting circuits

Trapped ions

Rydberg atoms

Lamata *et al.*, Adv. Phys. X 3, 1457981 (2018)

Korenblit *et al.*, New J. Phys. 14 095024 (2012)

Nguyen *et al.*, PRX 8, 011032 (2018)

Why magnetism on networks?

Networks enable variable degrees of frustration — ingredient of spin liquid

Antiferromagnetic Heisenberg model: $\hat{H} = \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$

Savary, Balents, Rep. Prog. Phys. '16

Zhou, Kanoda, Ng, RMP, '17

Knolle, Moessner, '18

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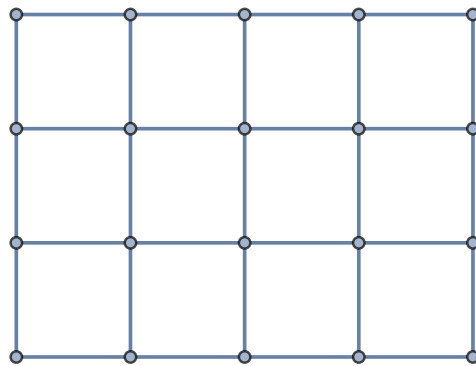
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Regular bipartite



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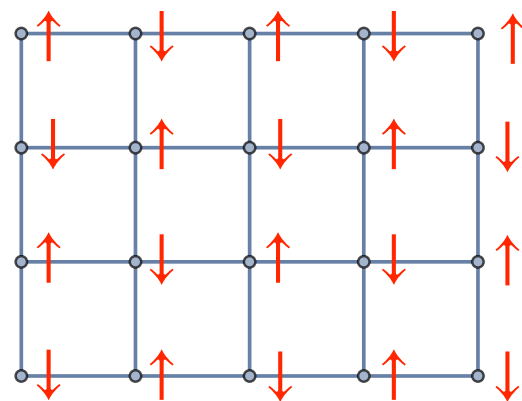
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$$S_{\text{total}} = 0$$

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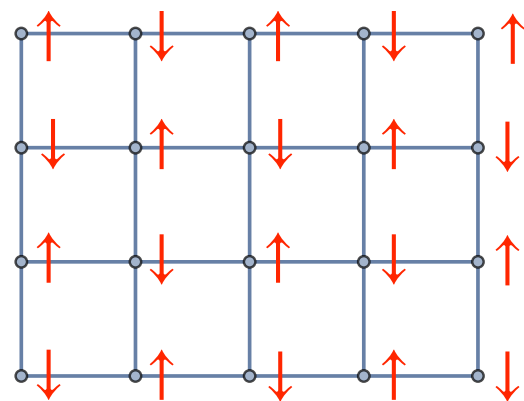
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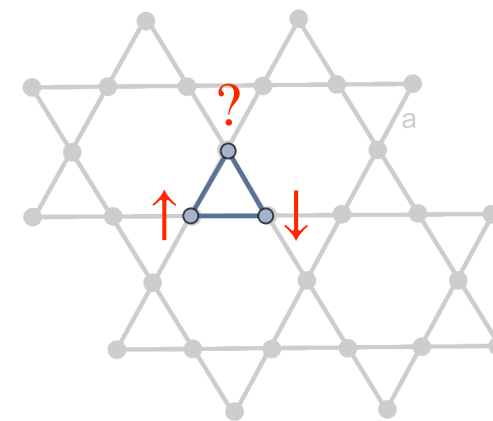
Knolle, Moessner, '18

Regular bipartite



$$S_{\text{total}} = 0$$

Regular nonbipartite



Frustrated

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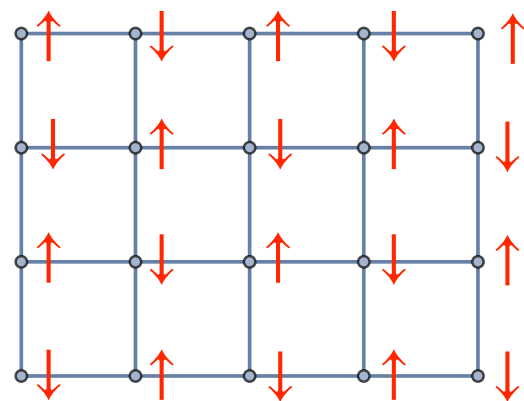
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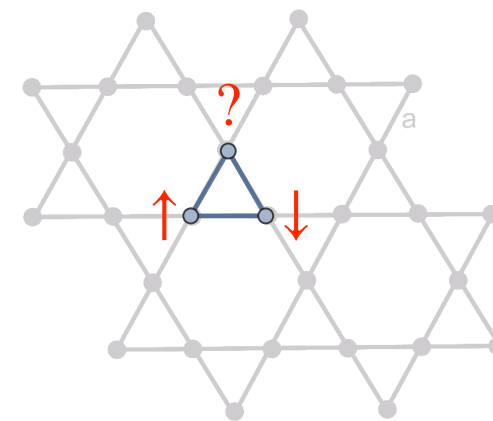
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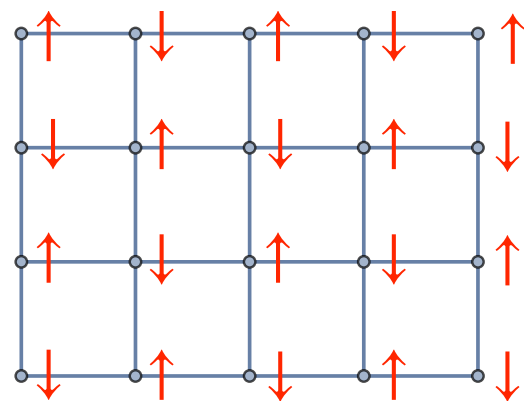
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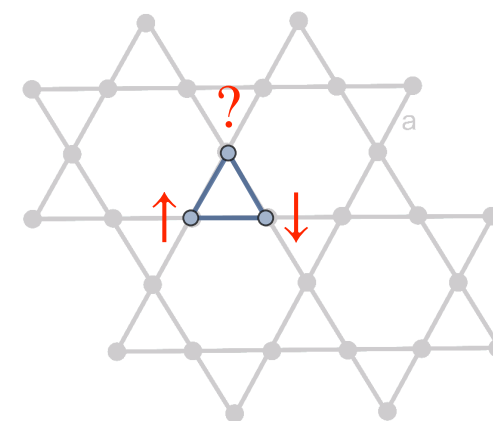
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Regular bipartite



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Regular nonbipartite

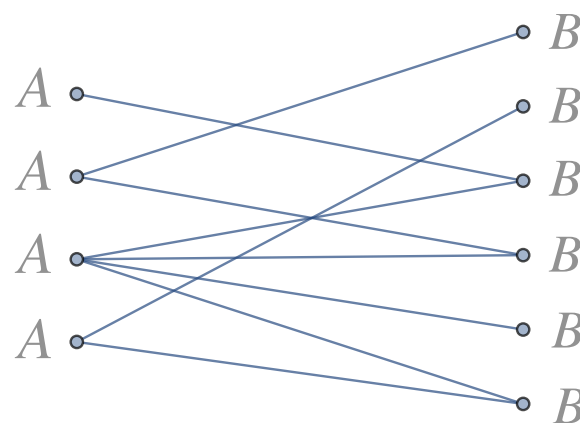


Frustrated

$$S_{\text{total}} = 0$$

Spin liquid

General bipartite



$$S_{\text{total}} = \frac{|N_A - N_B|}{2}$$

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Networks enable variable degrees of frustration — ingredient of spin liquid

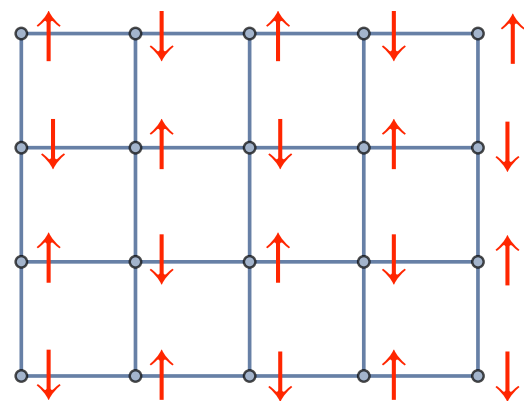
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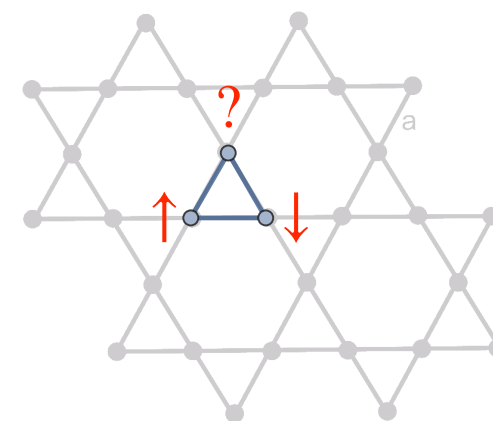
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$$S_{\text{total}} = 0$$

Regular nonbipartite

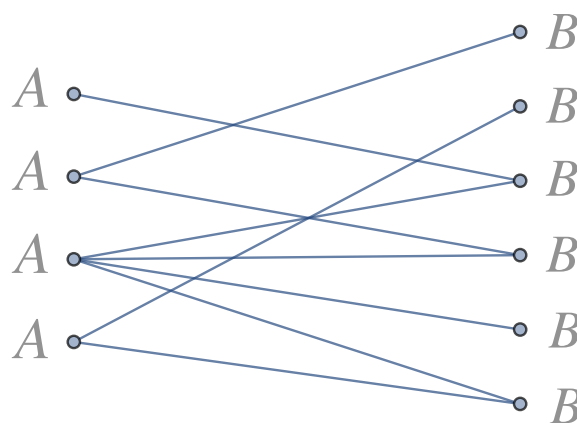


Frustrated

$$S_{\text{total}} = 0$$

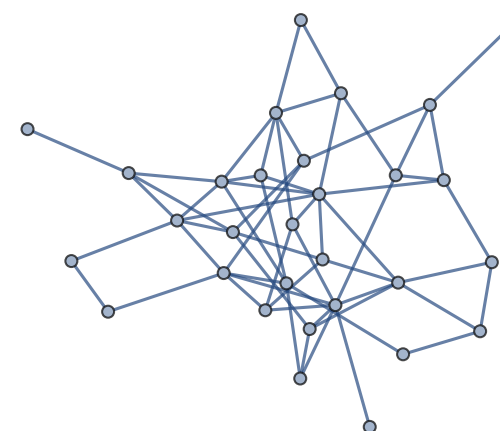
Spin liquid

General bipartite



$$S_{\text{total}} = \frac{|N_A - N_B|}{2}$$

General nonbipartite



Frustrated

$$S_{\text{total}} = ?$$

Ordering ?

Why magnetism on networks?

Networks enable variable degrees of frustration — ingredient of spin liquid

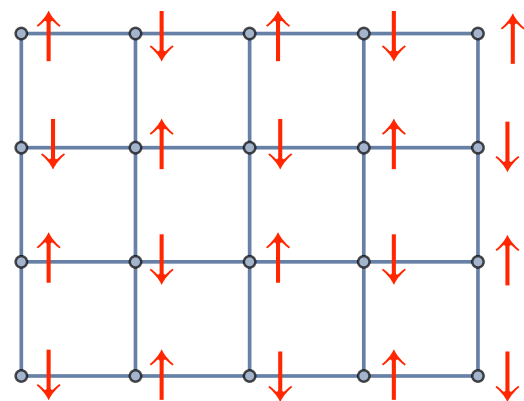
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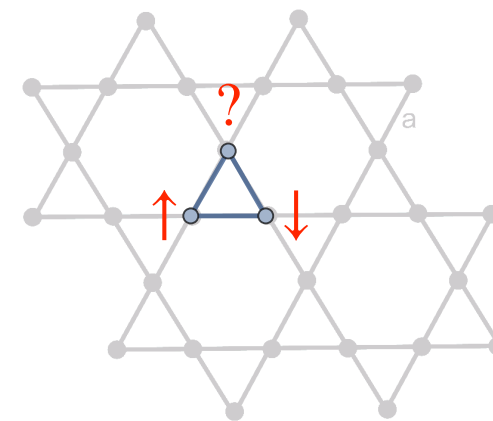
Knolle, Moessner, '18

Regular bipartite



$$S_{\text{total}} = 0$$

Regular nonbipartite

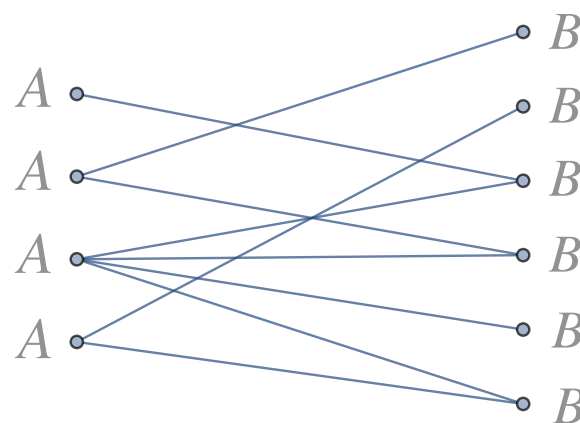


Frustrated

$$S_{\text{total}} = 0$$

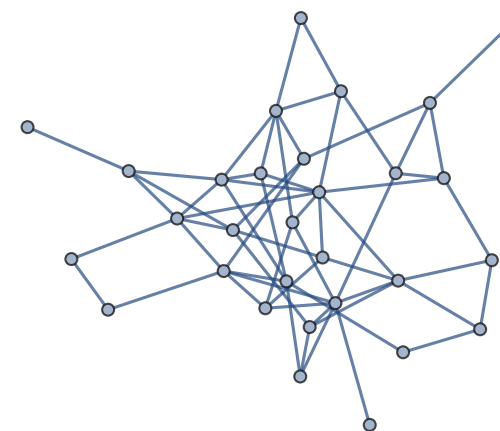
Spin liquid

General bipartite



$$S_{\text{total}} = \frac{|N_A - N_B|}{2}$$

General nonbipartite



Frustrated

$$S_{\text{total}} = ?$$

Ordering ?

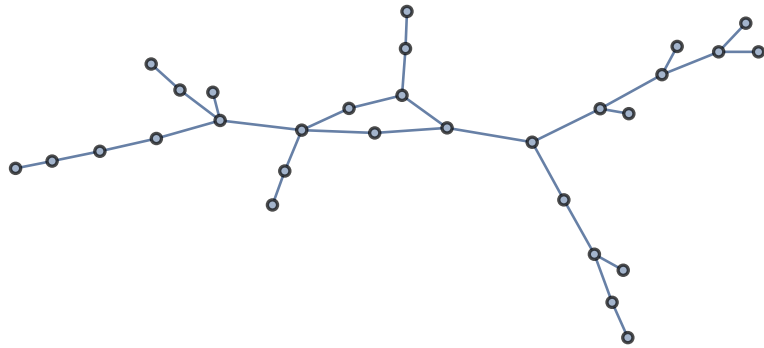
Q: How does network topology determine magnetic order? **Here: What sets S_{total} ?**

Random graphs

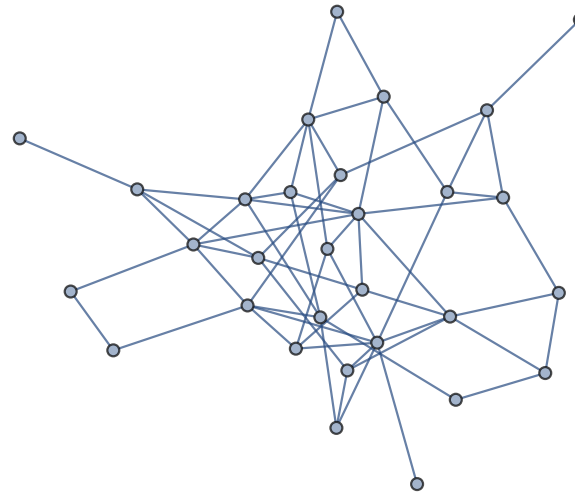
Random (connected) graphs

Random network of N spins & N_e bonds — generically nonbipartite

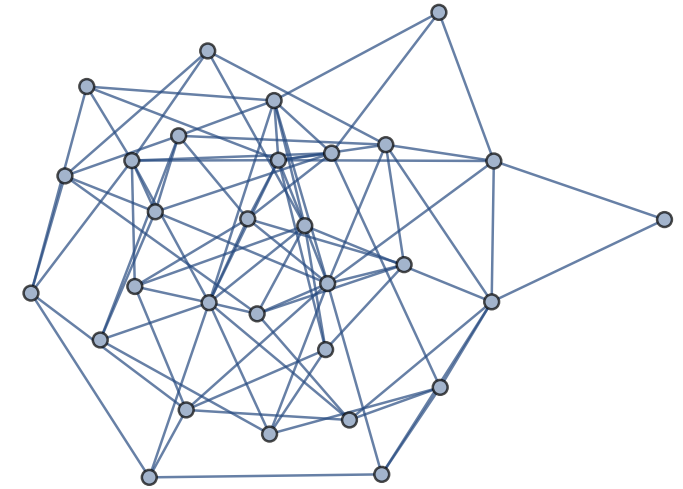
$$N = 30, N_e = 30$$



$$N = 30, N_e = 60$$



$$N = 30, N_e = 90$$

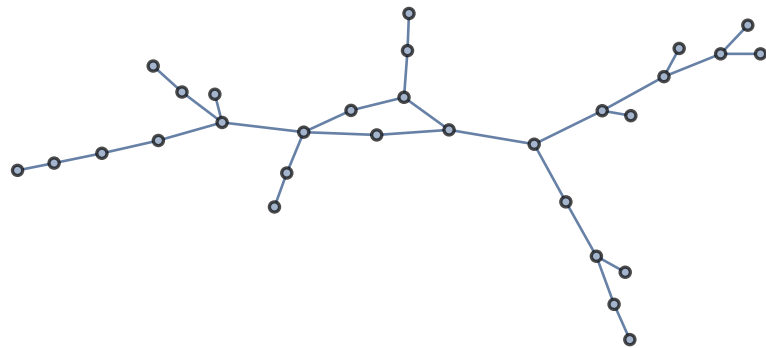


Avg # of neighbors (degree) $\bar{k} = 2N_e/N = 2, 4, 6$

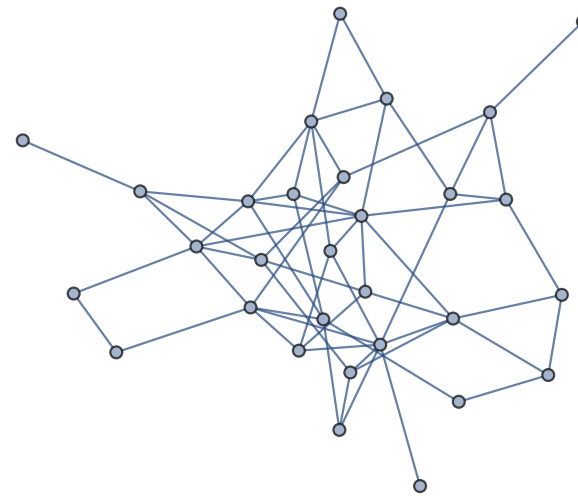
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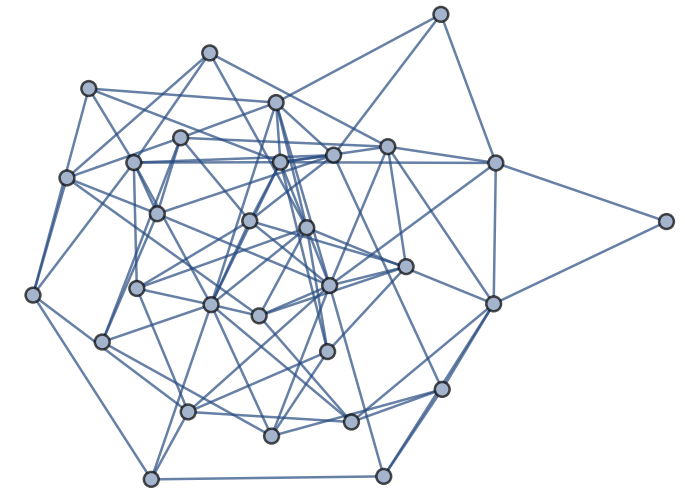
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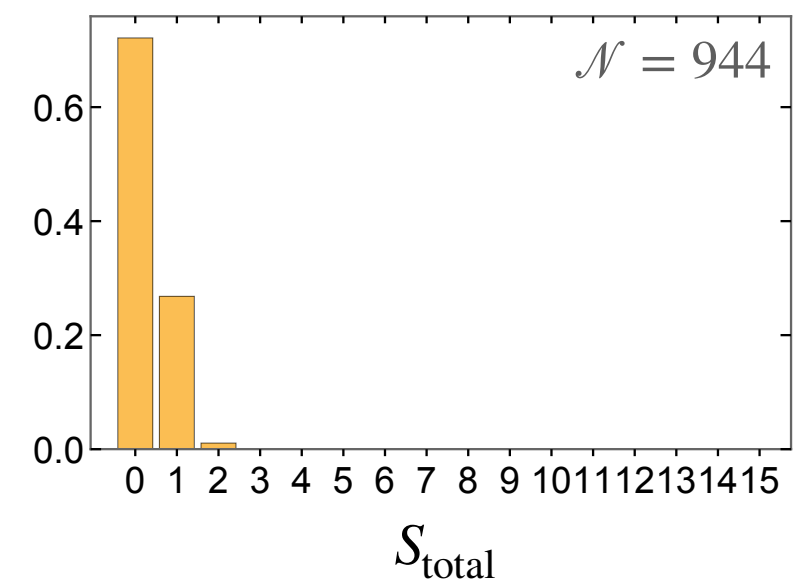
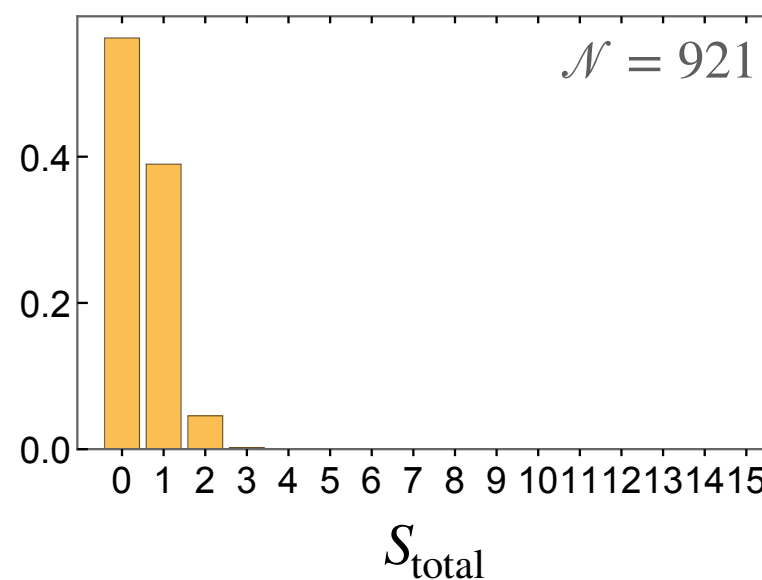
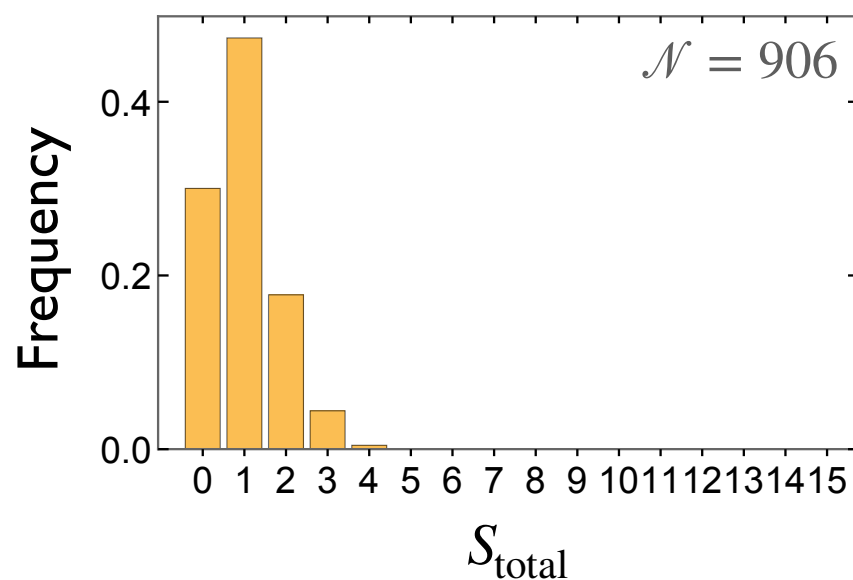


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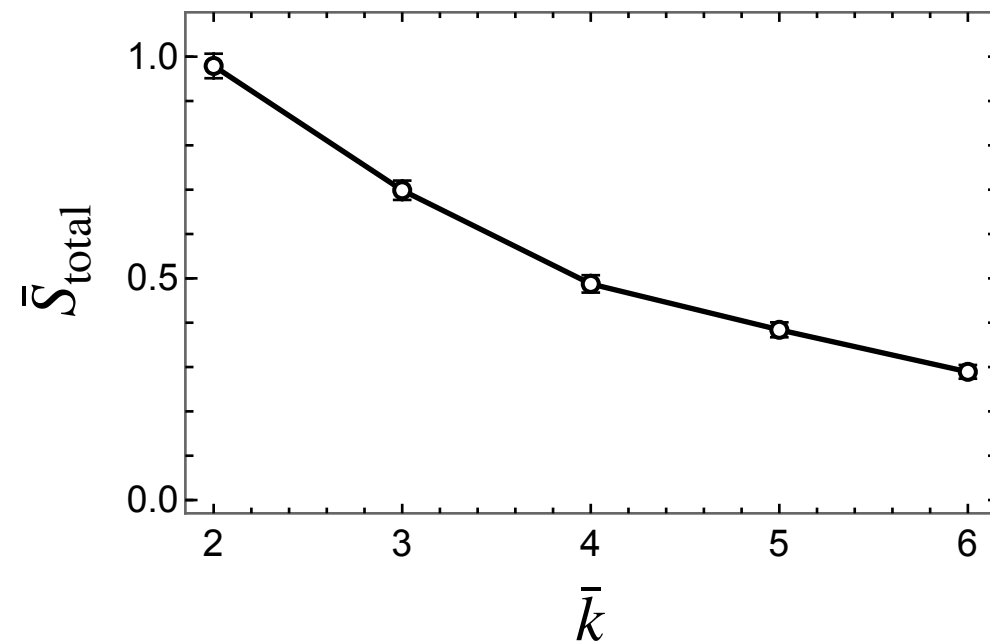
Avg # of neighbors (degree) $\bar{k} = 2N_e/N = 2, 4, 6$

S_{total} small — falls with increasing \bar{k} :



Random (connected) graphs

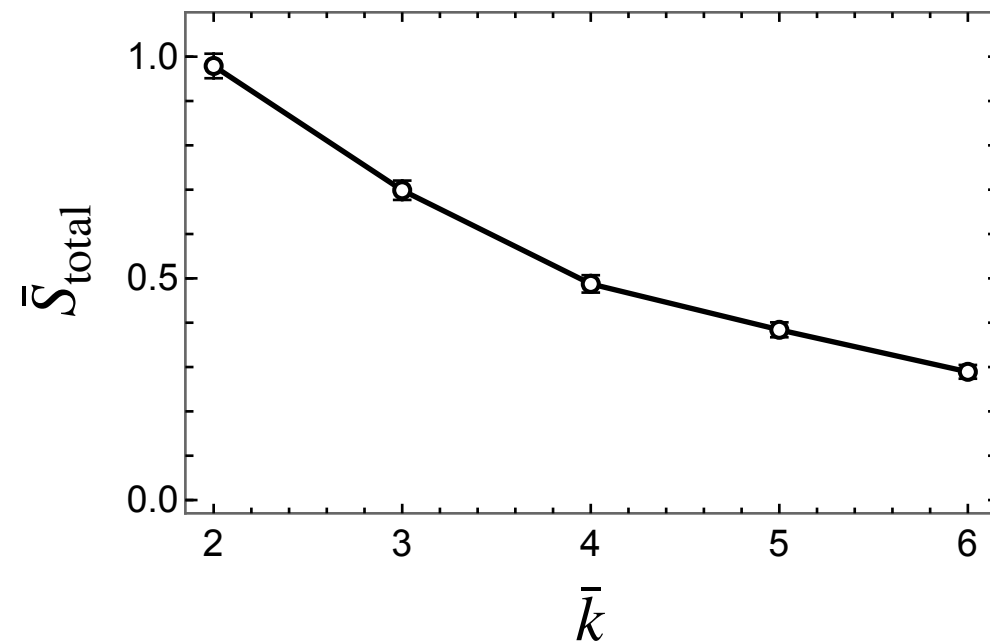
Random network of N spins & N_e bonds — generically nonbipartite



Magnetization falls with more neighbors

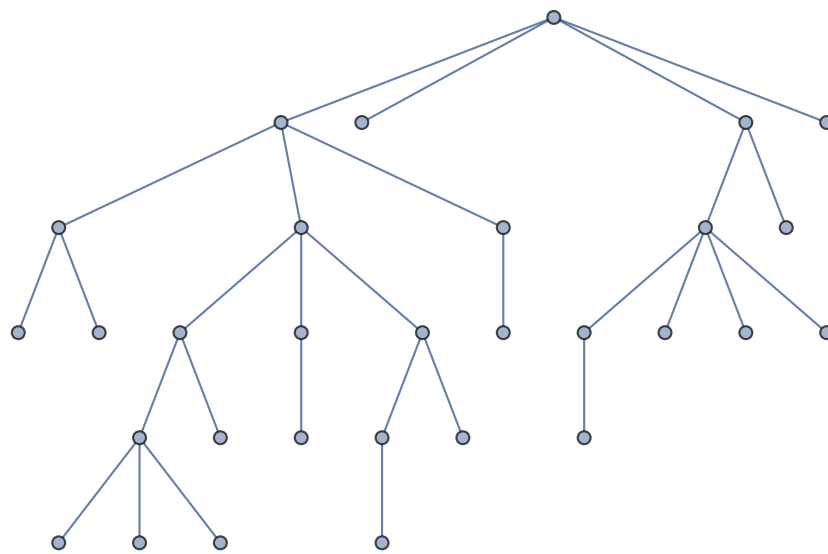
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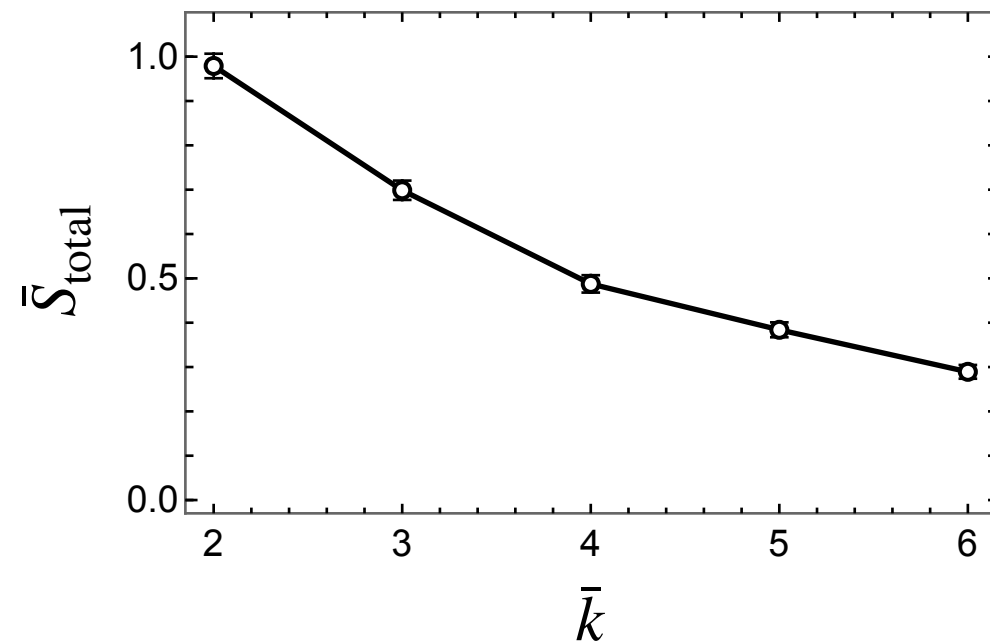
Magnetization falls with more neighbors

$$N_e^{\min} = N - 1$$



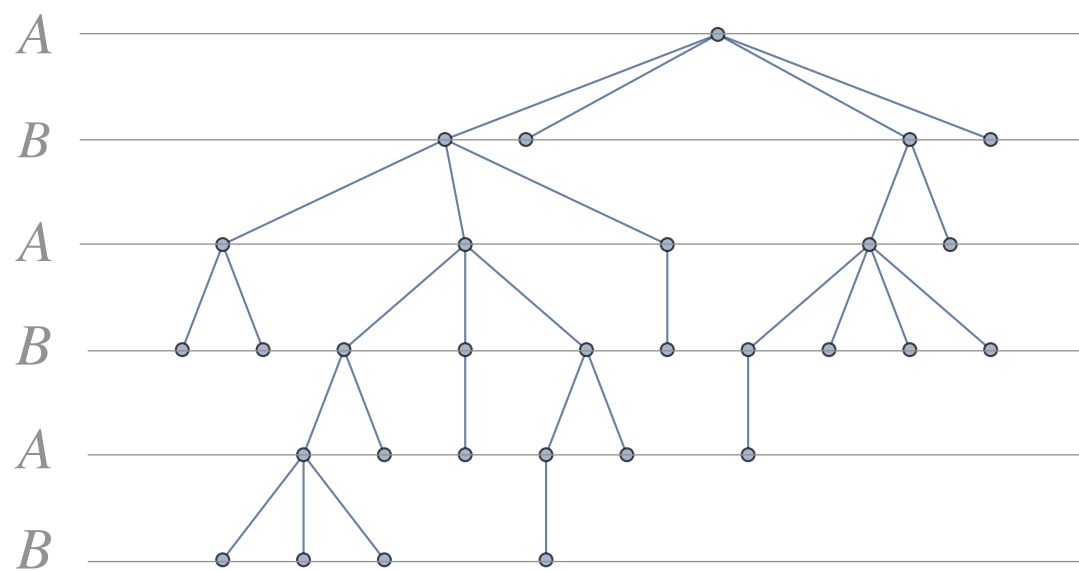
Random (connected) graphs

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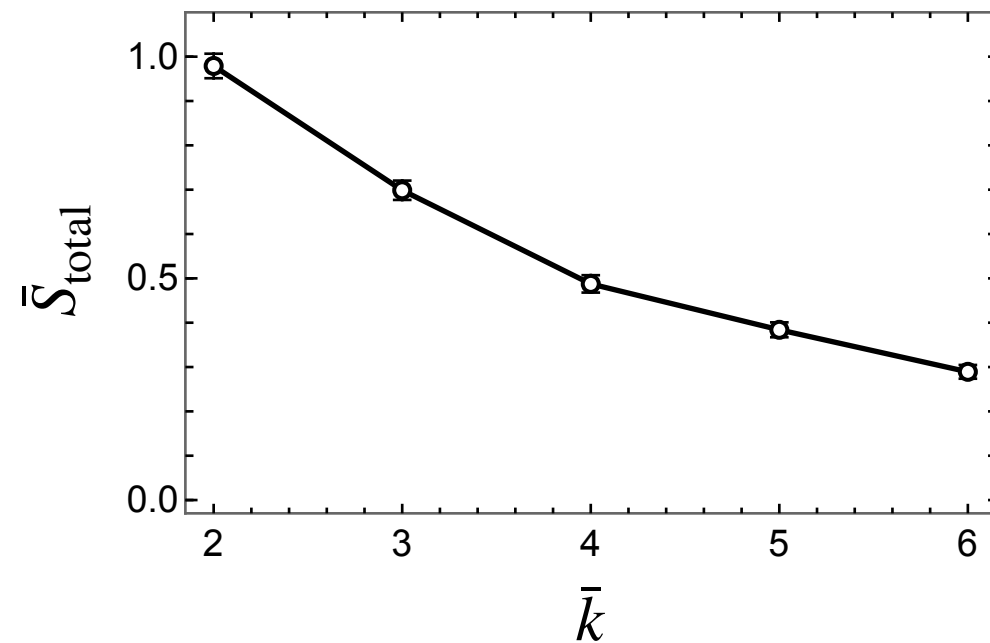
$$N_e^{\min} = N - 1$$



$$S_{\text{total}} = |N_A - N_B|/2 \sim \sqrt{N}$$

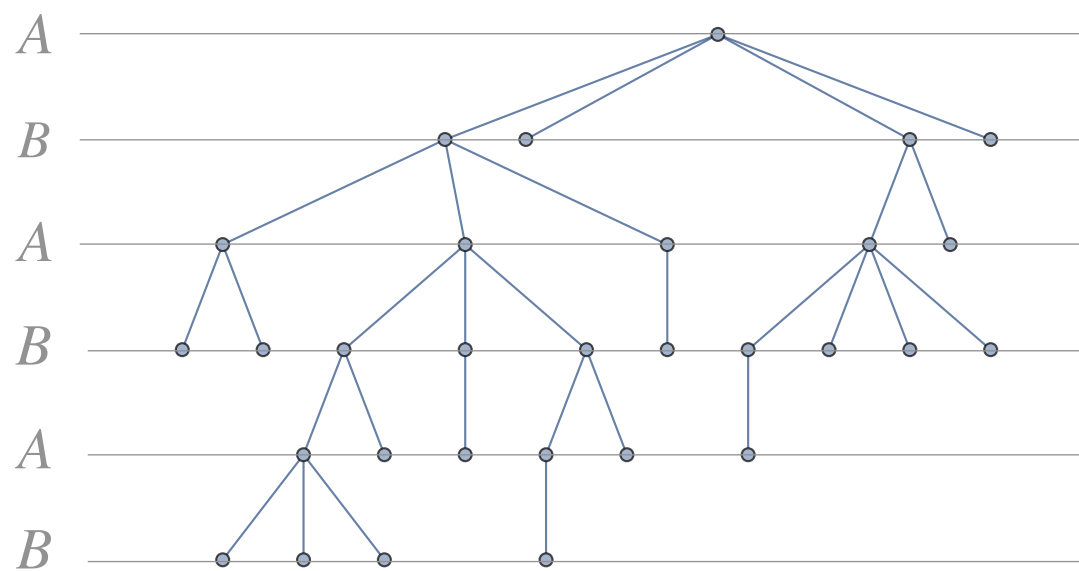
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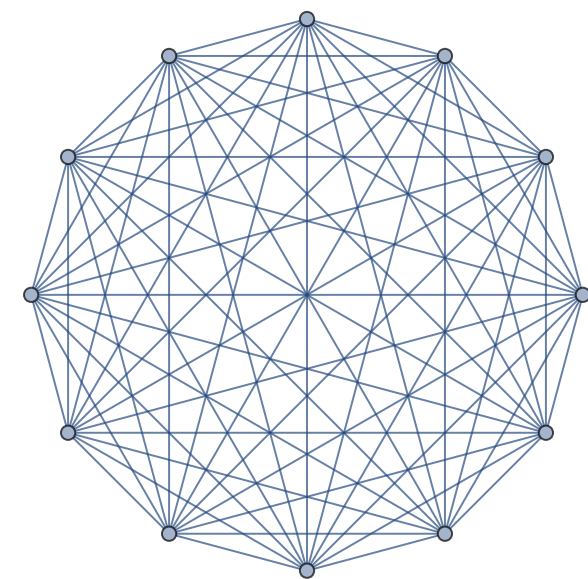
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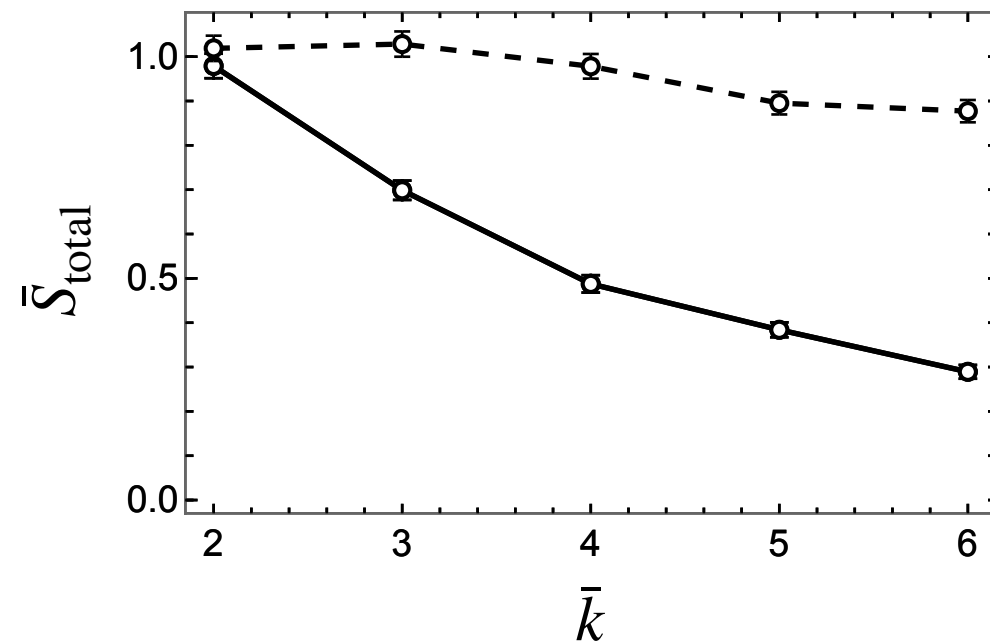
$$N_e^{\max} = N(N - 1)/2$$



$$S_{\text{total}} = 0$$

Random (connected) graphs

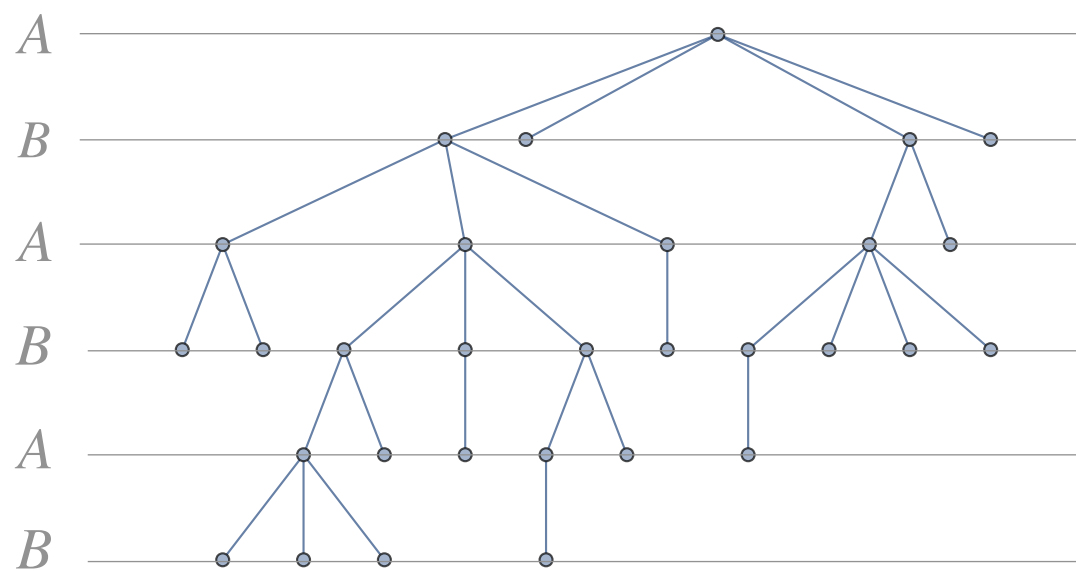
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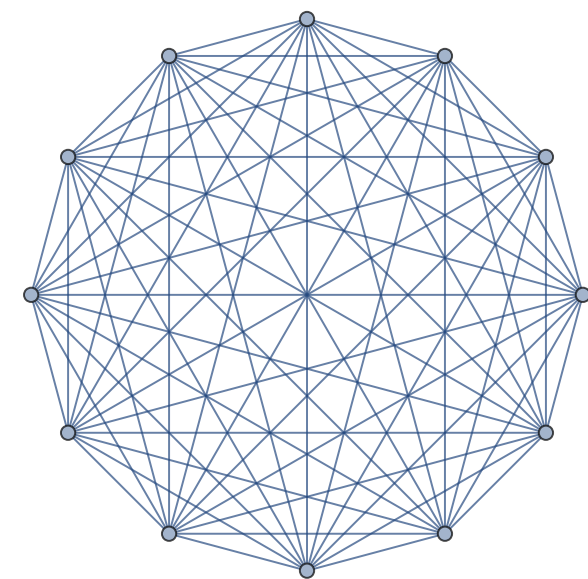
- - - Classical Ising spin glass — “max-cut”

$$N_e^{\min} = N - 1$$



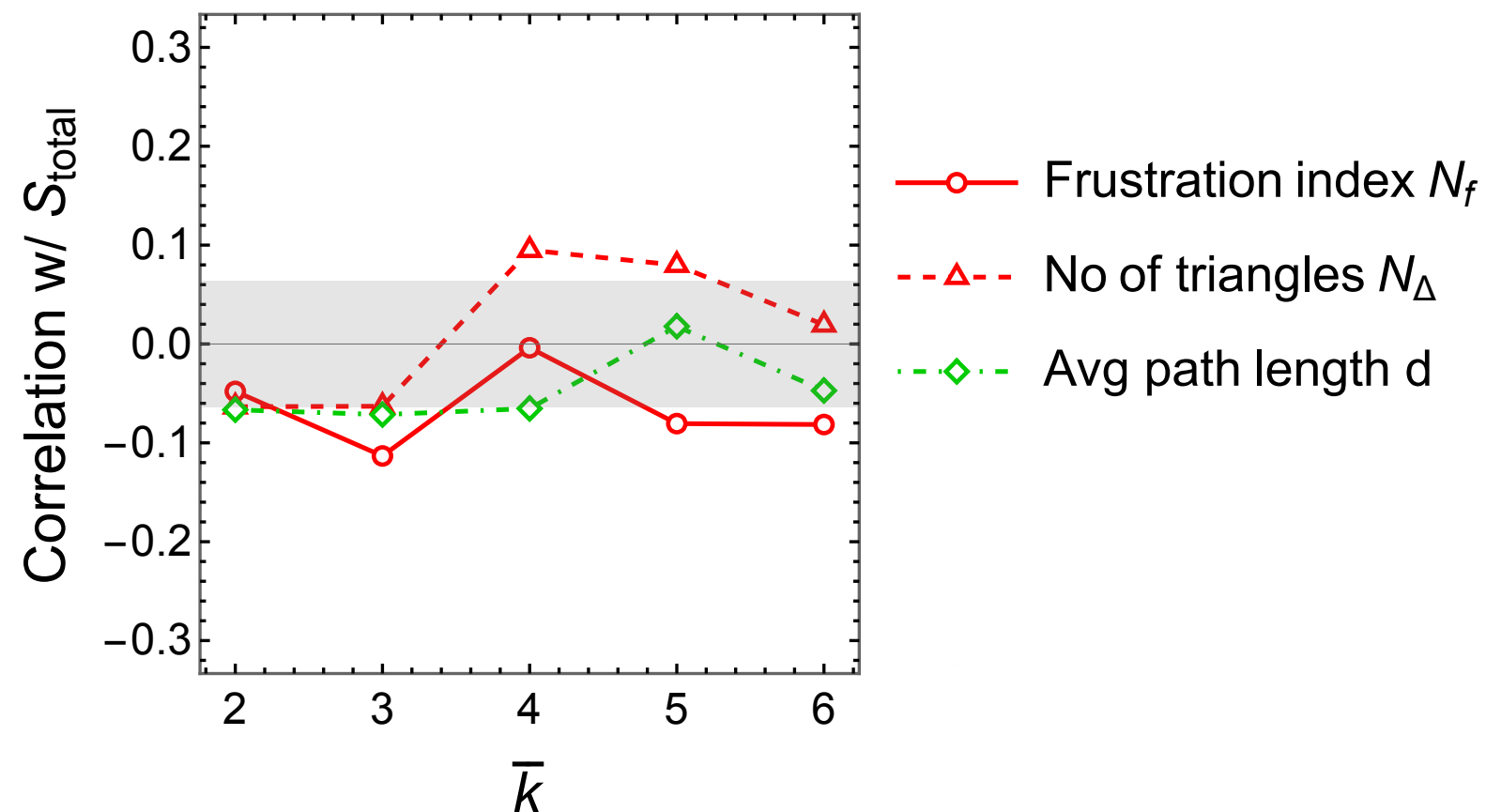
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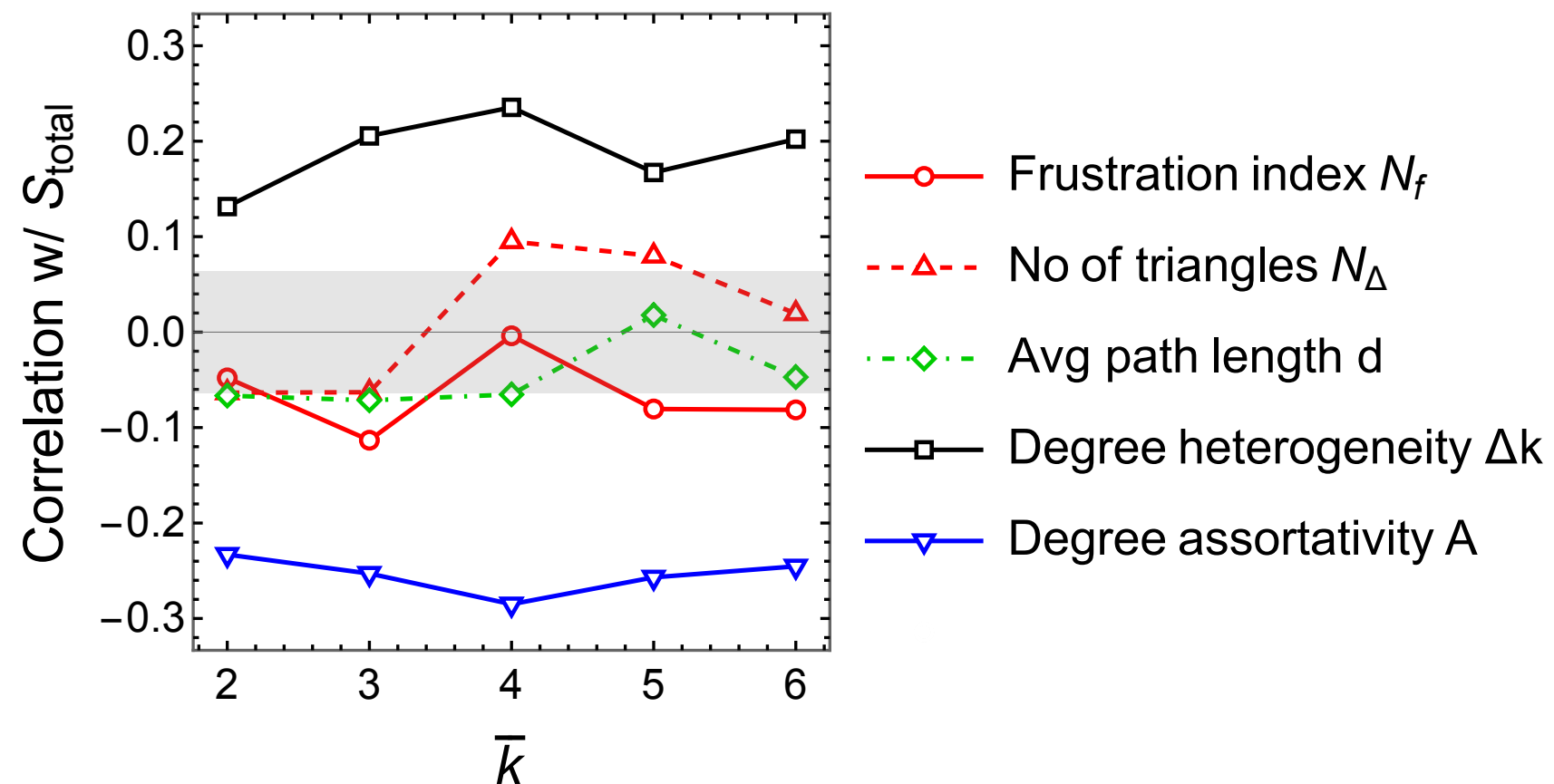
Random (connected) graphs — correlations



N_f : number of bonds to cut to make bipartite

Weak correlation w/ frustration

Random (connected) graphs — correlations



N_f : number of bonds to cut to make bipartite

$A \in [-1,1]$: +ve \implies high-degree nodes connect to high-degree nodes (& vice versa)

Newman, PRE 67, 026126 (2003)

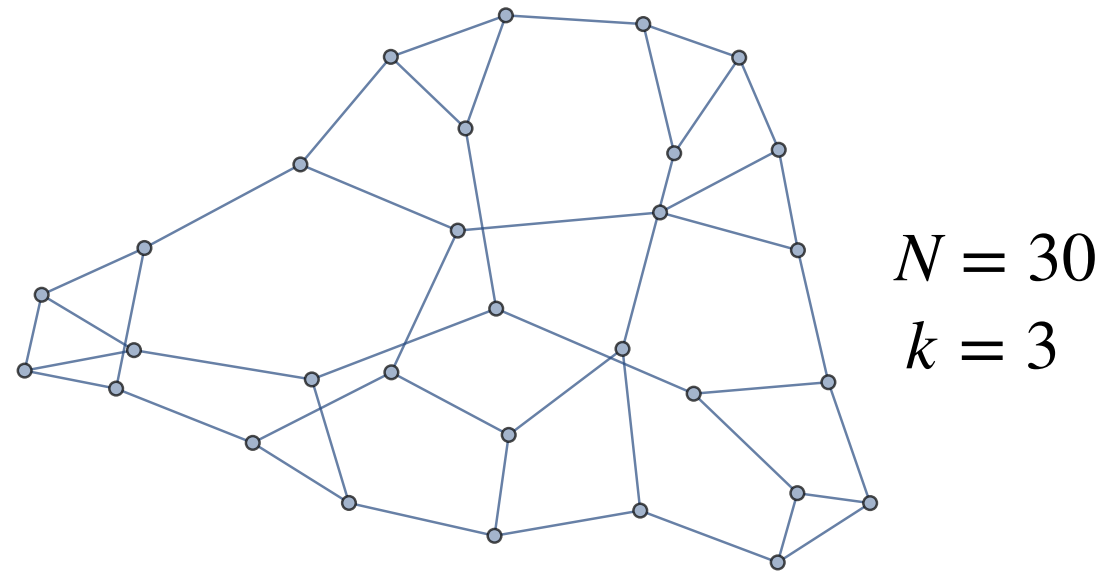
Weak correlation w/ frustration

Strong correlation w/ heterogeneity & assortativity

Heterogeneity

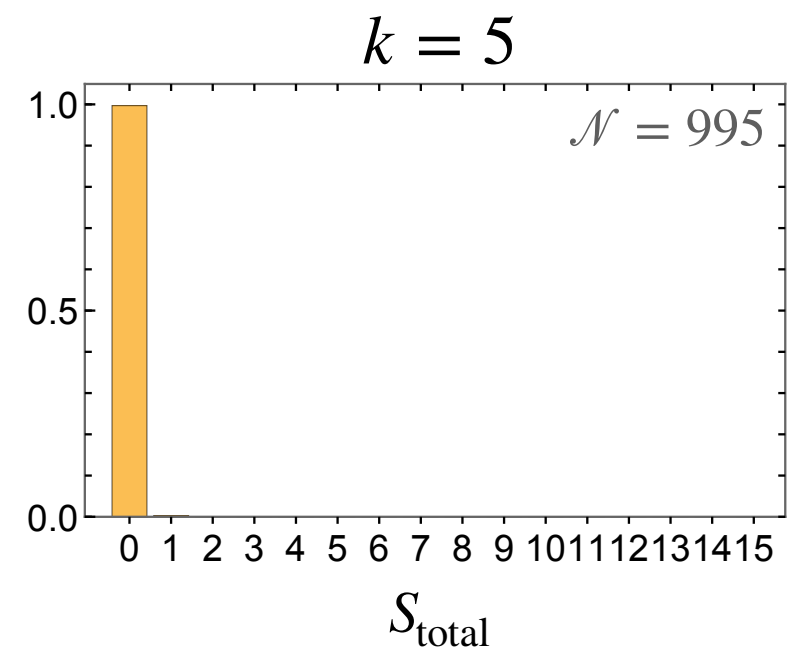
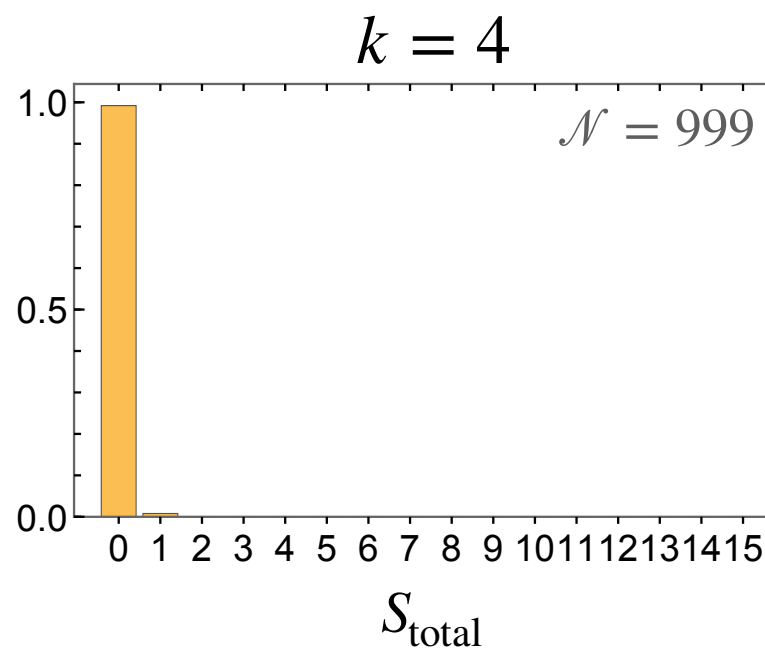
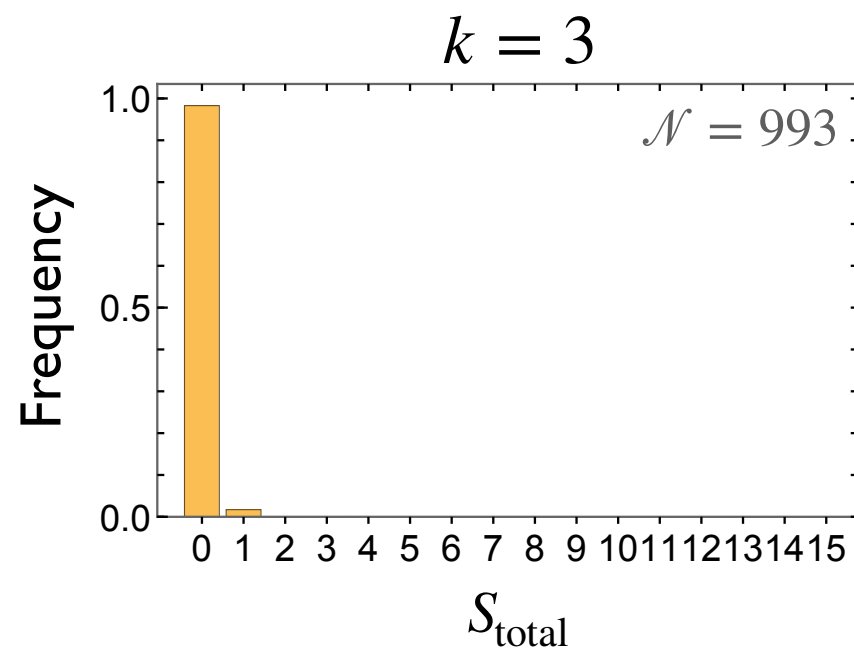
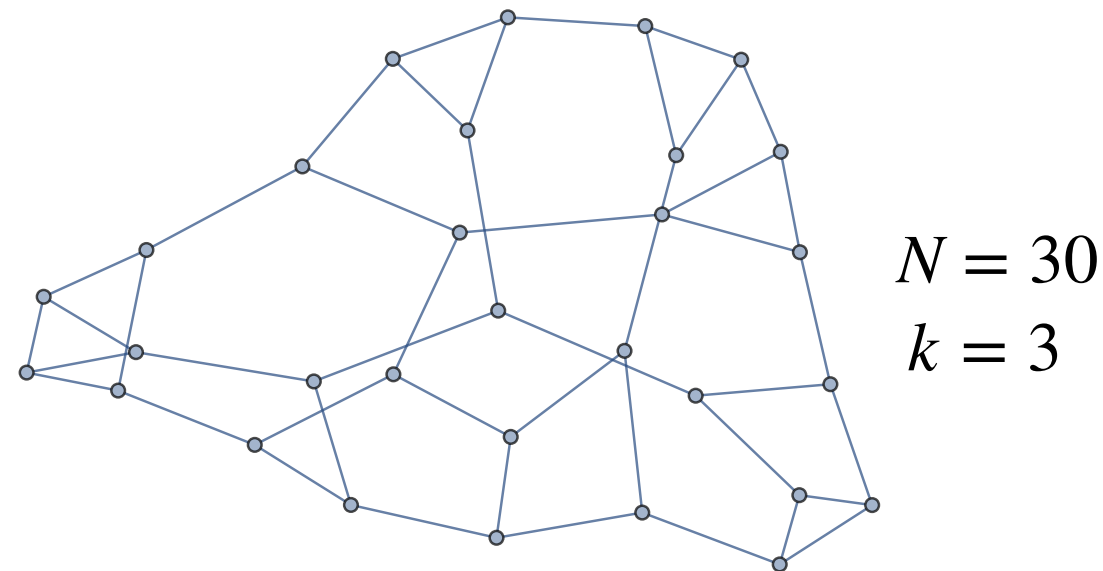
No heterogeneity: Random regular graphs

Every spin has k neighbors



No heterogeneity: Random regular graphs

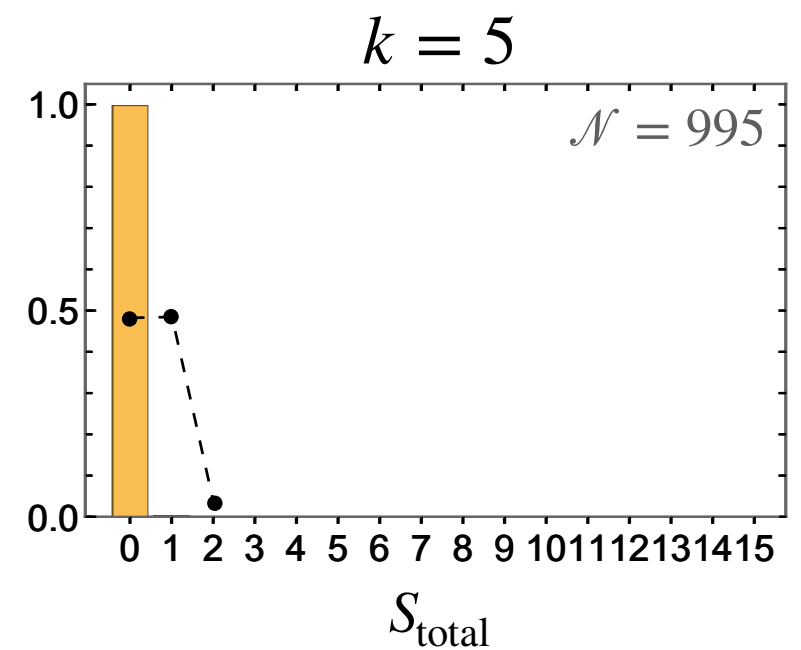
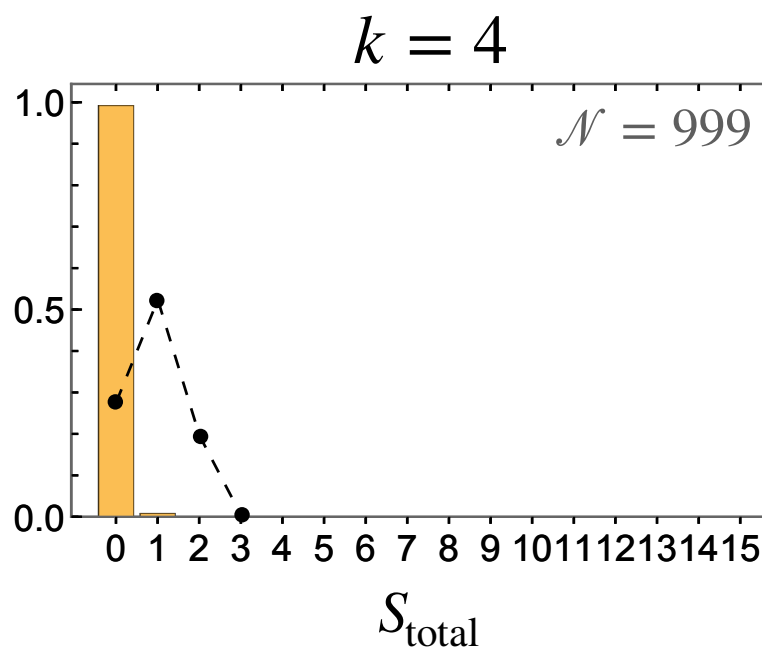
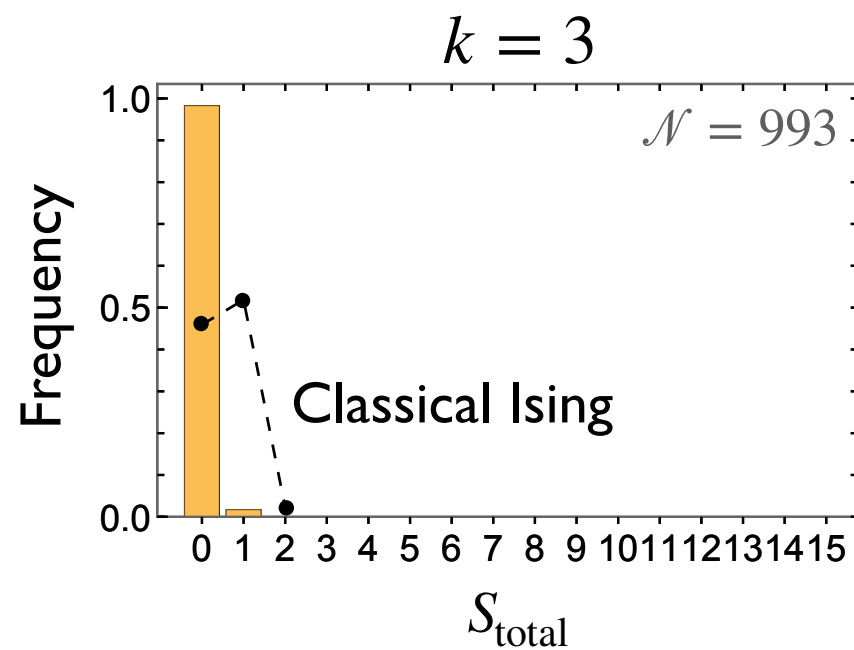
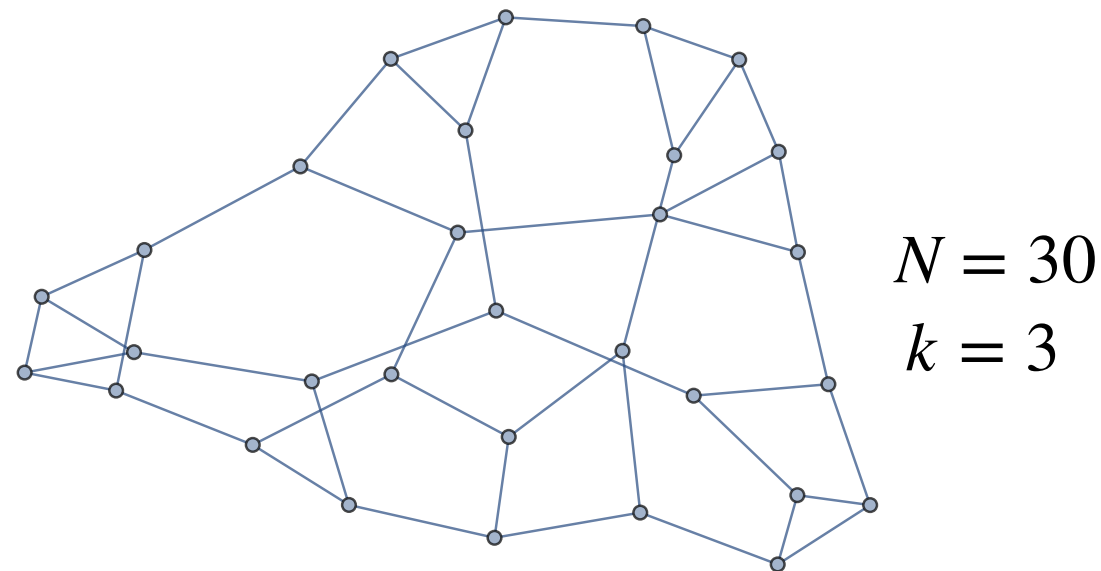
Every spin has k neighbors



\Rightarrow **Nonzero S_{total} requires spread in degree (# of neighbors)**

No heterogeneity: Random regular graphs

Every spin has k neighbors



\Rightarrow **Nonzero S_{total} requires spread in degree (# of neighbors)**

Increased heterogeneity: Scale-free graphs

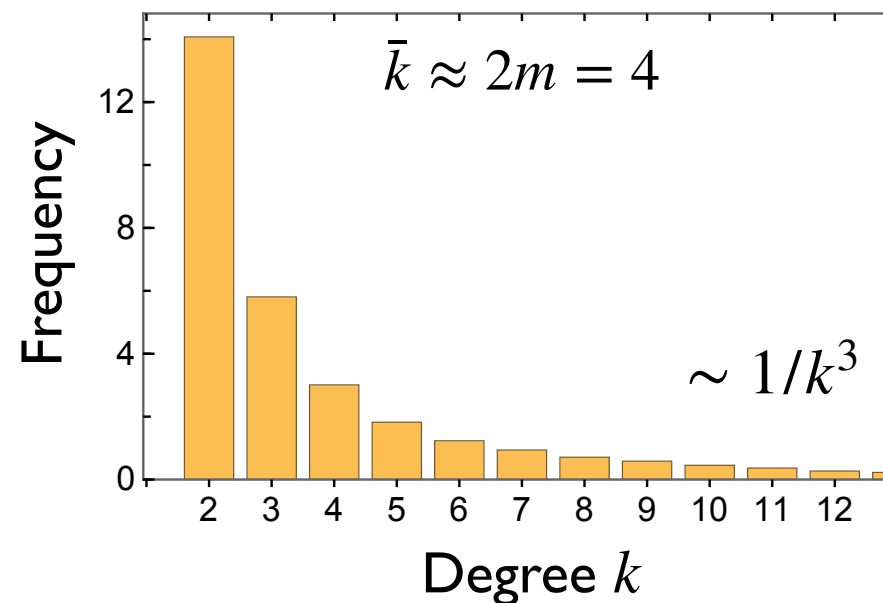
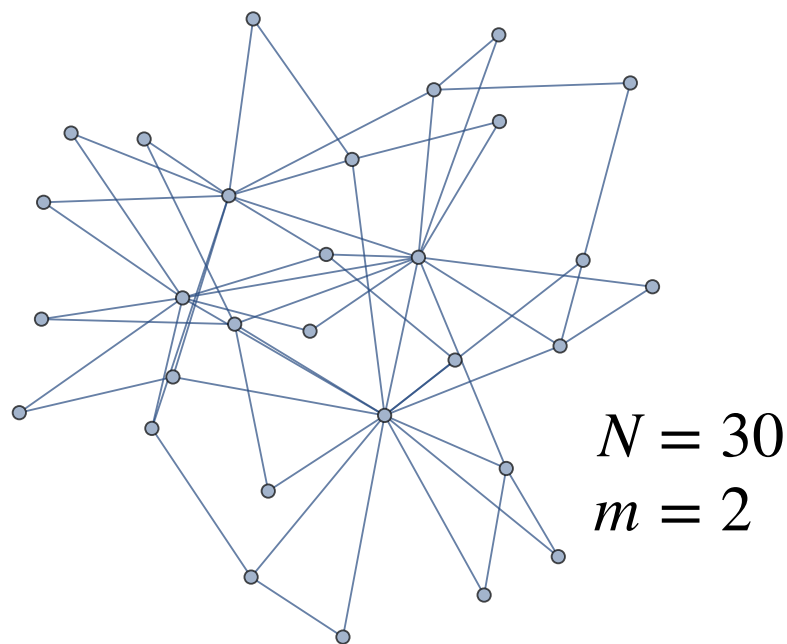
Power-law degree distribution

Increased heterogeneity: Scale-free graphs

Power-law degree distribution

Barabasi-Albert: each new node connects to m existing nodes following preferential attachment — $p_i \propto k_i$

RMP 74, 47 (2002)

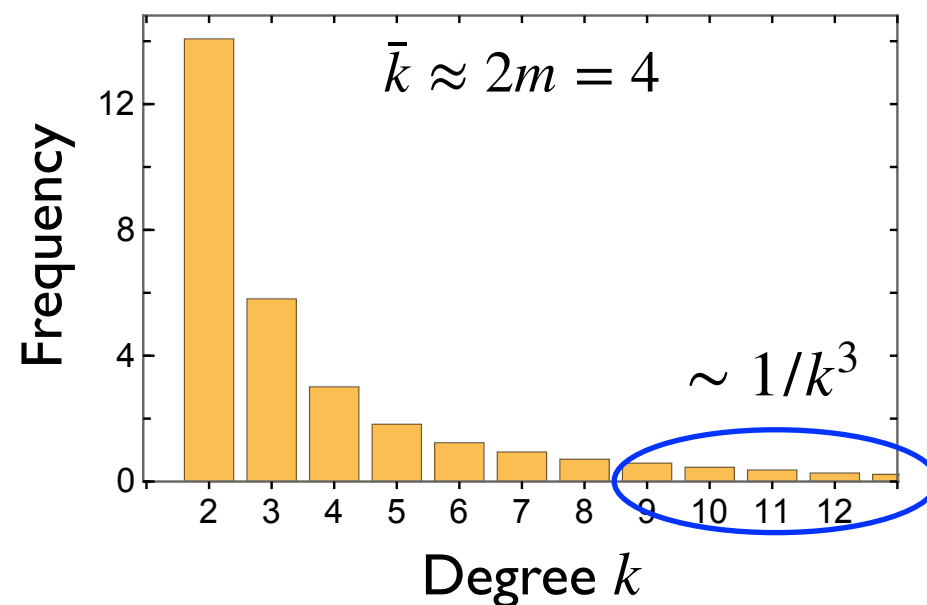
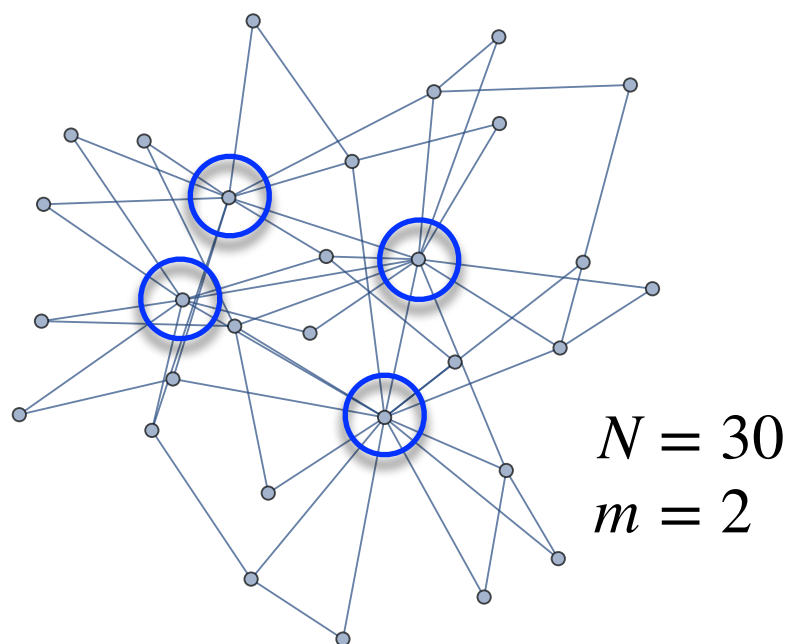


Increased heterogeneity: Scale-free graphs

Power-law degree distribution \Rightarrow Hubs

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RMP 74, 47 (2002)

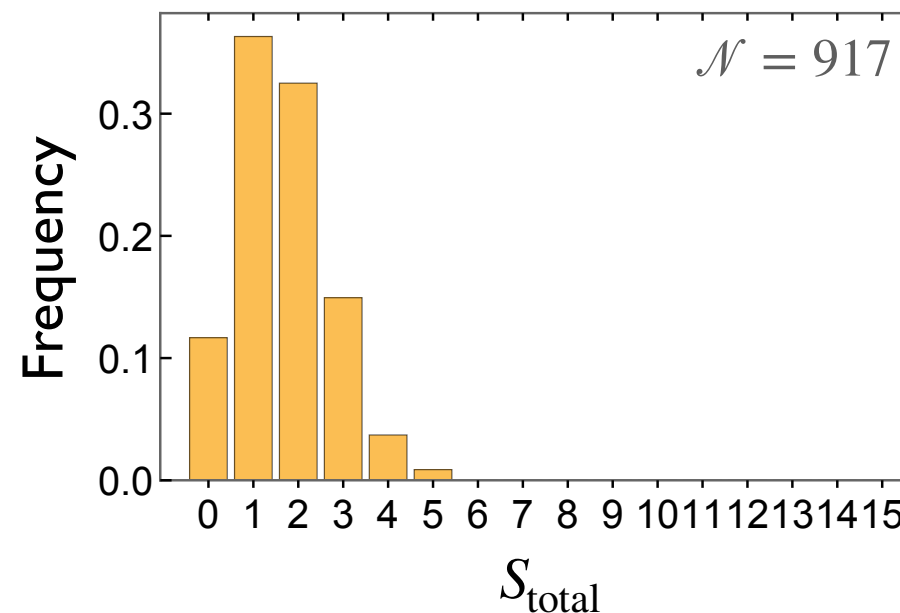
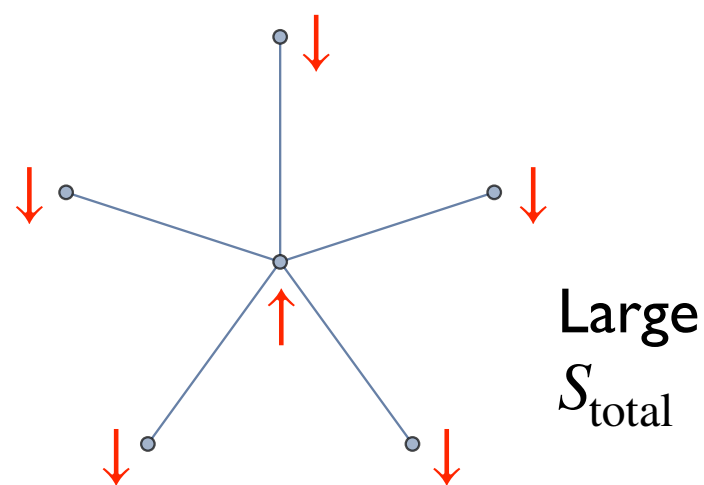
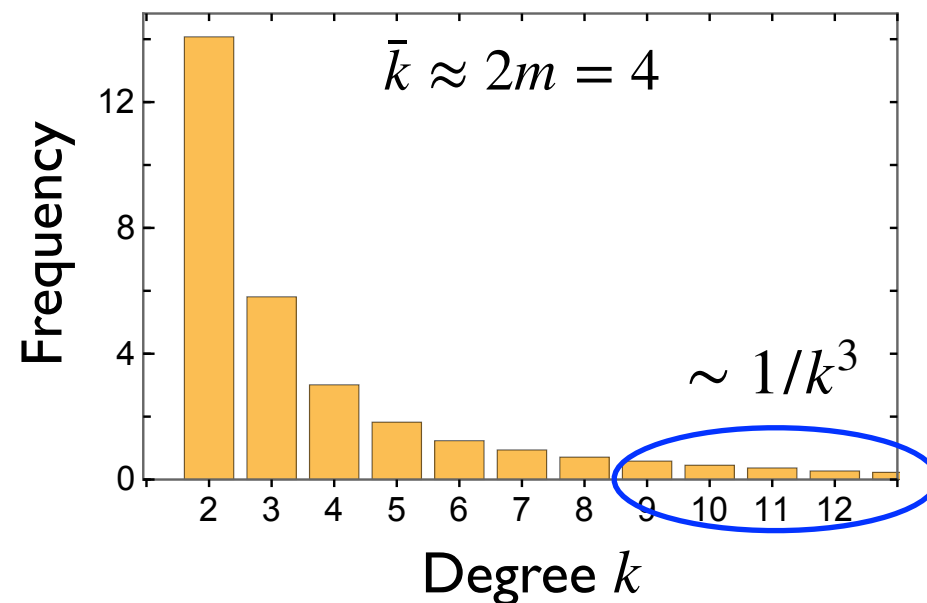
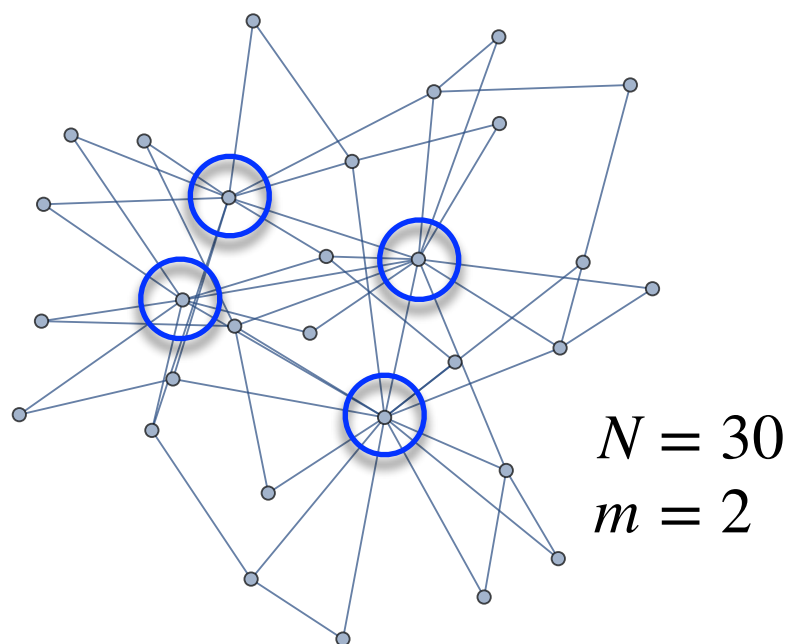


Increased heterogeneity: Scale-free graphs

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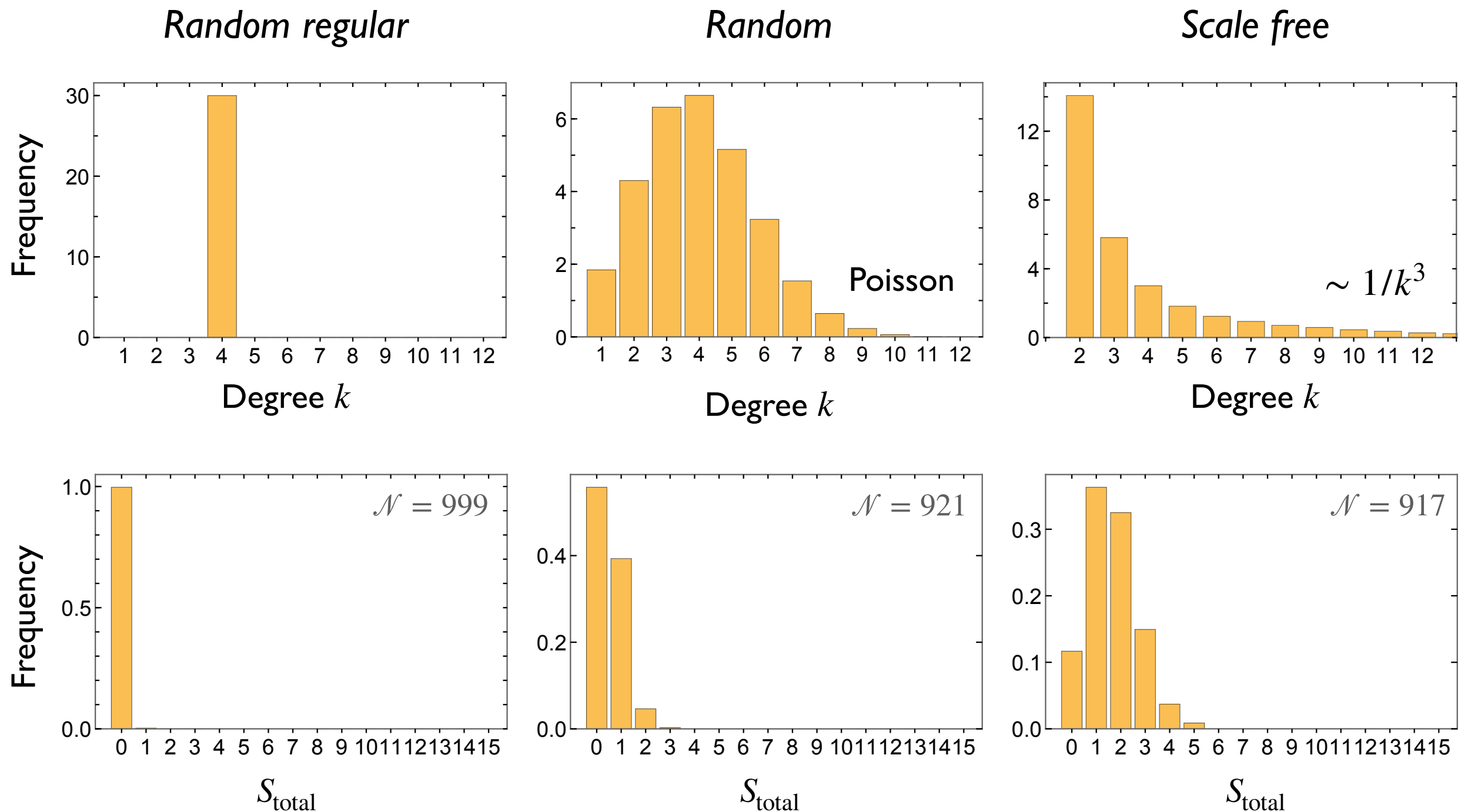
Barabasi-Albert: each new node connects to m existing nodes following preferential attachment — $p_i \propto k_i$

RMP 74, 47 (2002)



Enhanced magnetization

Summary: Magnetization grows w/ heterogeneity



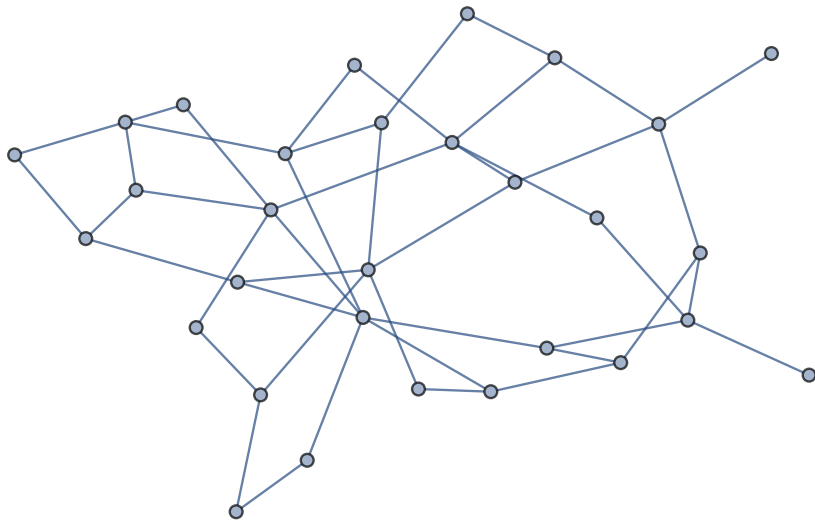
Results for: $N = 30, \bar{k} = 4$

Frustration level

Insensitivity to frustration level

Remove all triangles

Bayati, Montanari, Saberi,
arXiv:0811.2853

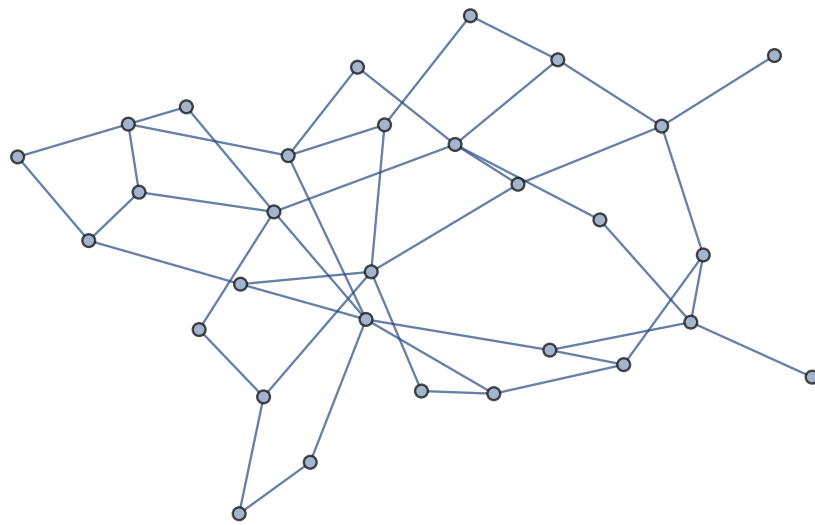


$$N = 30, N_e = 45, N_{\Delta} = 0$$

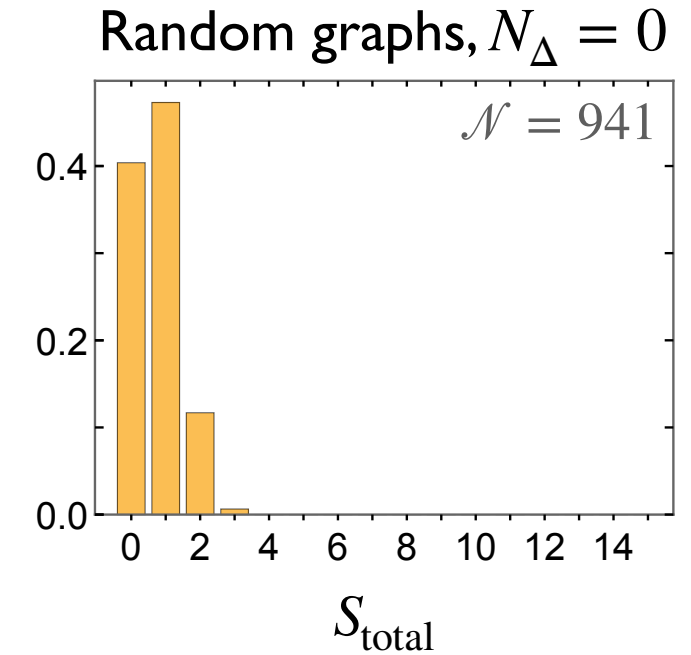
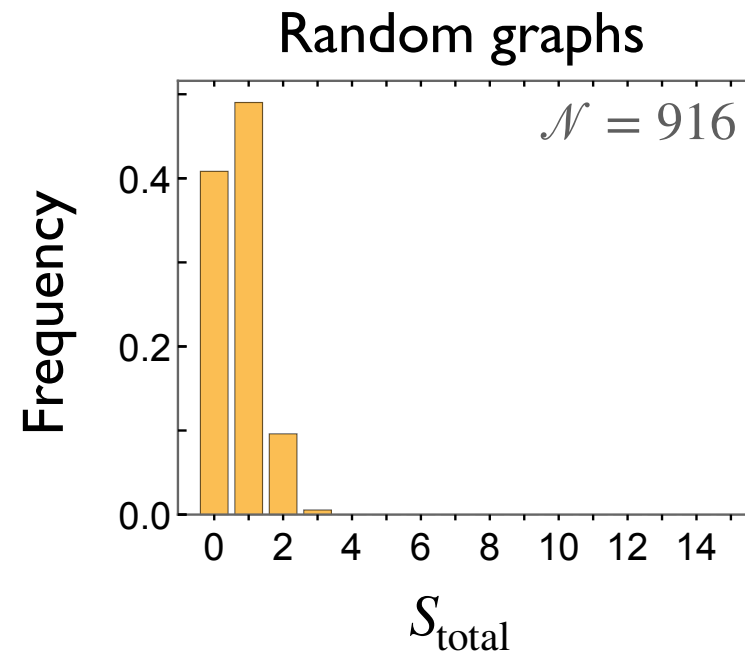
Insensitivity to frustration level

Remove all triangles \implies Spin distribution unaffected

Bayati, Montanari, Saberi,
arXiv:0811.2853



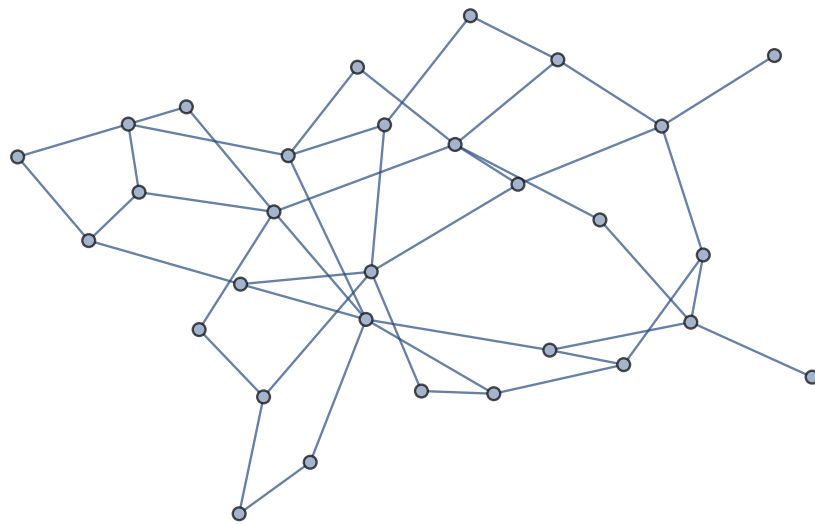
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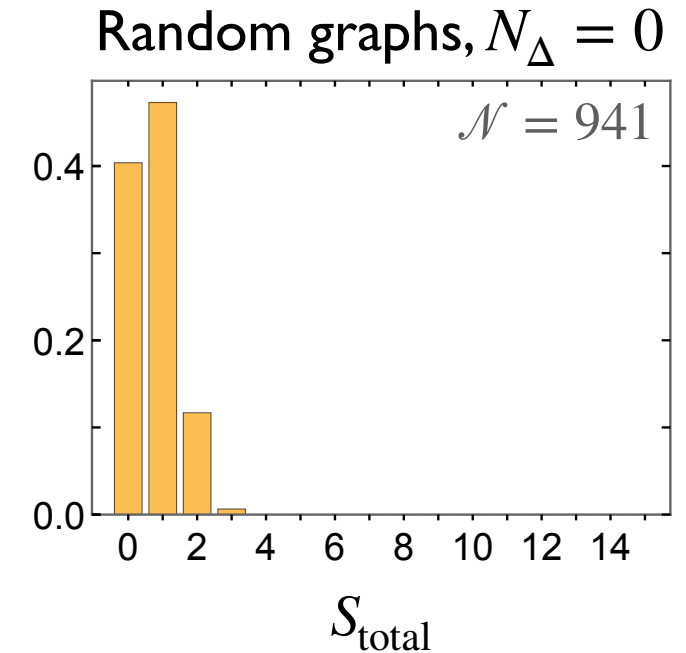
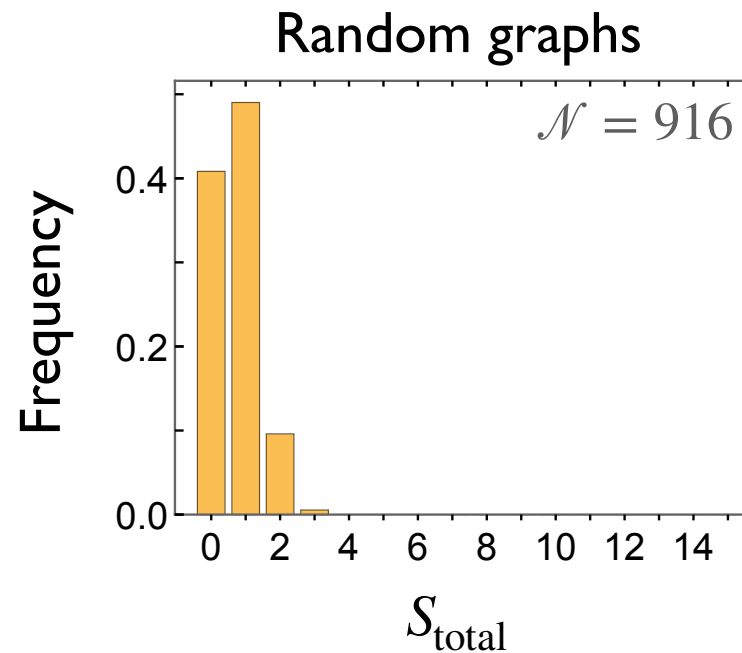
Insensitivity to frustration level

Remove all triangles \implies Spin distribution unaffected

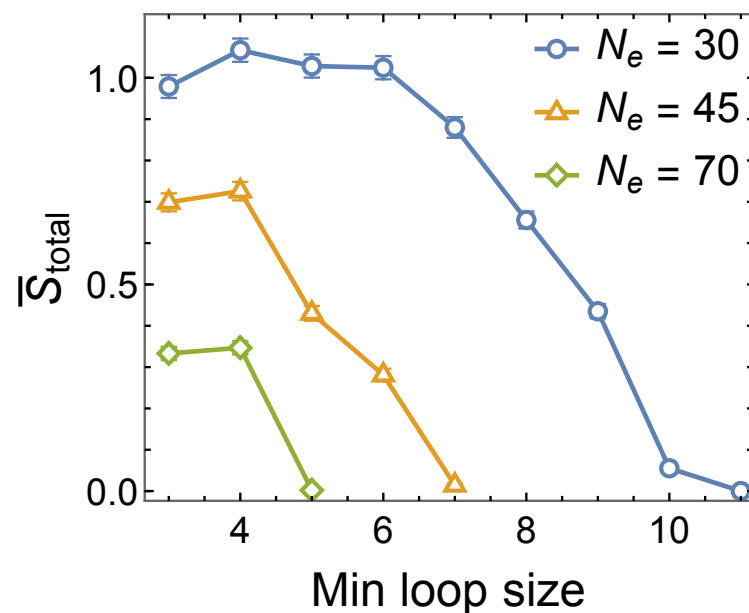
Bayati, Montanari, Saberi,
arXiv:0811.2853



$N = 30, N_e = 45, N_\Delta = 0$



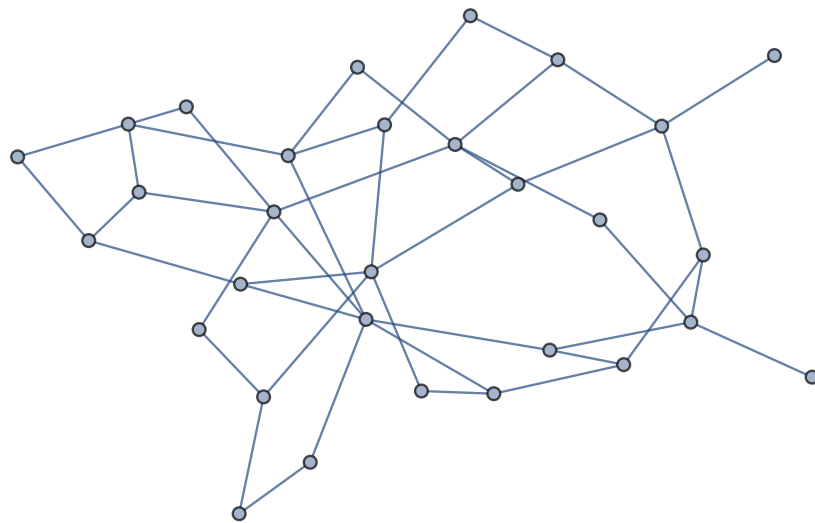
Remove short loops



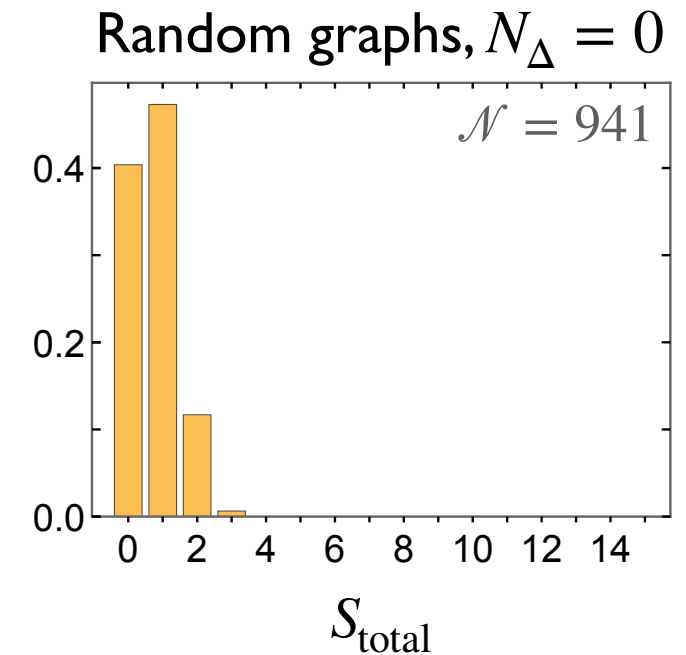
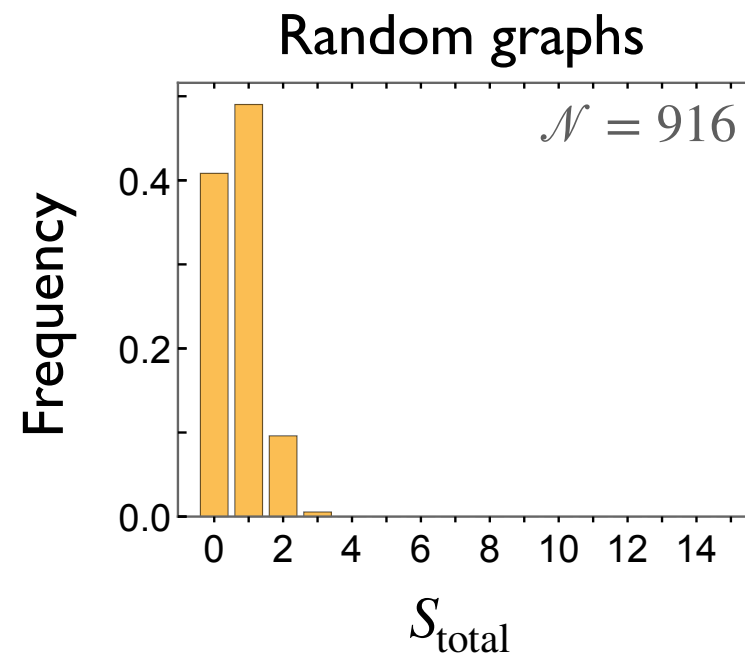
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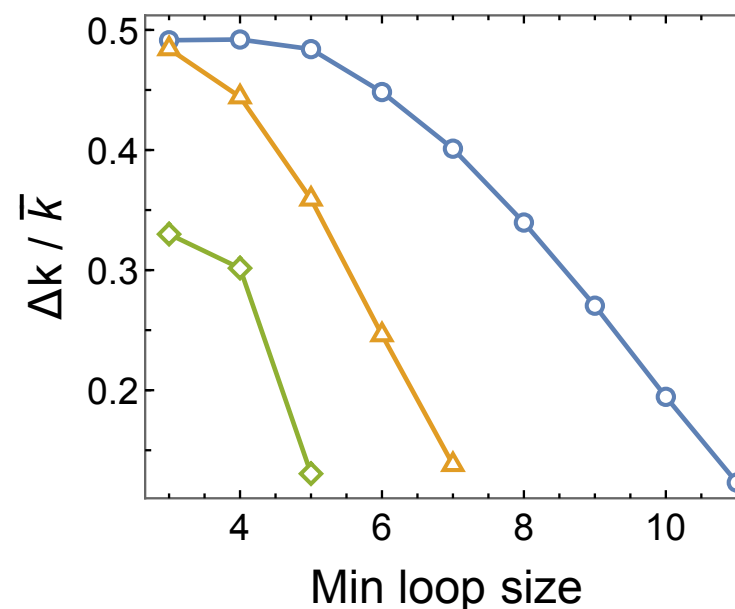
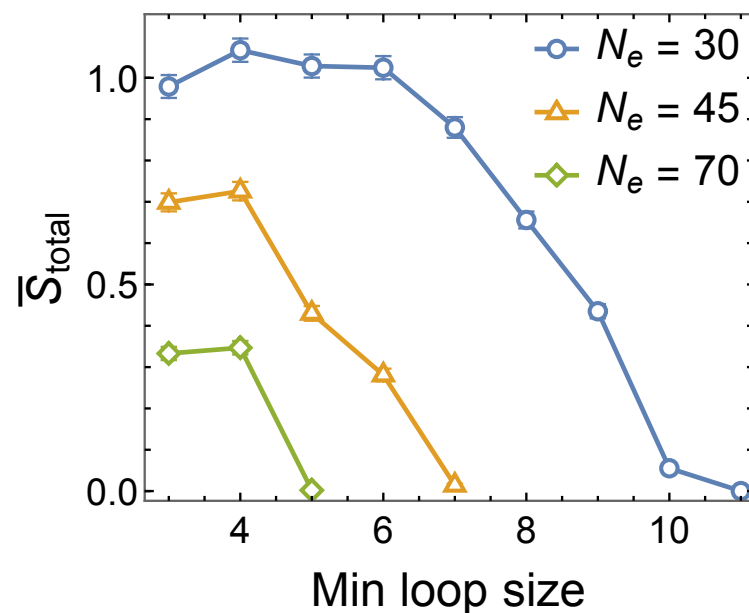
Bayati, Montanari, Saberi,
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$N = 30, N_e = 45, N_\Delta = 0$



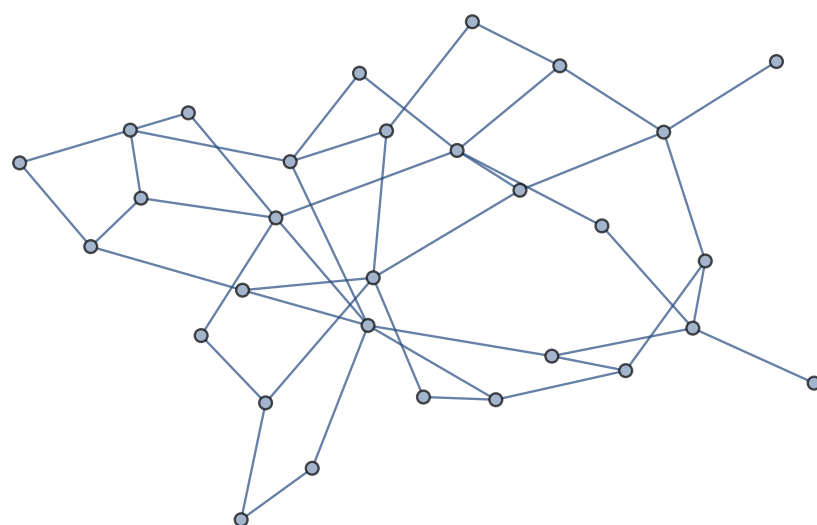
Remove short loops \implies Magnetization follows heterogeneity



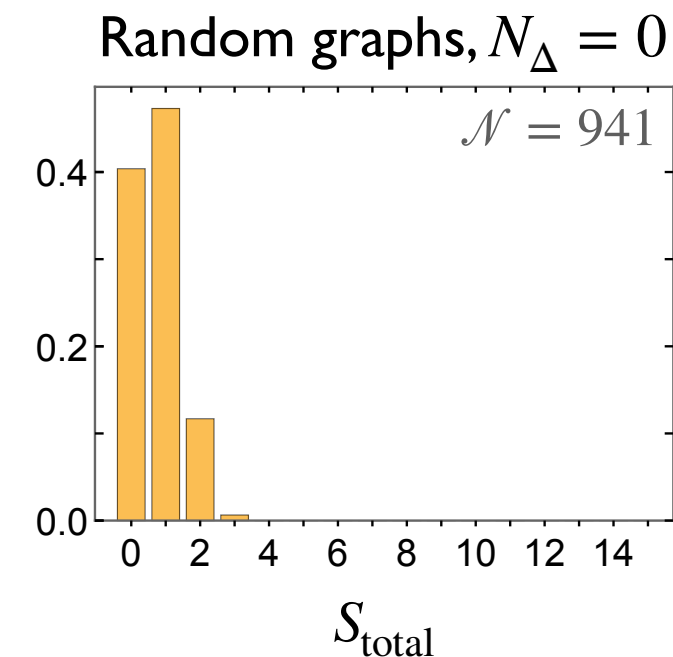
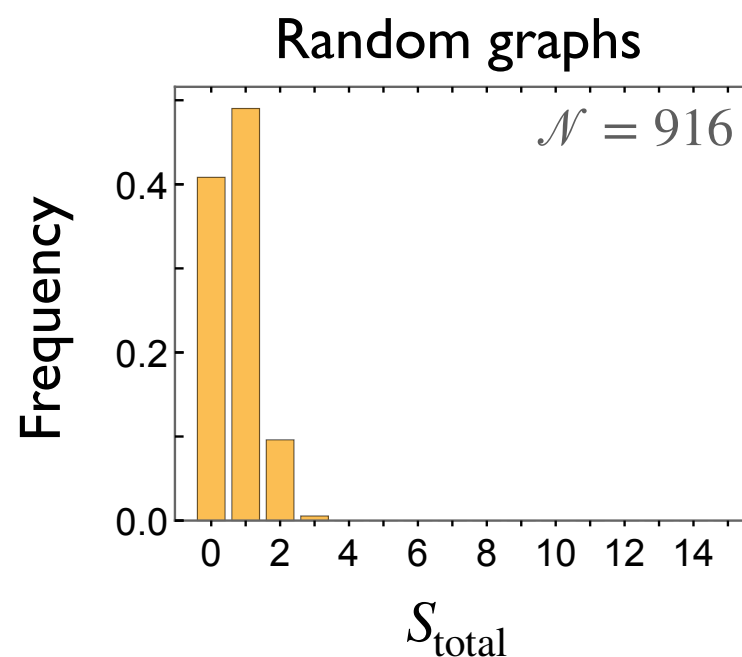
Insensitivity to frustration level

Remove all triangles \implies Spin distribution unaffected

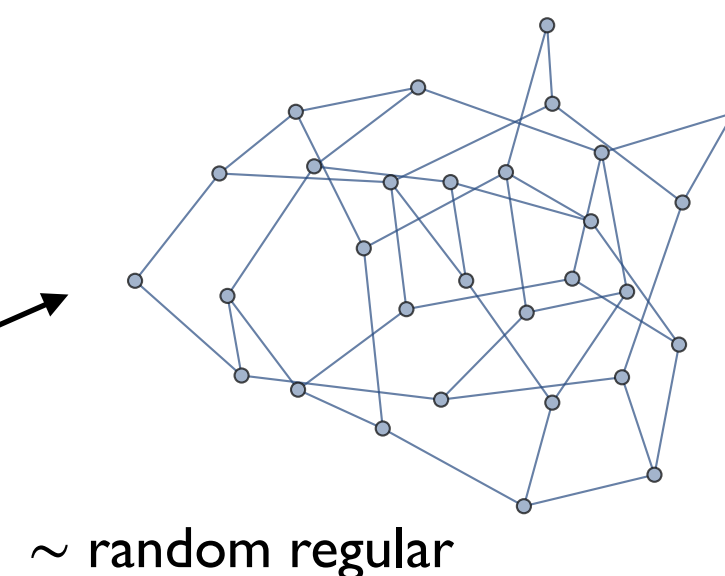
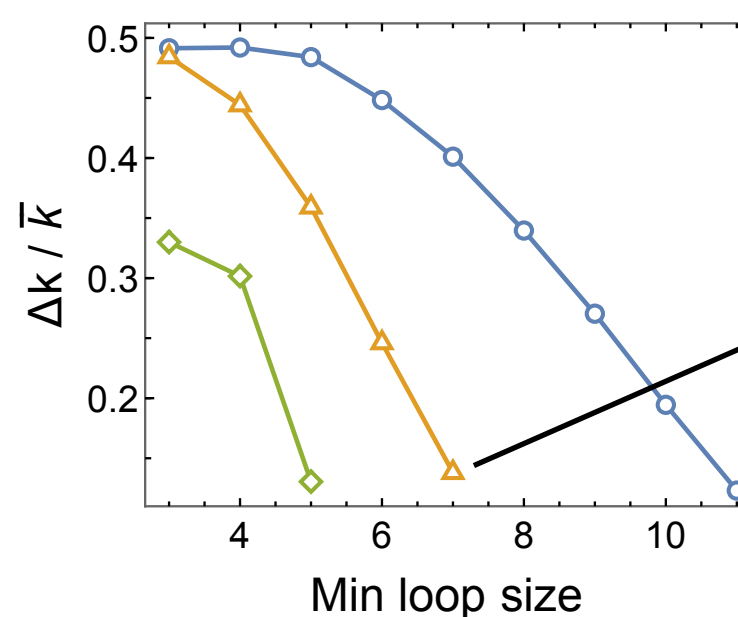
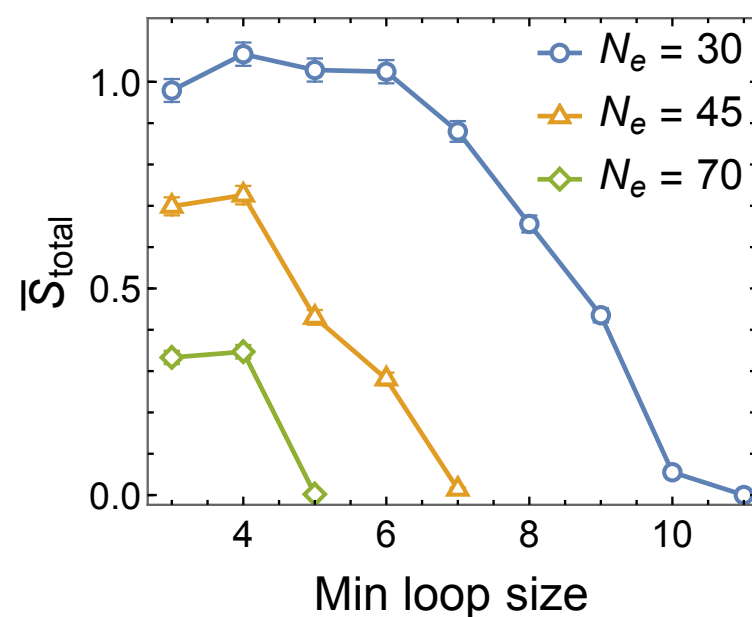
Bayati, Montanari, Saberi,
arXiv:0811.2853



$N = 30, N_e = 45, N_\Delta = 0$



Remove short loops \implies Magnetization follows heterogeneity



Insensitivity to frustration level

Tune N_{Δ} without changing degree distribution ($\sim 1/k^3$)

Holme and Kim,
PRE 65, 026107 (2002)

Insensitivity to frustration level

Tune N_{Δ} without changing degree distribution ($\sim 1/k^3$)

Holme and Kim,
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For every new node:

- (1) Connect to existing node i with prob $p_i \propto k_i$
- (2) With prob p connect to a neighbor of i , else repeat (1)

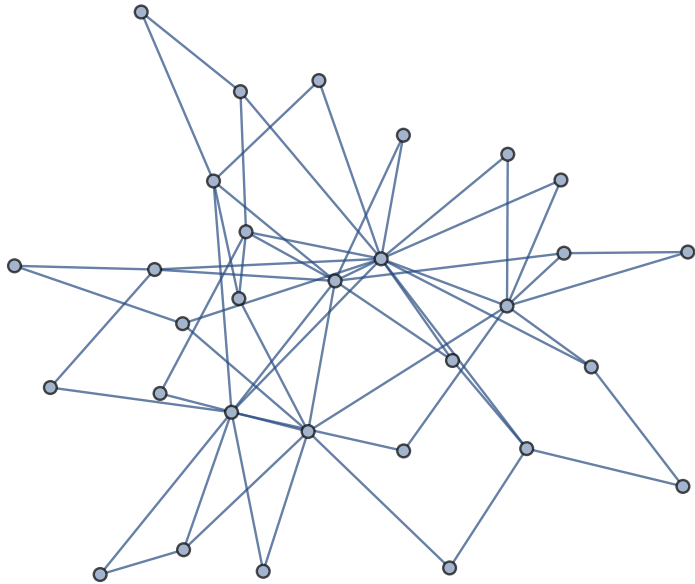
Insensitivity to frustration level

Tune N_{Δ} without changing degree distribution ($\sim 1/k^3$)

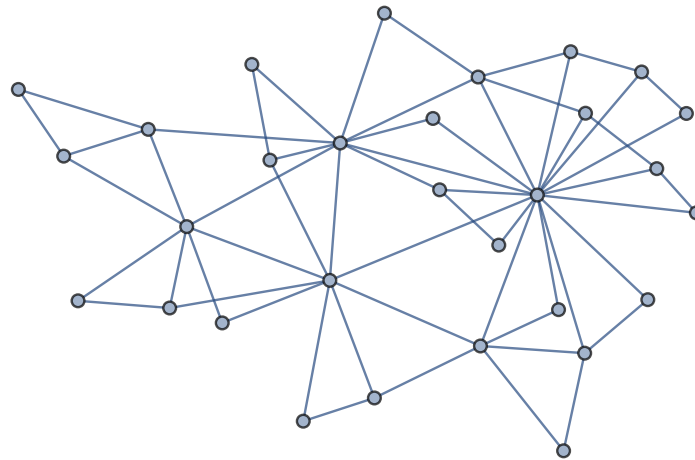
Holme and Kim,
PRE 65, 026107 (2002)

For every new node:

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Fewer triangles



More triangles

Insensitivity to frustration level

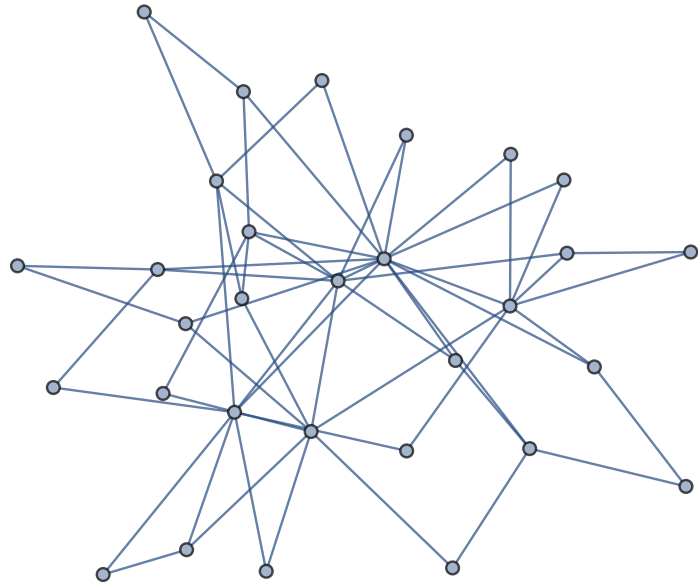
Tune N_{Δ} without changing degree distribution ($\sim 1/k^3$)

\Rightarrow **Weak variation of S_{total}**

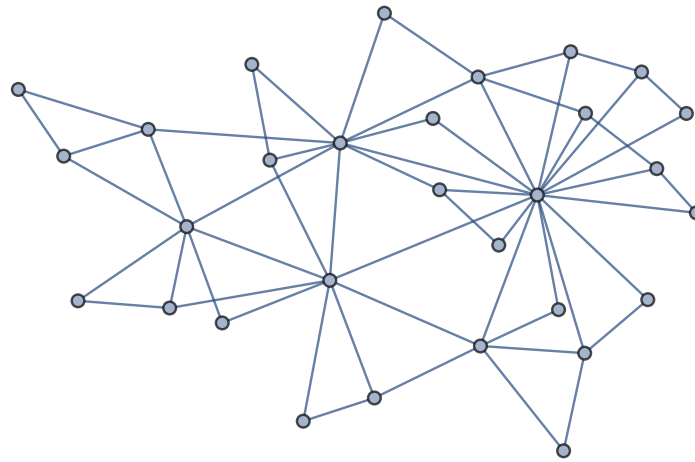
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For every new node:

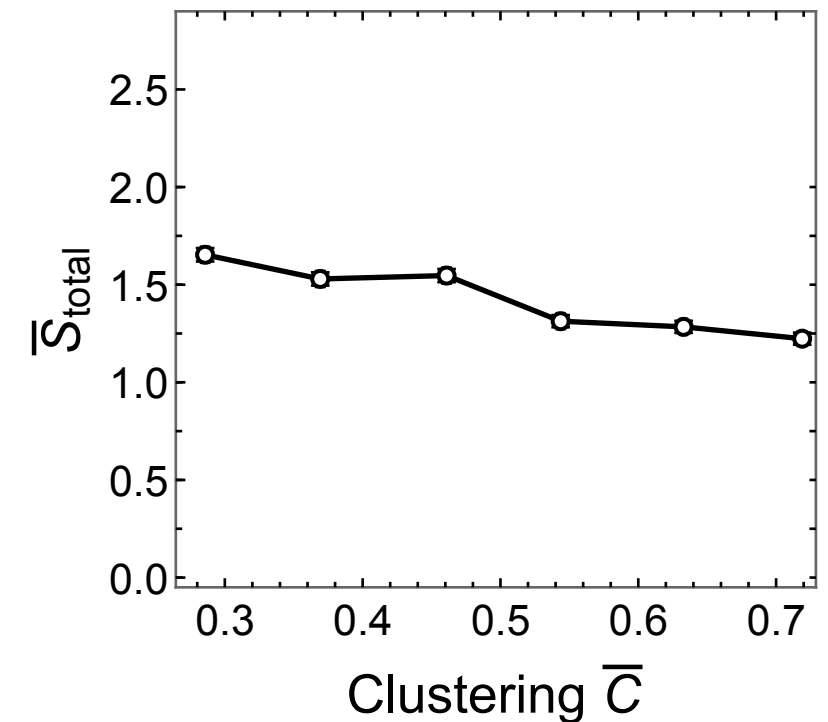
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Fewer triangles



More triangles



Insensitivity to frustration level

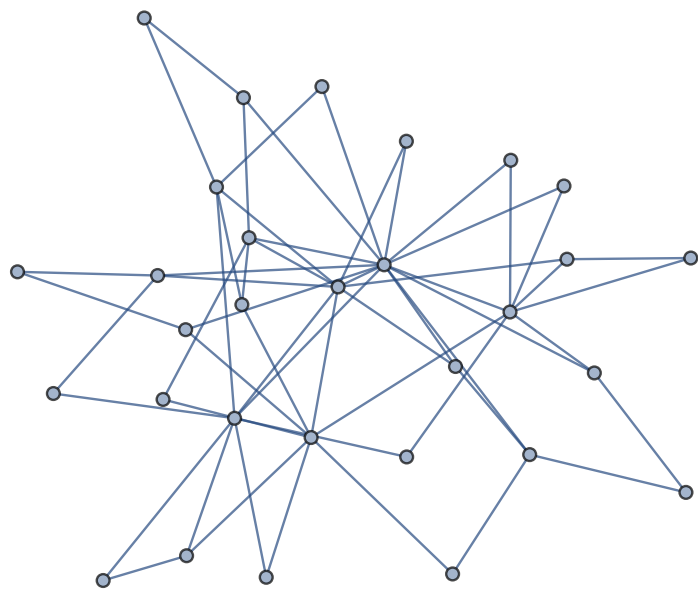
Tune N_{Δ} without changing degree distribution ($\sim 1/k^3$)

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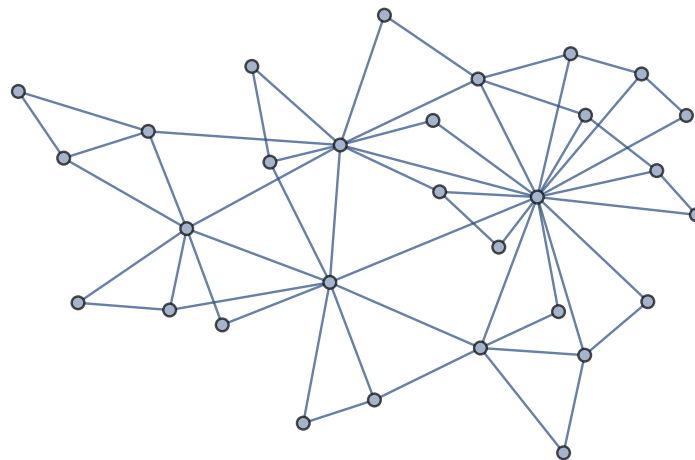
Holme and Kim,
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For every new node:

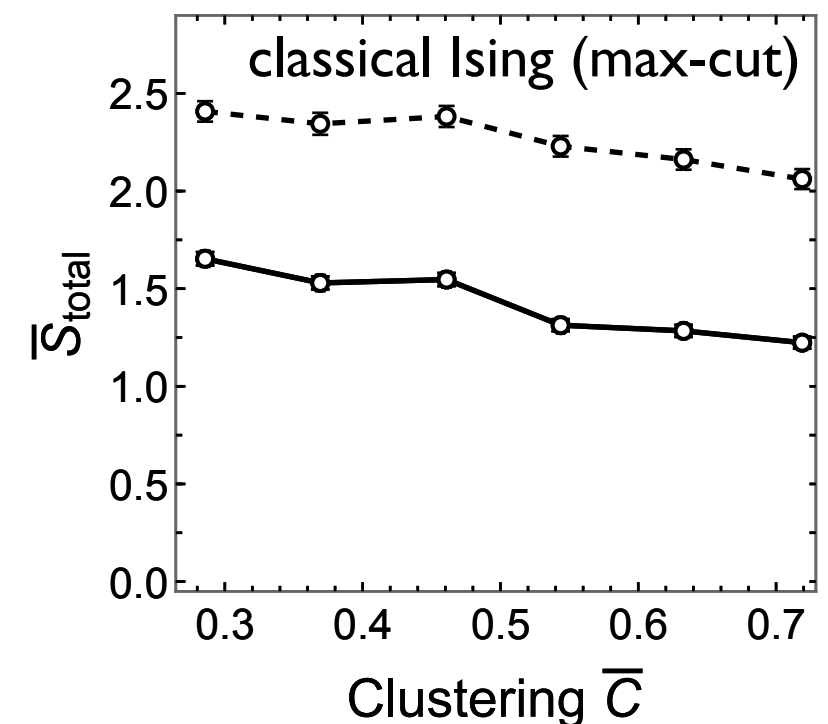
- (1) Connect to existing node i with prob $p_i \propto k_i$
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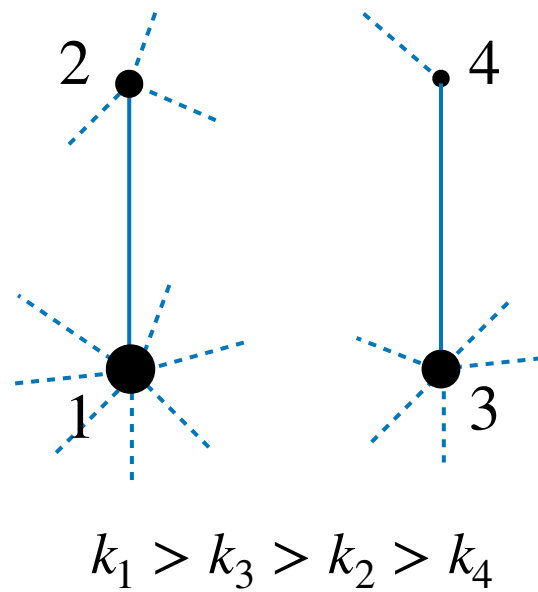


Assortativity

Tuning assortativity

Van Mieghem *et al*,
EPJ-B 76, 643 (2010)

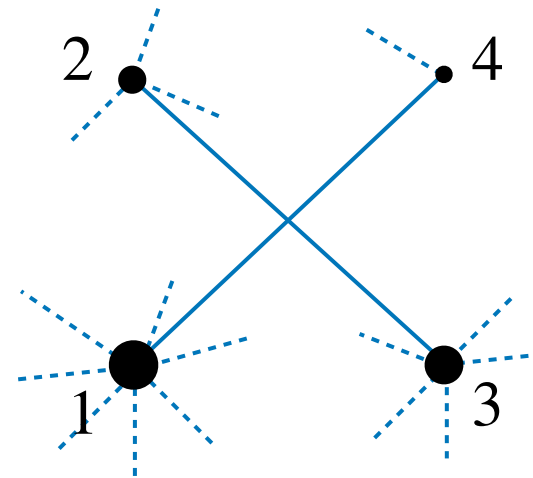
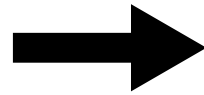
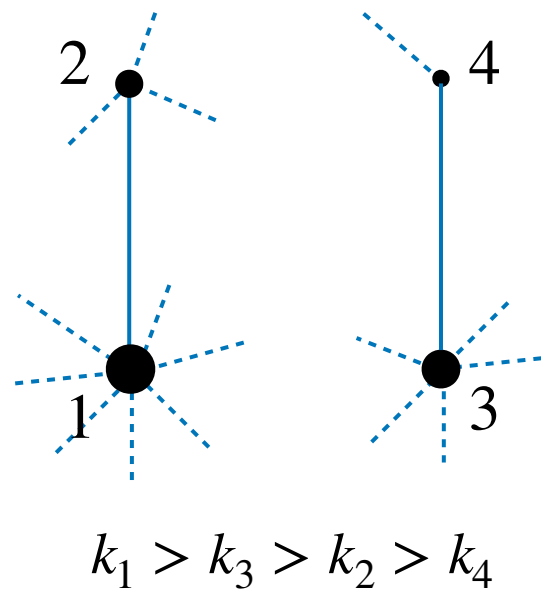
Degree-preserving rewiring



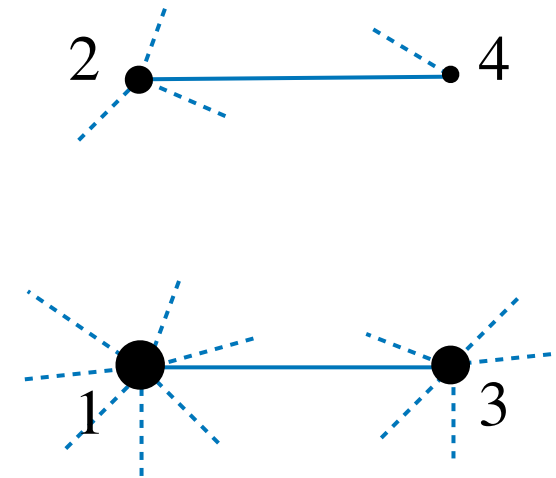
Tuning assortativity

Van Mieghem et al,
EPJ-B 76, 643 (2010)

Degree-preserving rewiring



Disassortative

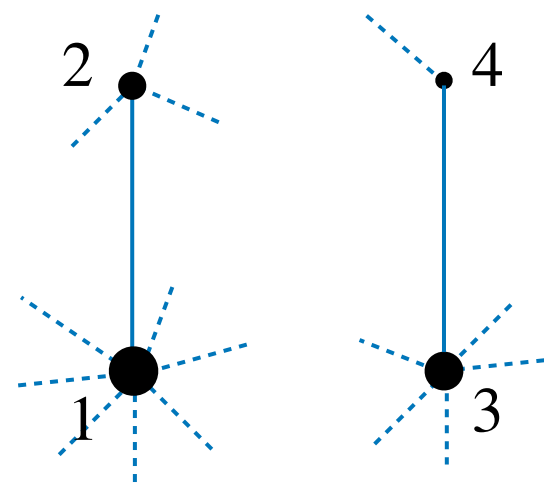


Assortative

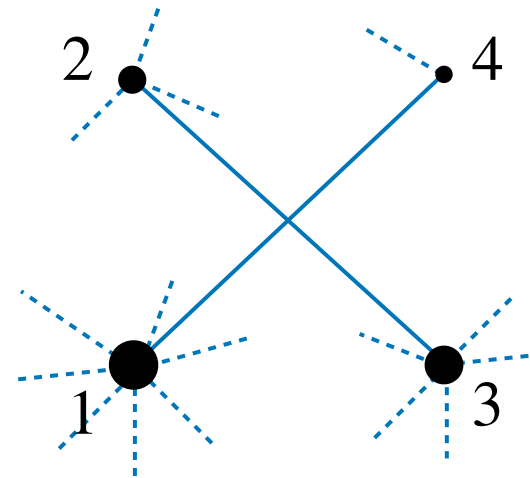
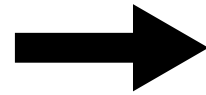
Tuning assortativity

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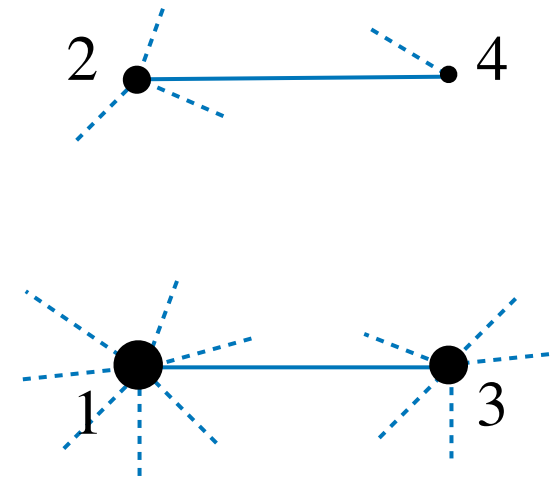
Degree-preserving rewiring



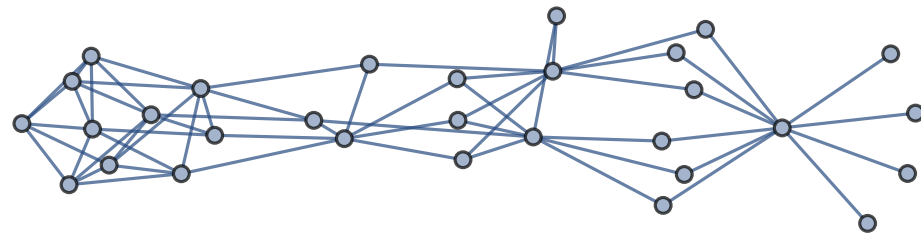
$$k_1 > k_3 > k_2 > k_4$$



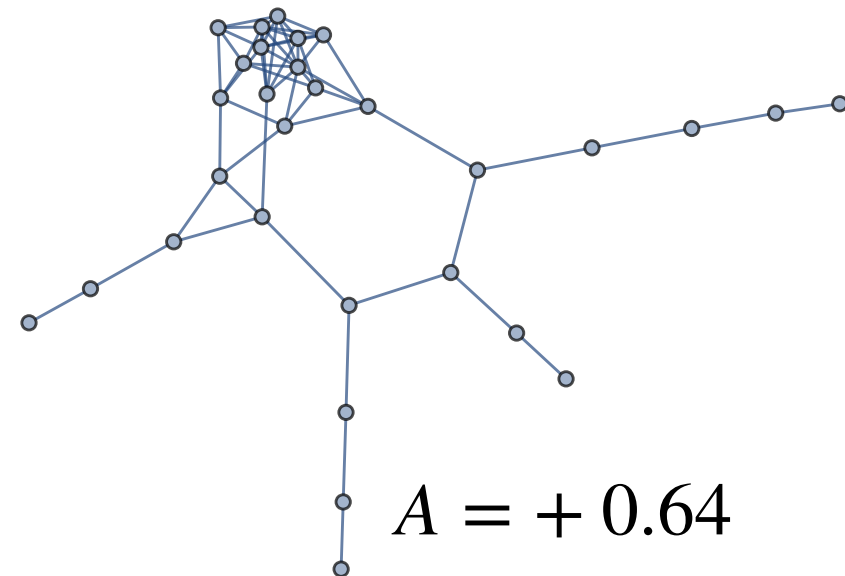
Disassortative



Assortative



$$A = -0.95$$

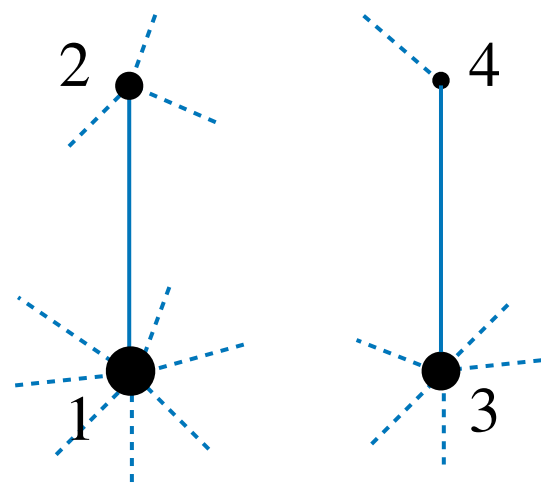


$$A = +0.64$$

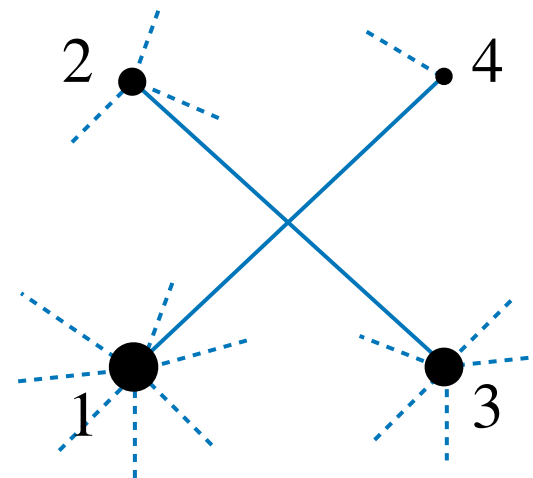
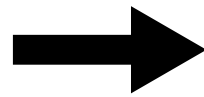
Tuning assortativity

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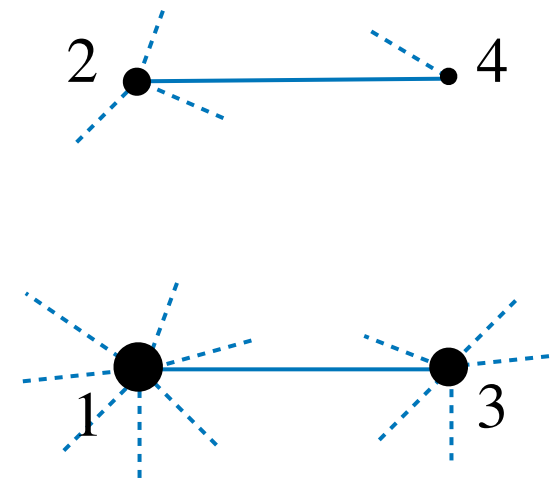
Degree-preserving rewiring



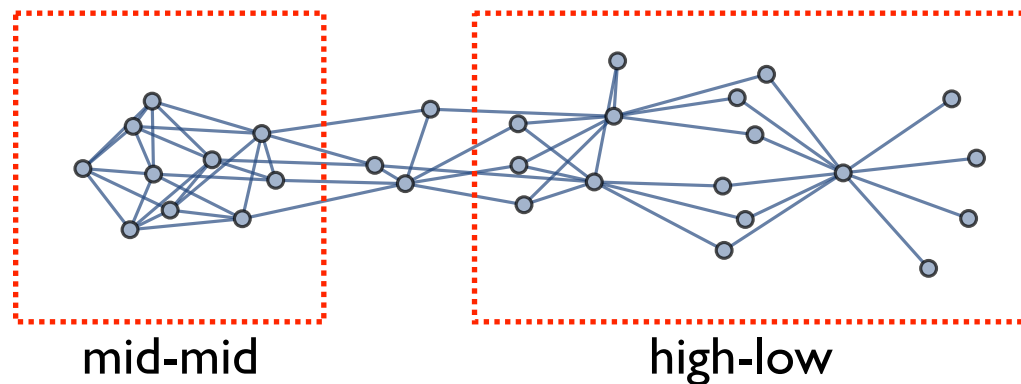
$$k_1 > k_3 > k_2 > k_4$$



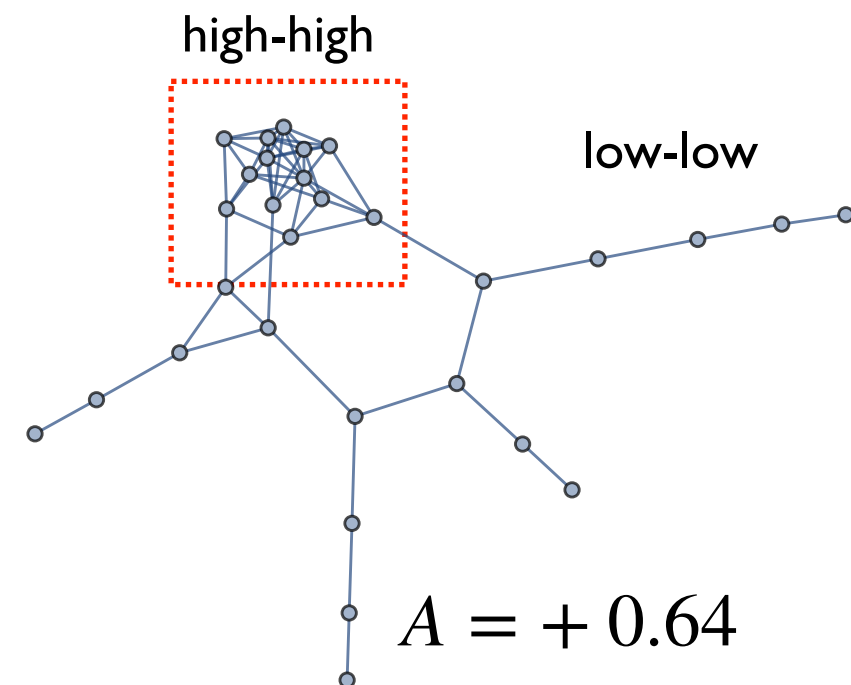
Disassortative



Assortative



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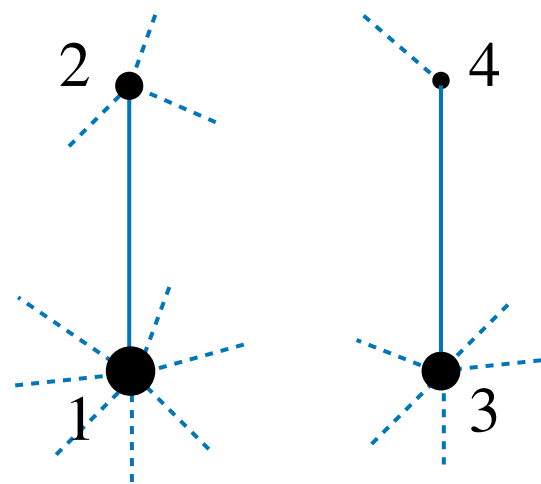


$$A = +0.64$$

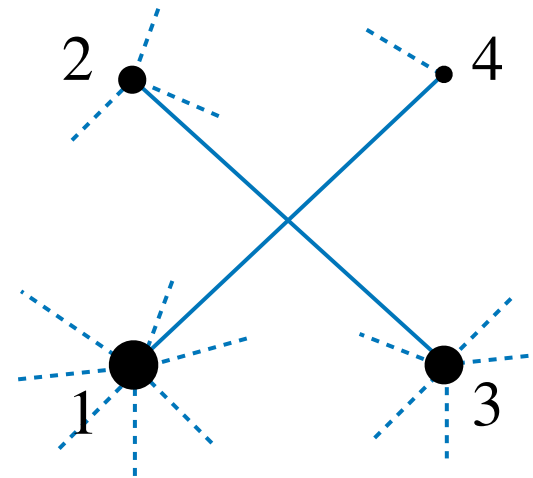
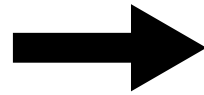
Tuning assortativity

Van Mieghem et al,
EPJ-B 76, 643 (2010)

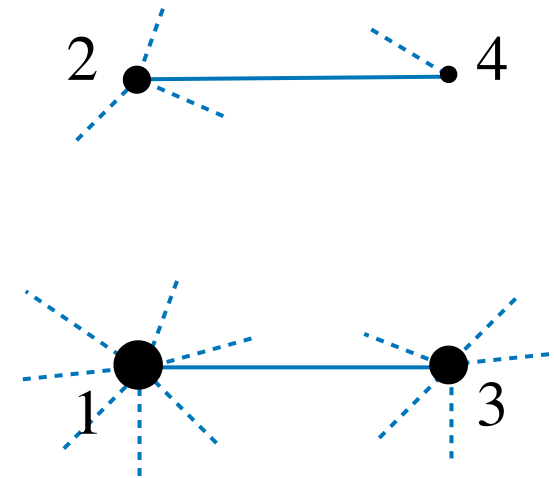
Degree-preserving rewiring



$$k_1 > k_3 > k_2 > k_4$$

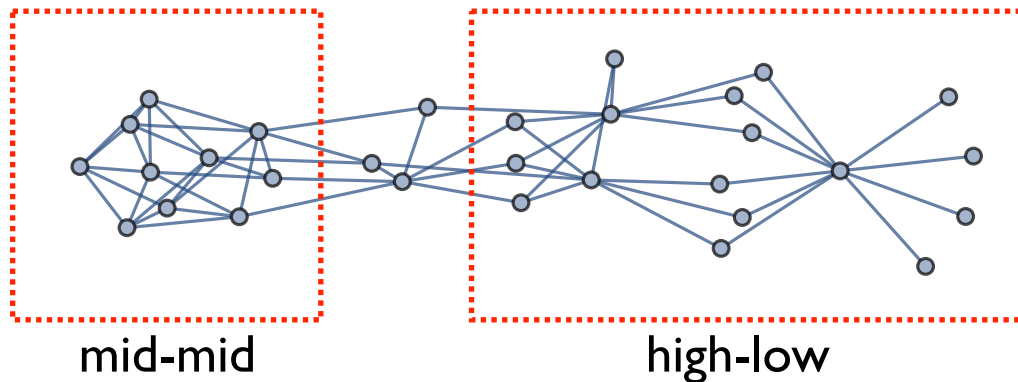


Disassortative

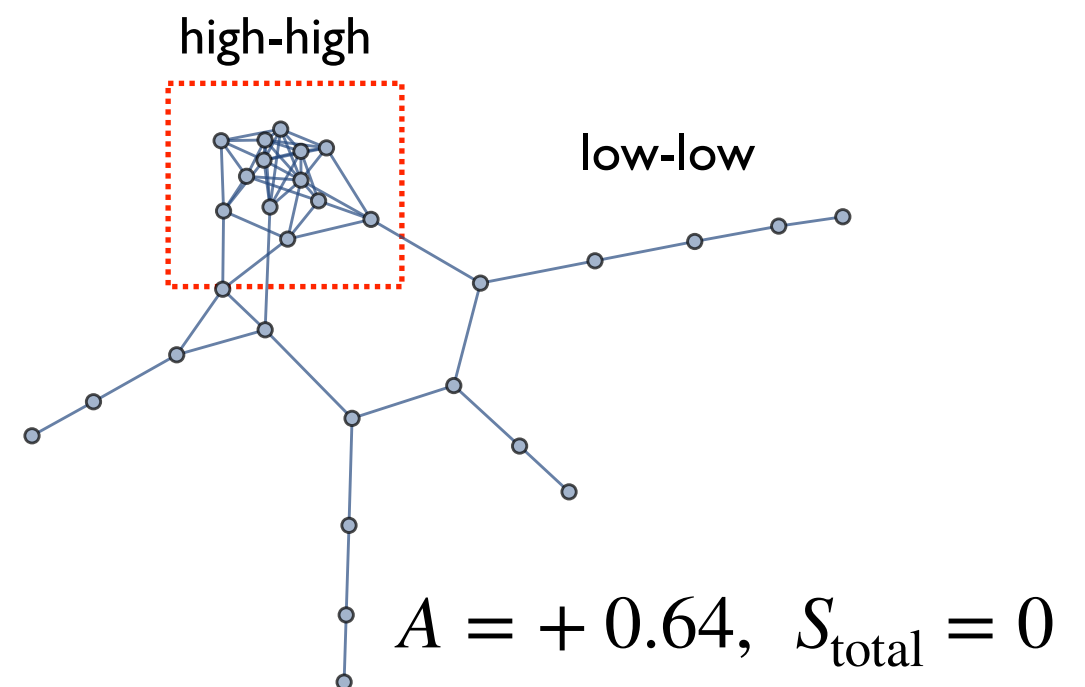


Assortative

Approximately bipartite w/
large sublattice imbalance

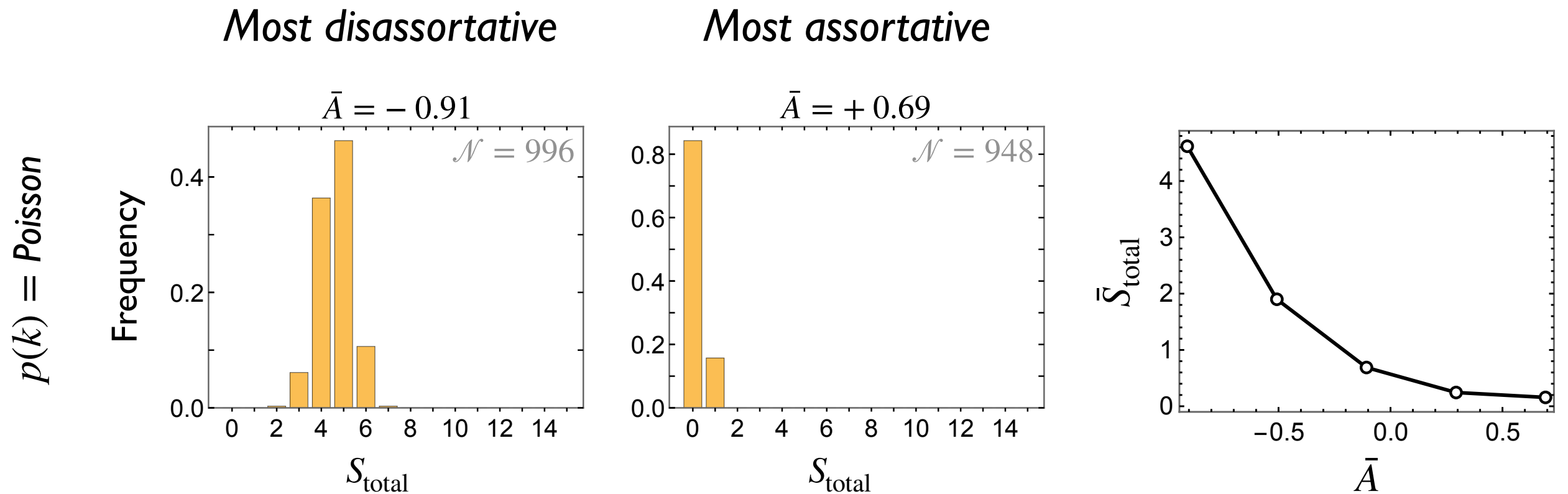


$$A = -0.95, S_{\text{total}} = 7$$



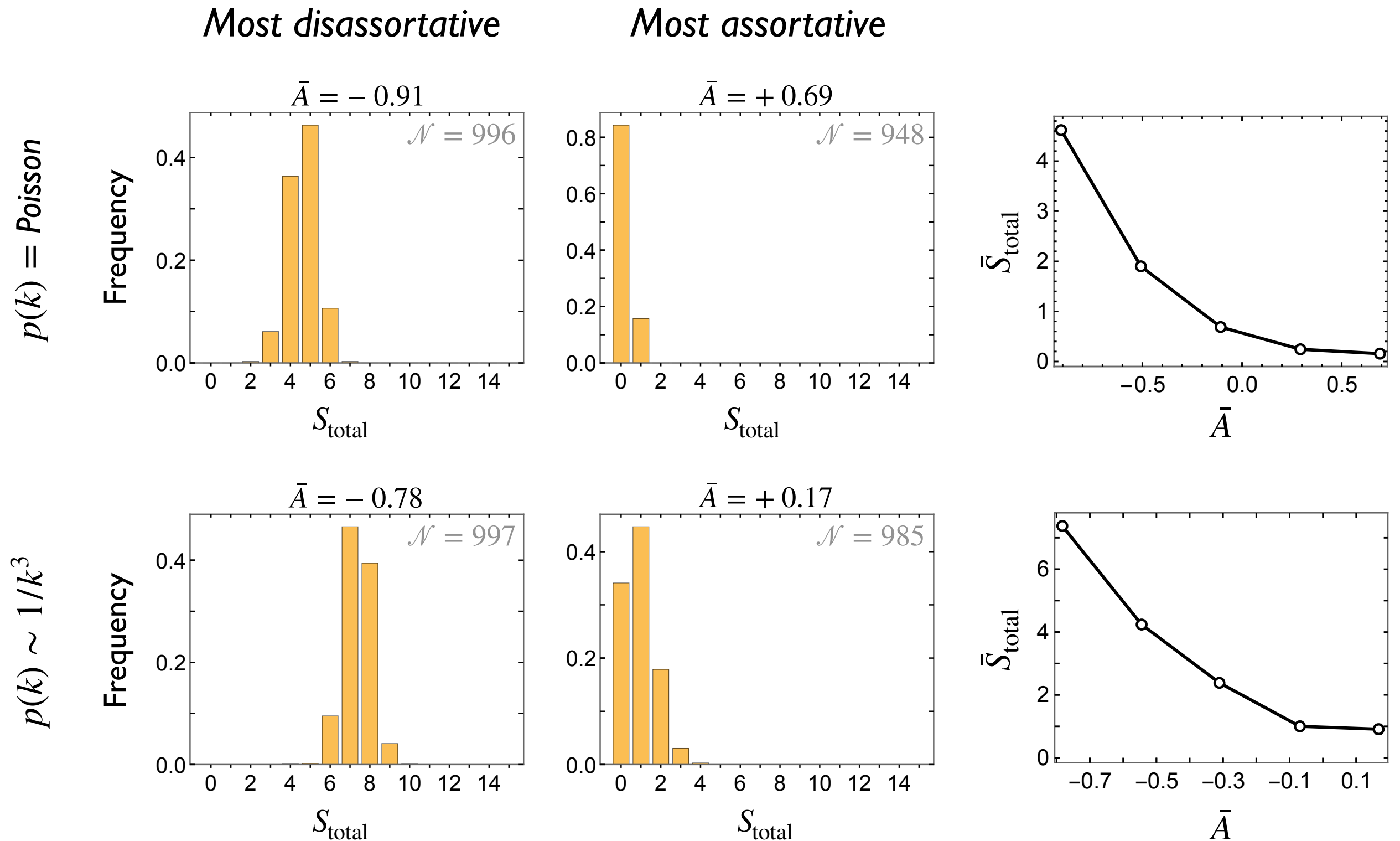
$$A = +0.64, S_{\text{total}} = 0$$

Magnetization falls w/ assortativity



Results for: $N = 30, \bar{k} = 4$

Magnetization falls w/ assortativity



Results for: $N = 30, \bar{k} = 4$

Putting together:
Tunable spin distribution

Embedding disassortative hubs: “Copy model”

Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

Alam, Perumalla, and Sanders
Data Sci. Eng. 4, 61 (2019)

Embedding disassortative hubs: “Copy model”

Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

Alam, Perumalla, and Sanders
Data Sci. Eng. 4, 61 (2019)

For every new node j :

- Randomly pick an existing node i
- With prob p connect (i, j)
- With prob $1 - p$ connect to a neighbor of i w/ $p_{i'} \propto k_{i'}$
- Repeat m times

Embedding disassortative hubs: “Copy model”

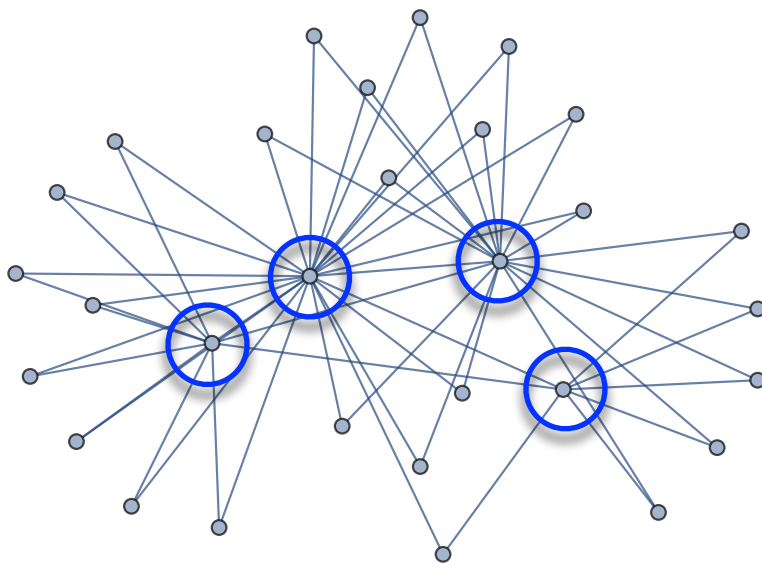
Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

Alam, Perumalla, and Sanders
Data Sci. Eng. 4, 61 (2019)

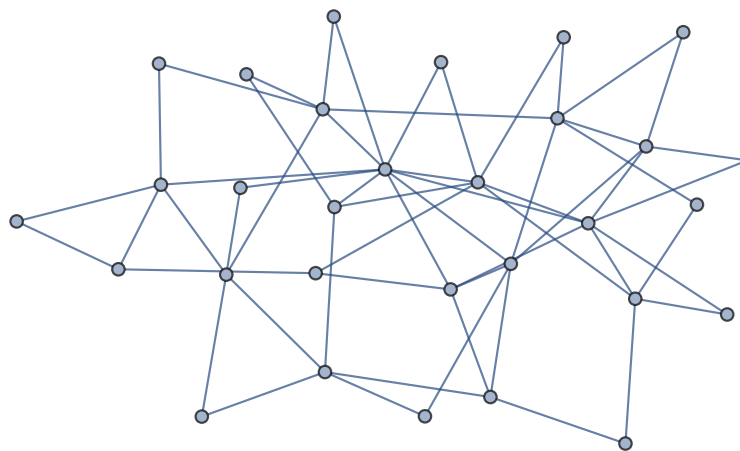
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- Repeat m times

$p = 0$: embedded hubs



$p = 1$: no hub



$N = 30, m = 2$

Embedding disassortative hubs: “Copy model”

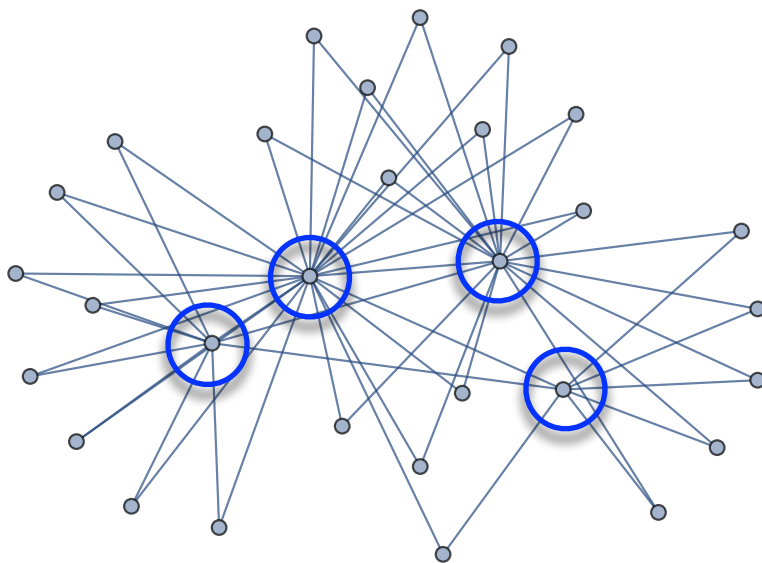
Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

Alam, Perumalla, and Sanders
Data Sci. Eng. 4, 61 (2019)

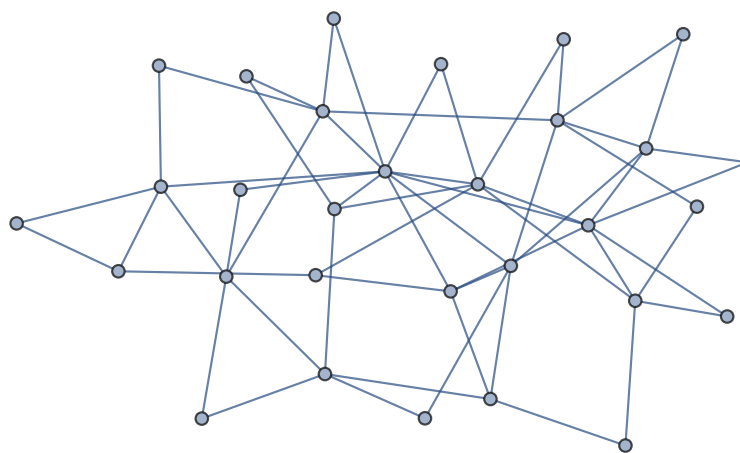
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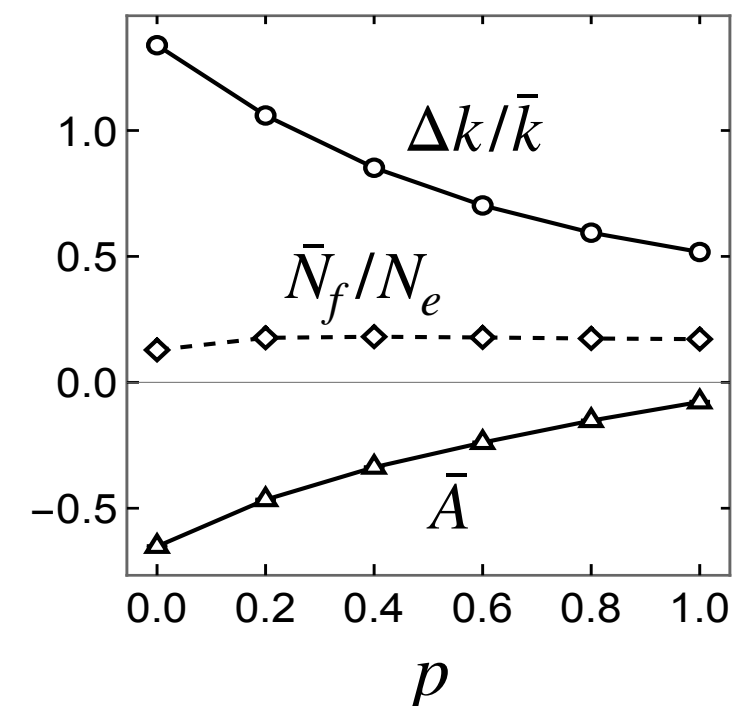
$p = 0$: embedded hubs



$p = 1$: no hub



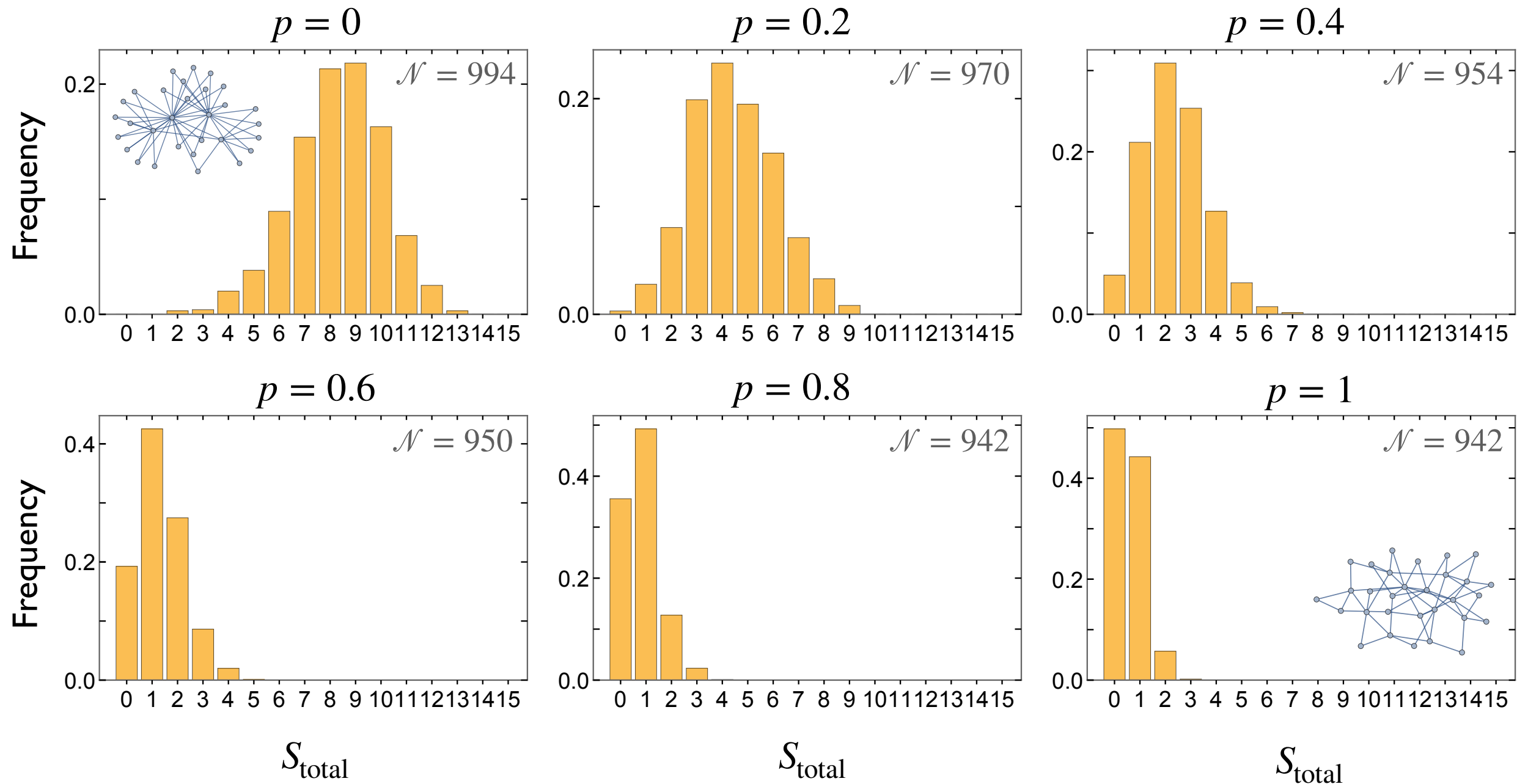
Heterogeneity + Assortativity



$N = 30, m = 2$

Embedding disassortative hubs: “Copy model”

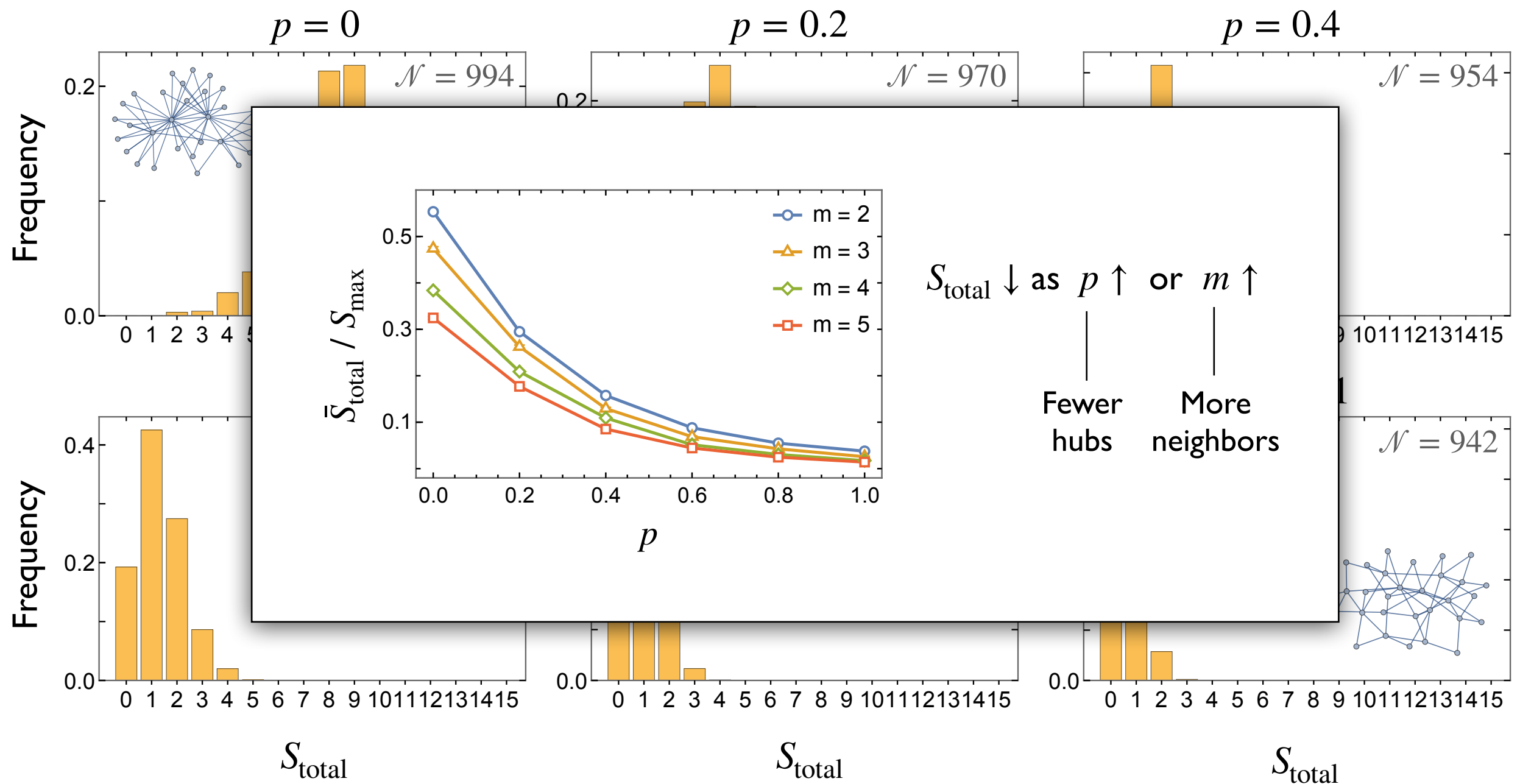
Tunable spin distribution:



$$N = 30, m = 2$$

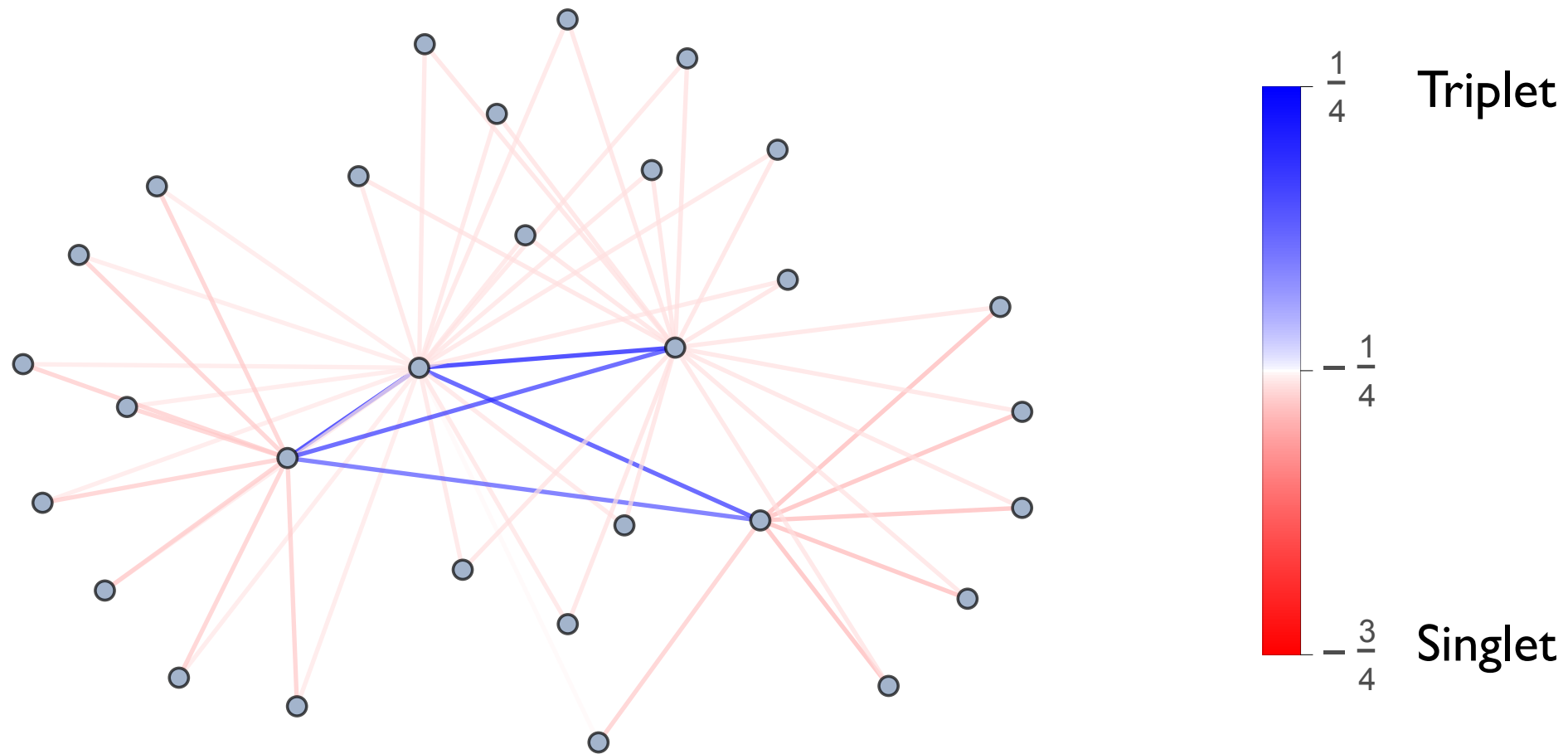
Embedding disassortative hubs: “Copy model”

Tunable spin distribution:



Embedding disassortative hubs: “Copy model”

Pairwise alignment: $\langle \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j \rangle$



Hubs aligned opposite to other nodes

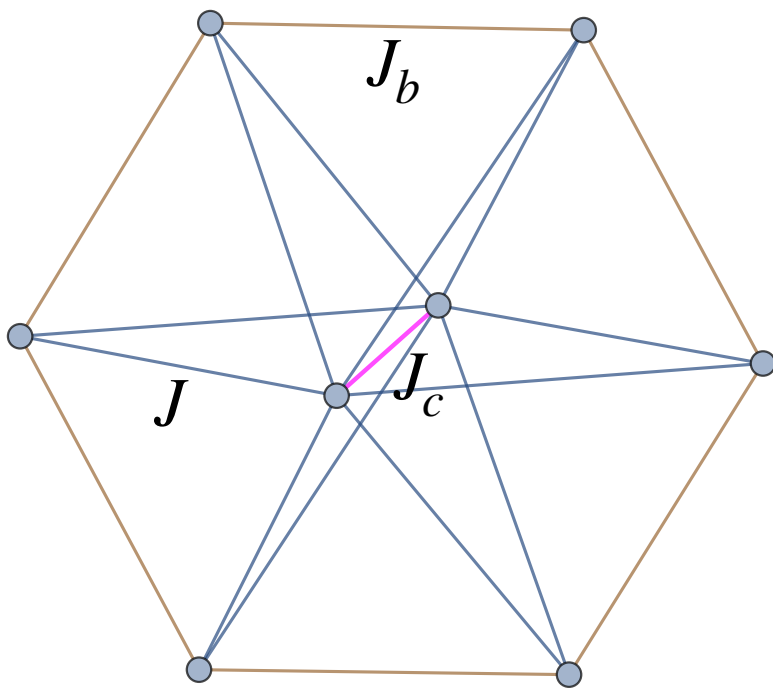
Can we tune S_{total} in a
non-random (frustrated) graph?

Tuning S_{total} in a wheel

N_c central spins (fully connected) + N_b outer spins

$$J, J_b, J_c > 0$$

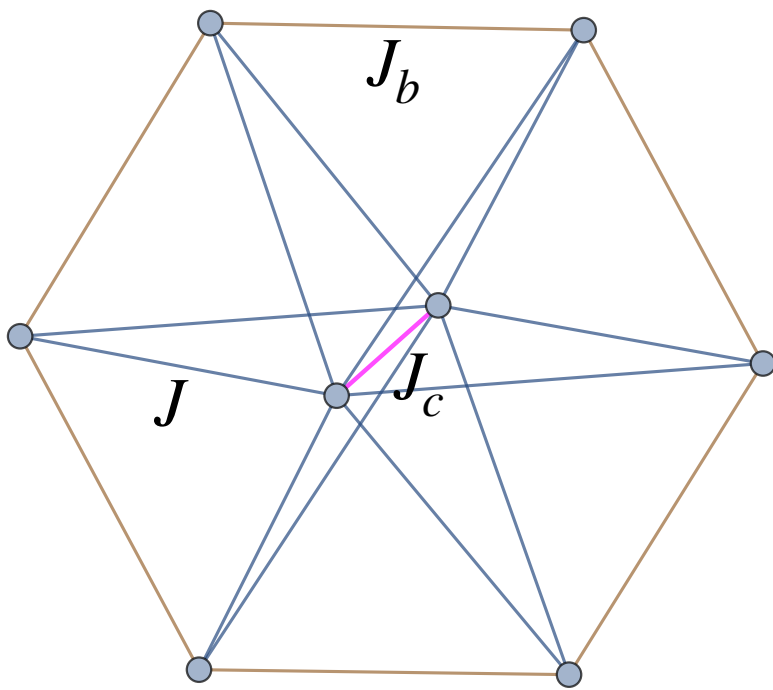
$$\hat{H} = (J_c/2) \hat{S}_c^2 + J \hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$



Tuning S_{total} in a wheel

N_c central spins (fully connected) + N_b outer spins

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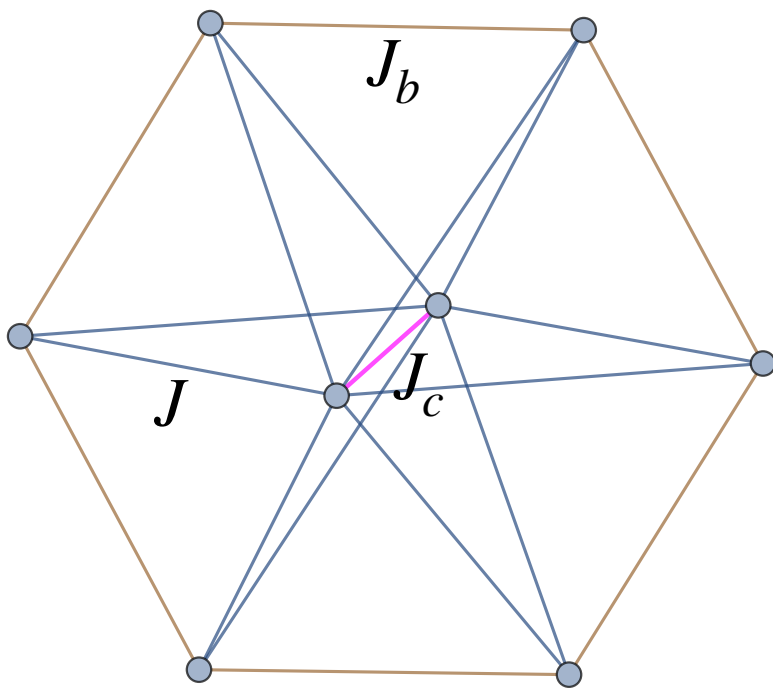
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$$\bullet J_c \gg J : S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$$

Tuning S_{total} in a wheel

N_c central spins (fully connected) + N_b outer spins

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$$\hat{H} = (J_c/2) \hat{S}_c^2 + J \hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

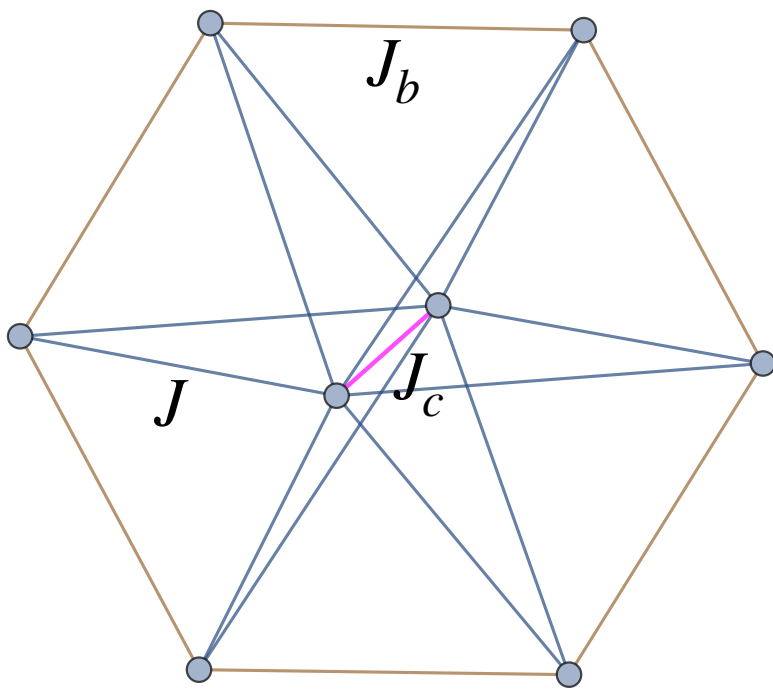
- $J_c \gg J : S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$

- Lower $J_c : S_c \sim 1 \implies$ if $J_b \ll J : S_b = S_b^{\text{max}} = N_b/2$
 $S_{\text{total}} \sim (N_b - 1)/2$

Tuning S_{total} in a wheel

N_c central spins (fully connected) + N_b outer spins

$$J, J_b, J_c > 0$$



$$\hat{H} = (J_c/2) \hat{S}_c^2 + J \hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

- $J_c \gg J : S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$

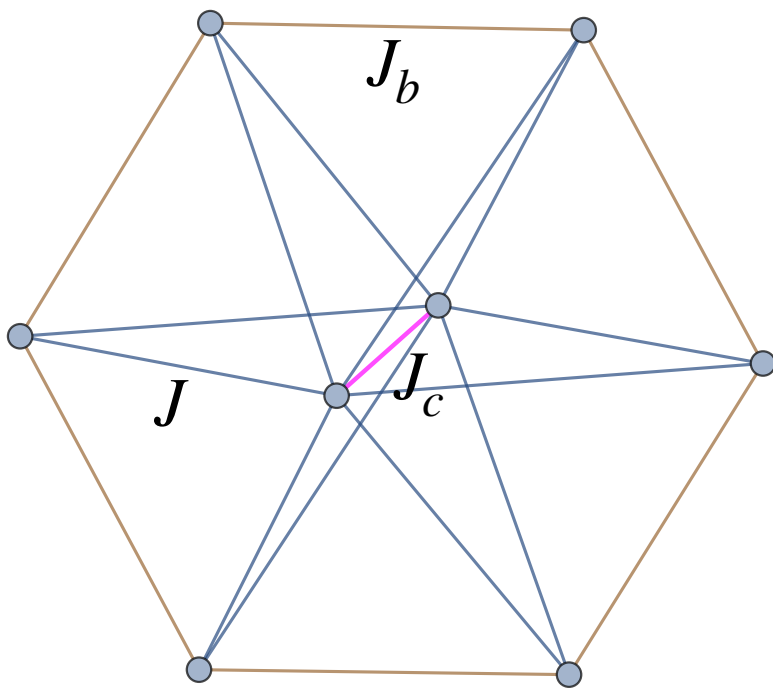
- Lower $J_c : S_c \sim 1 \implies$ if $J_b \ll J : S_b = S_b^{\text{max}} = N_b/2$
 $S_{\text{total}} \sim (N_b - 1)/2$

- $J_c/J \downarrow \implies S_c \uparrow, J_b/J \uparrow \implies S_b \downarrow$ —Variable S_{total}

Tuning S_{total} in a wheel

N_c central spins (fully connected) + N_b outer spins

$$J, J_b, J_c > 0$$



$$\hat{H} = (J_c/2) \hat{S}_c^2 + J \hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

$$\bullet J_c \gg J : S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$$

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$$S_{\text{total}} \sim (N_b - 1)/2$$

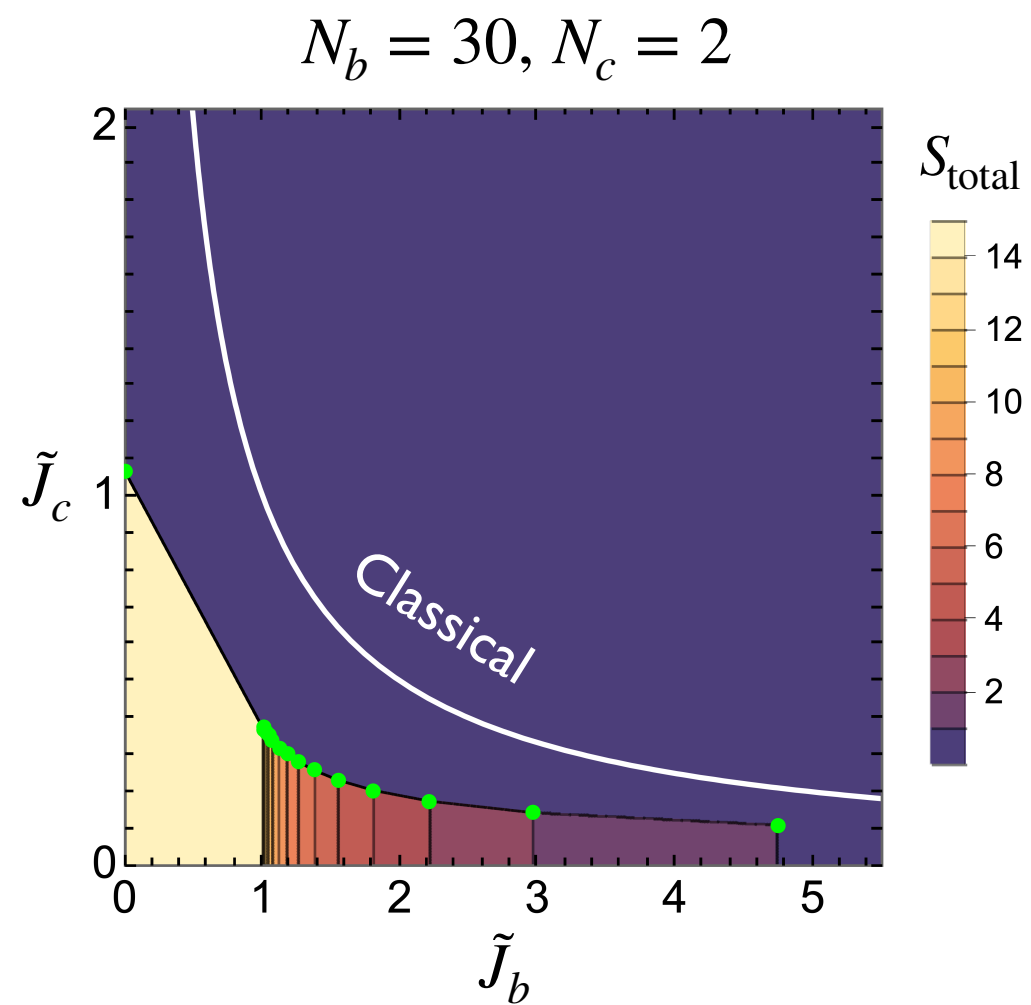
$$\bullet J_c/J \downarrow \implies S_c \uparrow, J_b/J \uparrow \implies S_b \downarrow \text{ --- Variable } S_{\text{total}}$$

Exactly solvable:

$S_b, S_c, S_{\text{total}}$ good quantum numbers — Energy minimized for $S_{\text{total}} = S_{bc} := |S_b - S_c|$

$$\implies E(S_b, S_c) = \frac{J}{2} S_{bc}(S_{bc} + 1) + \frac{J_c - J}{2} S_c(S_c + 1) - \frac{J}{2} S_b(S_b + 1) + J_b \underbrace{E_{\text{min}}^{\text{XXX}}(N_b, S_b)}_{\text{Bethe Ansatz}}$$

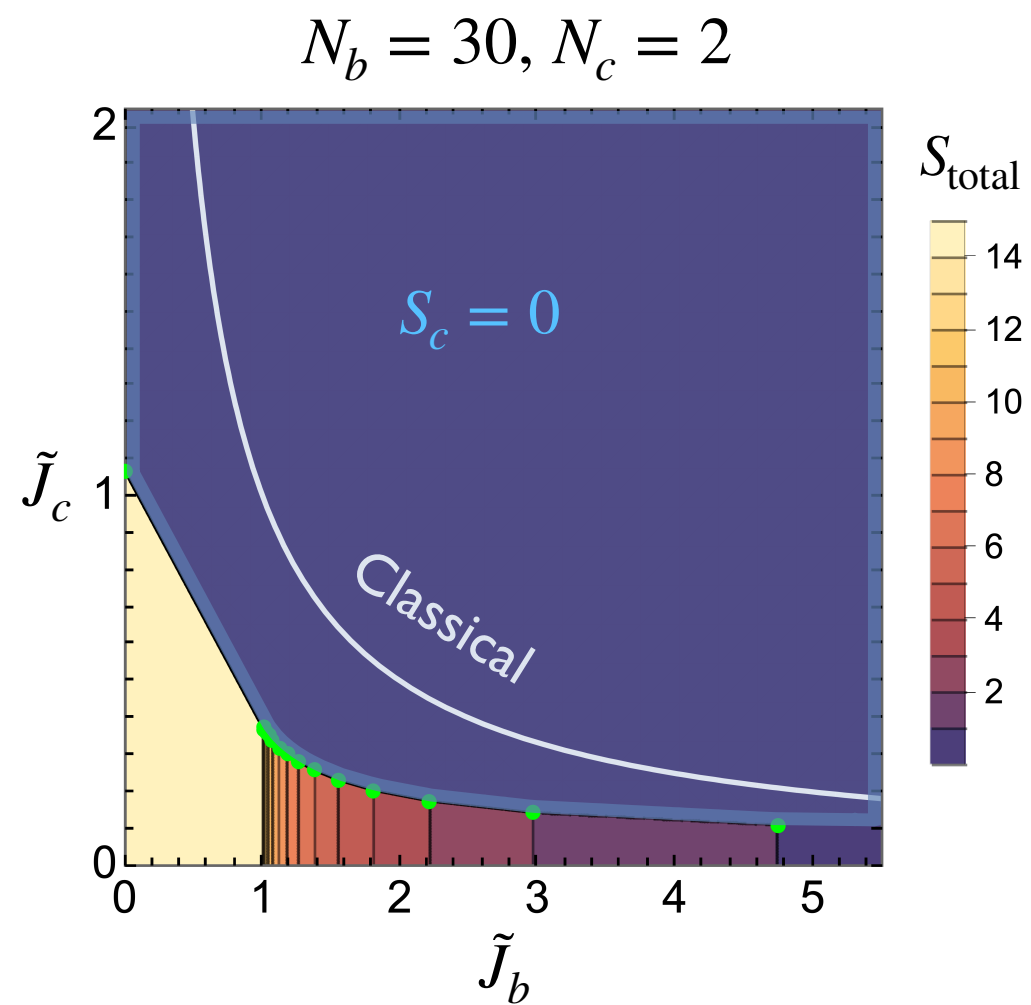
Tuning S_{total} in a wheel



$$\tilde{J}_b := \frac{4J_b}{JN_c}$$

$$\tilde{J}_c := \frac{J_c N_c}{JN_b}$$

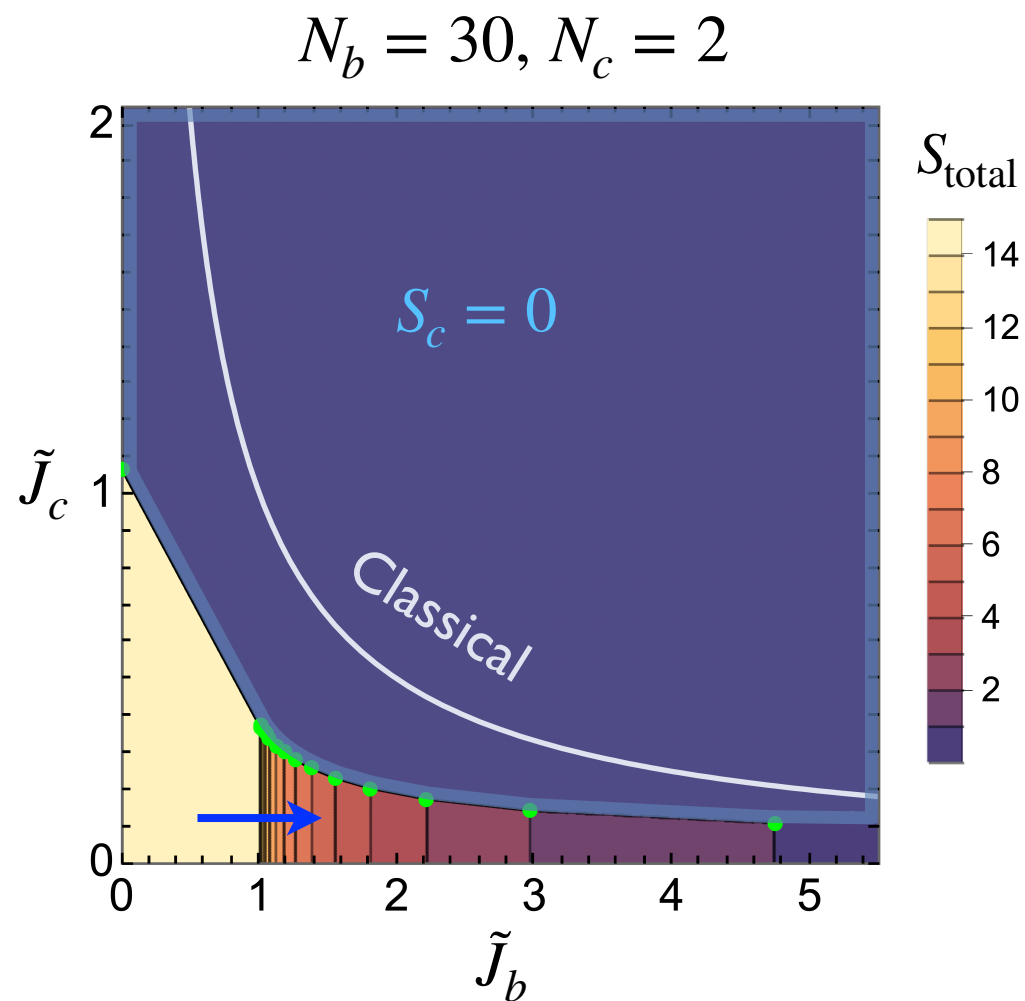
Tuning S_{total} in a wheel



$$\tilde{J}_b := \frac{4J_b}{JN_c}$$

$$\tilde{J}_c := \frac{J_c N_c}{JN_b}$$

Tuning S_{total} in a wheel



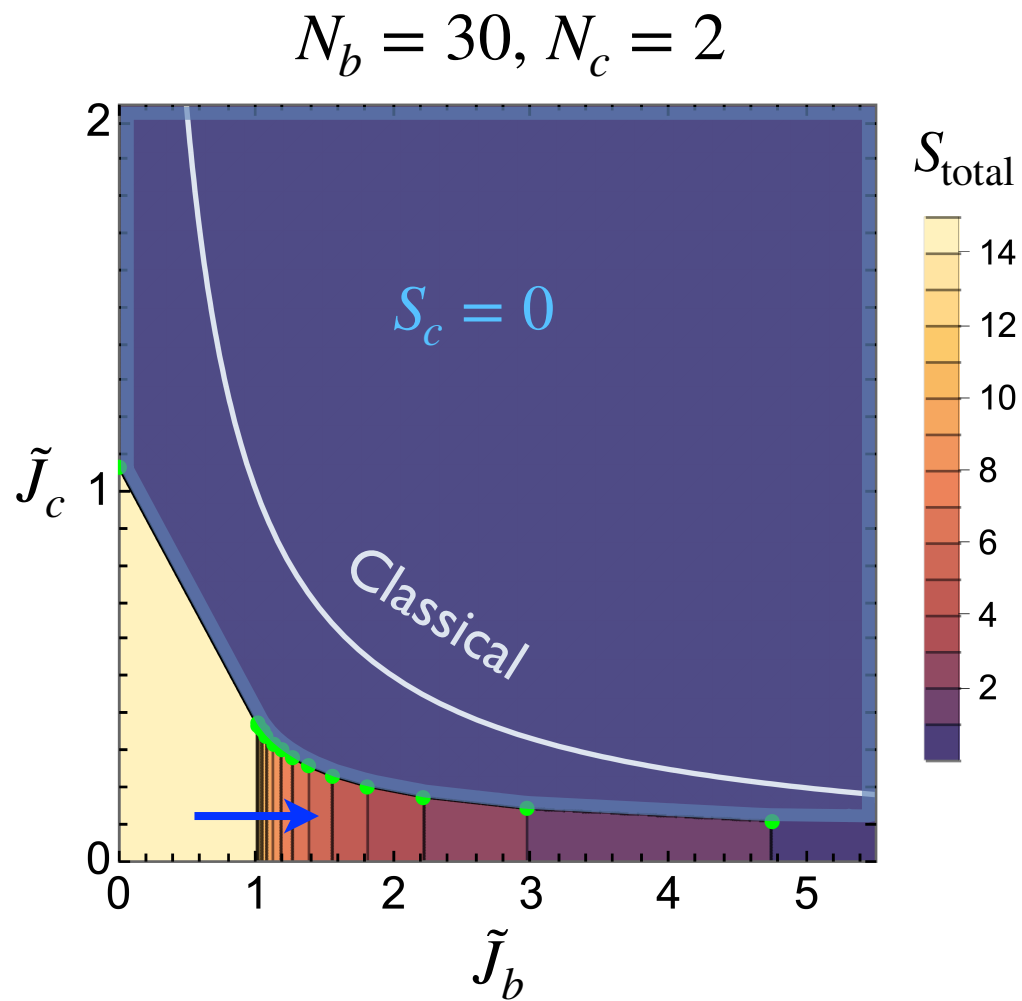
$$\tilde{J}_b := \frac{4J_b}{JN_c}$$

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J_c small: $S_c = 1$

$$\Rightarrow E = J_b \underbrace{E_{\min}^{\text{XXX}}(N_b, S_b)}_{\sim S_b^2} - JS_b$$

Tuning S_{total} in a wheel

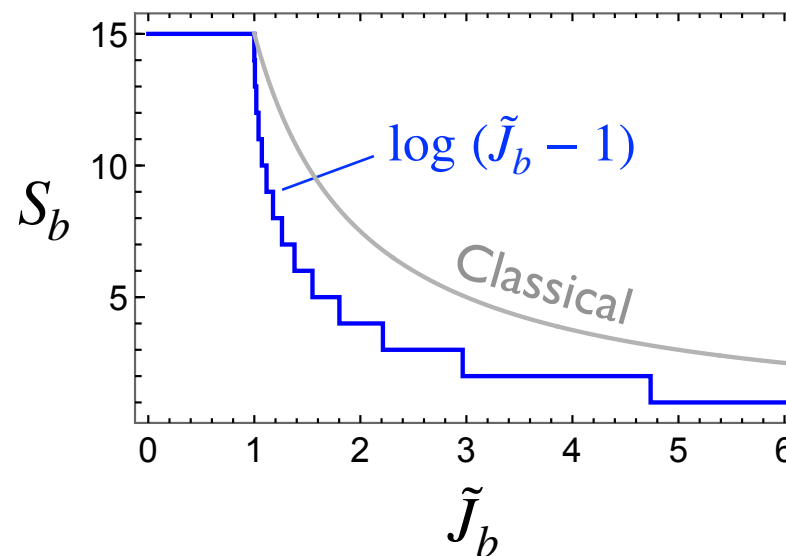


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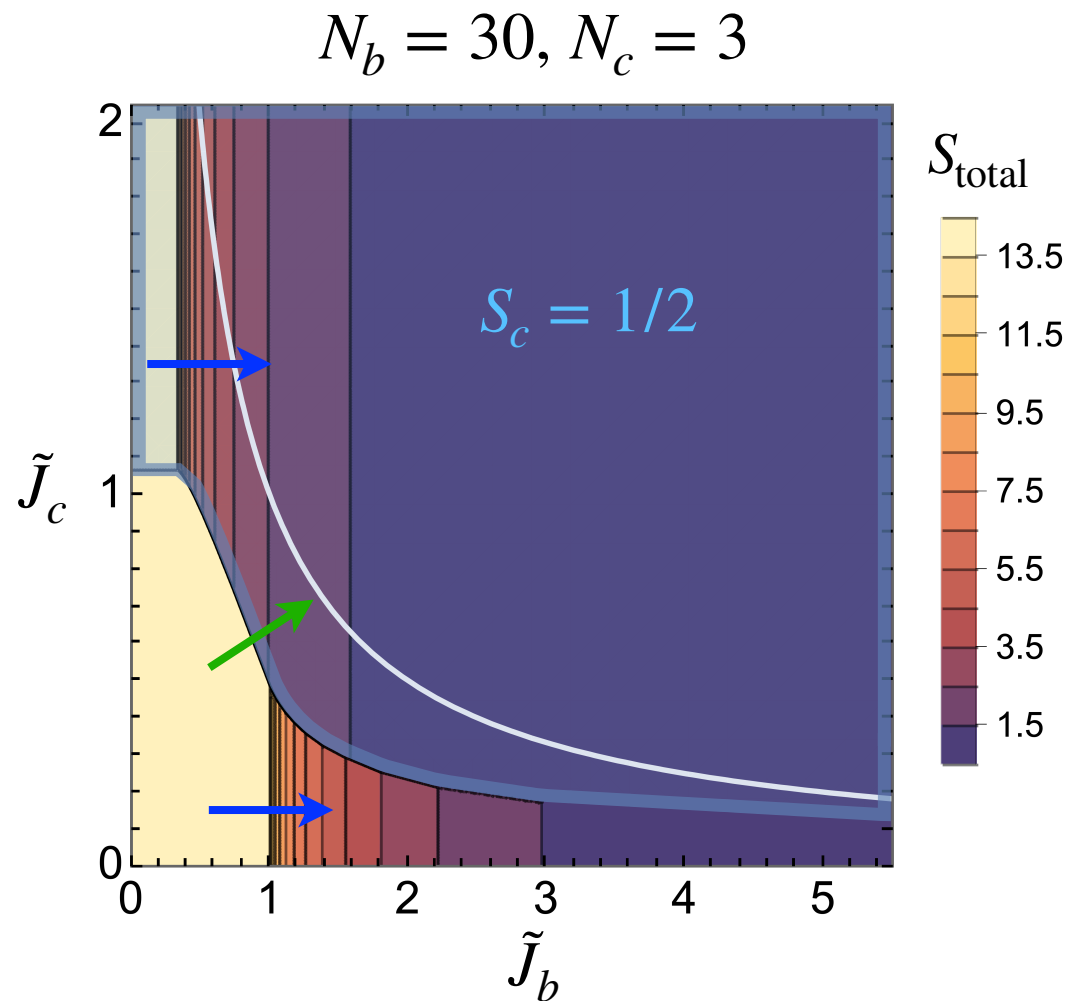
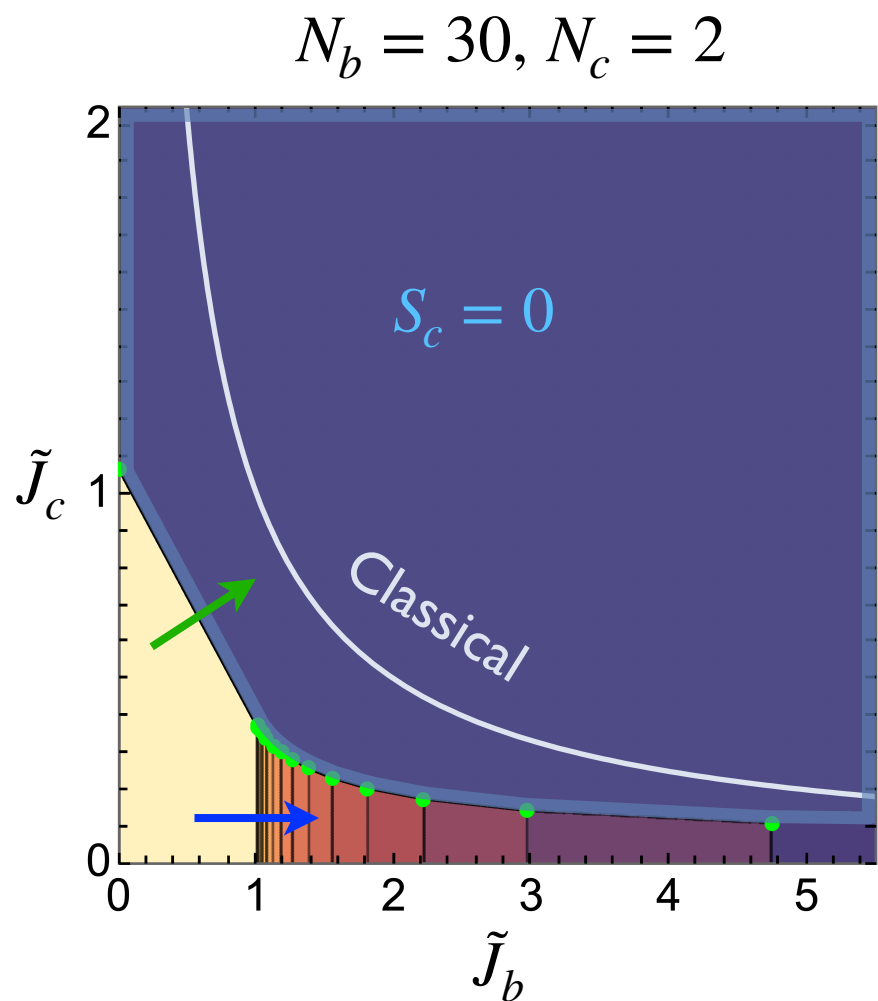
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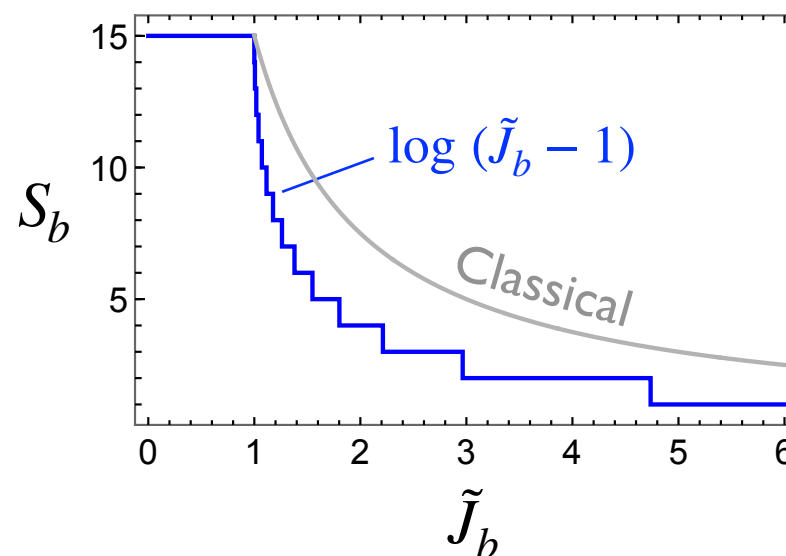


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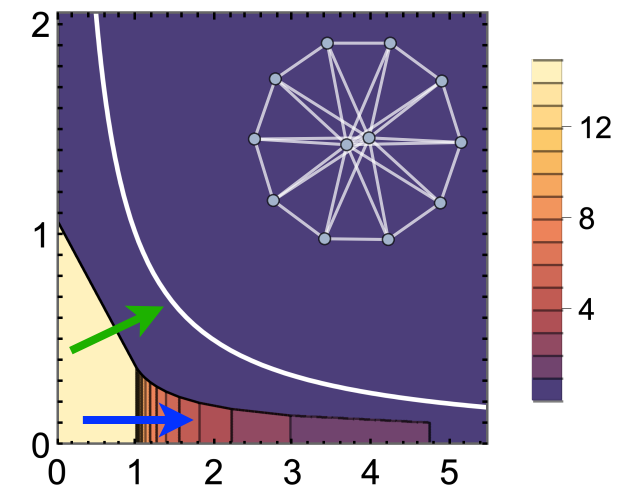
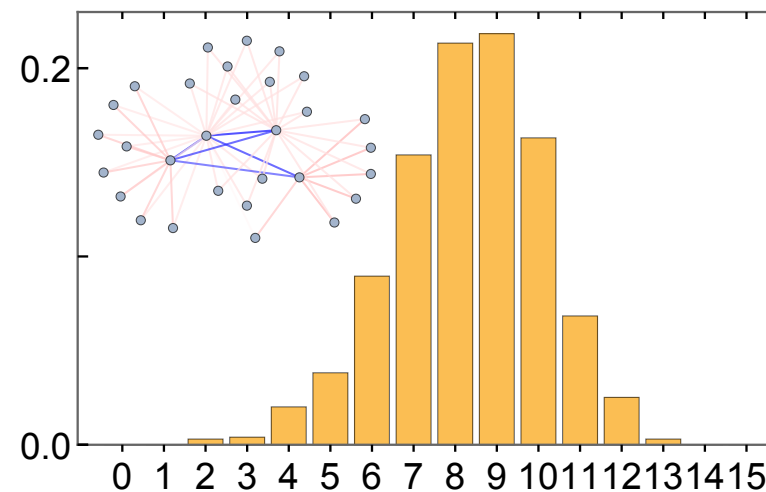
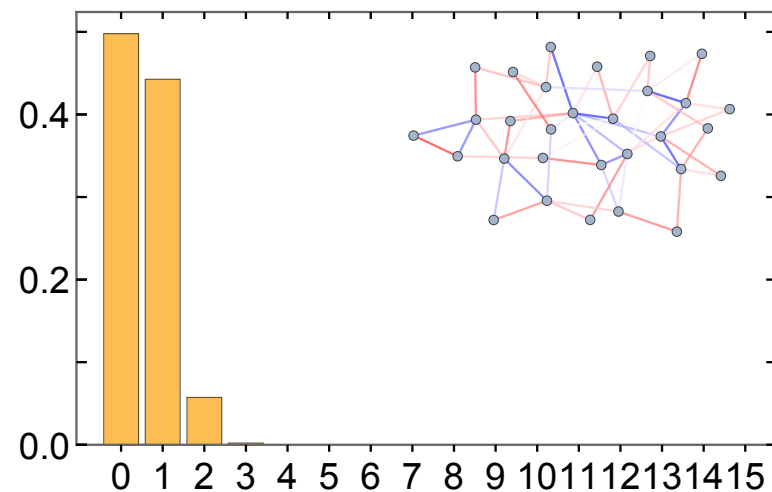
$$\Rightarrow E = J_b \underbrace{E_{\min}^{\text{XXX}}(N_b, S_b)}_{\sim S_b^2} - JS_b$$



S_{total} tunable over entire range across **discontinuous** & **continuous** transitions

Summary

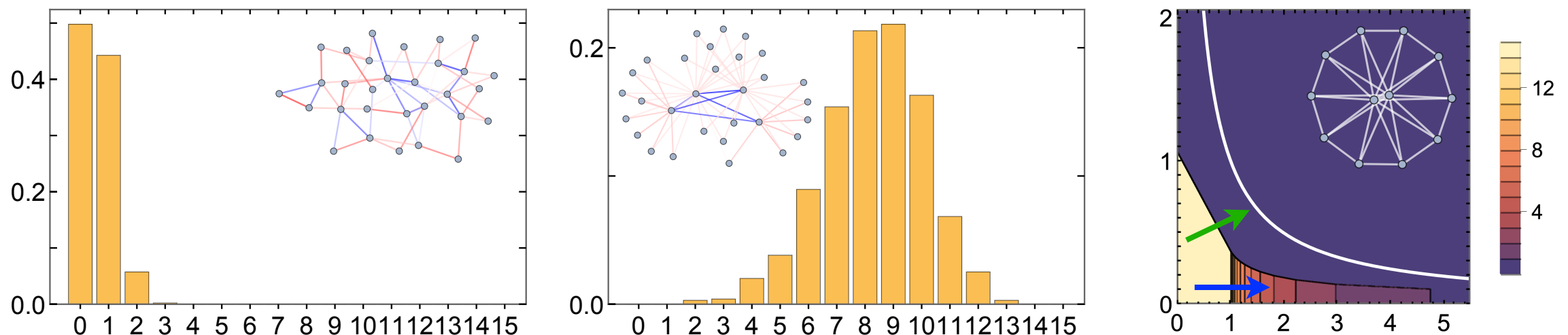
- Degree mismatch — disassortative hubs — essential for nonzero S_{total}
- S_{total} not sensitive to frustration level & falls w/ more neighbors
- S_{total} tunable over full range in nonbipartite graphs



Preethi G and SD, [arXiv:2403.09116](https://arxiv.org/abs/2403.09116)

Summary

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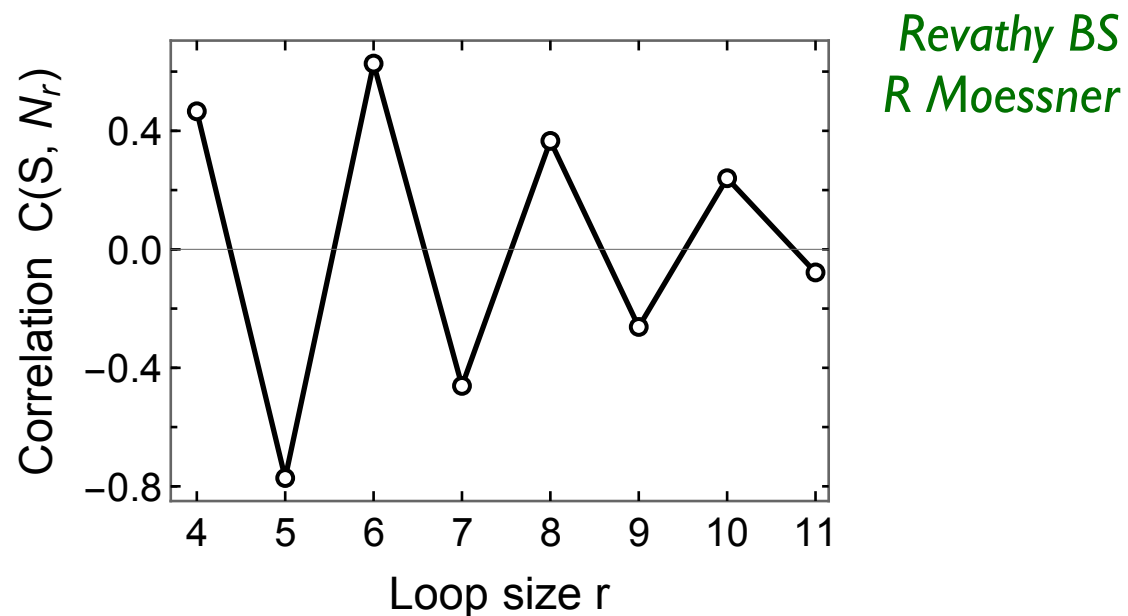
Open questions:

- Structure of ground state — spin glass? spin liquids?
- Dynamical properties? Importance of other motifs?
- Why insensitive to frustration — Contrast w/ kinetic magnetism

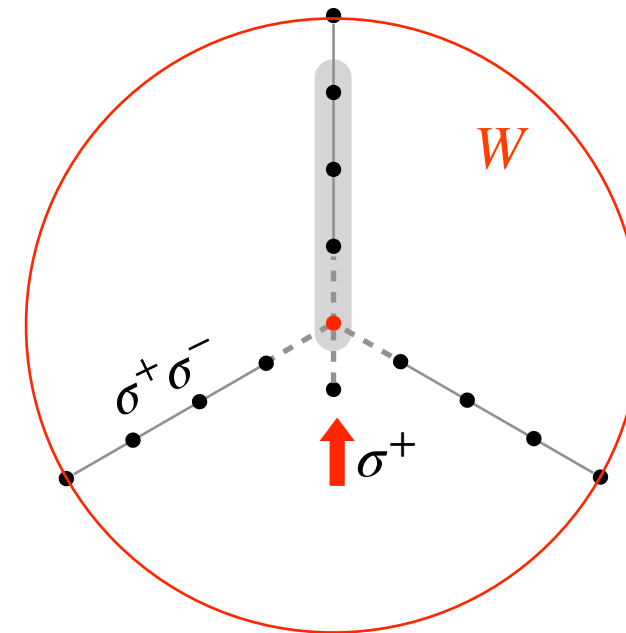
Preethi G and SD, [arXiv:2403.09116](https://arxiv.org/abs/2403.09116)

Other recent work

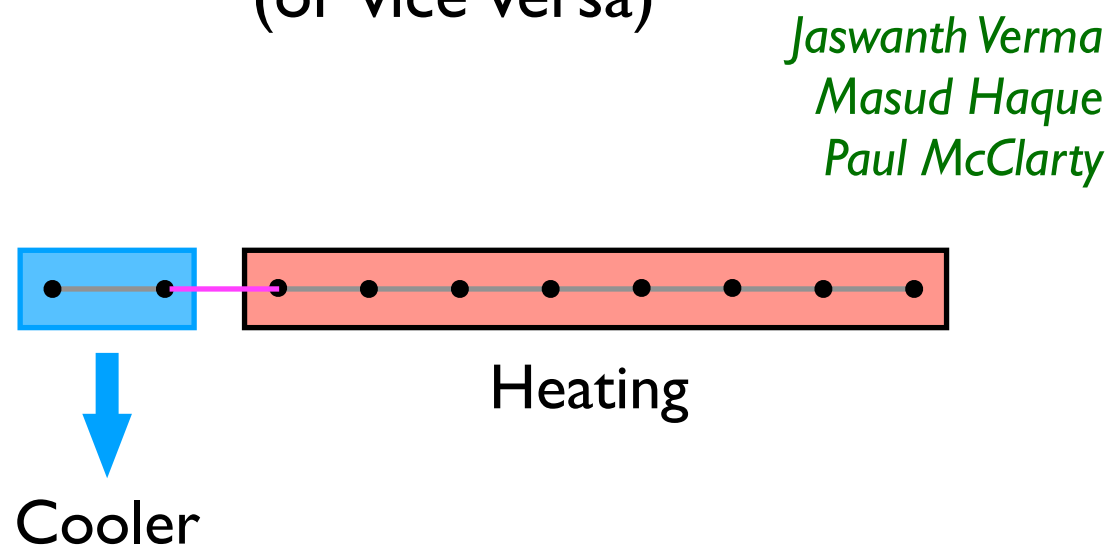
Nagaoka physics on general graph Frustration level key!



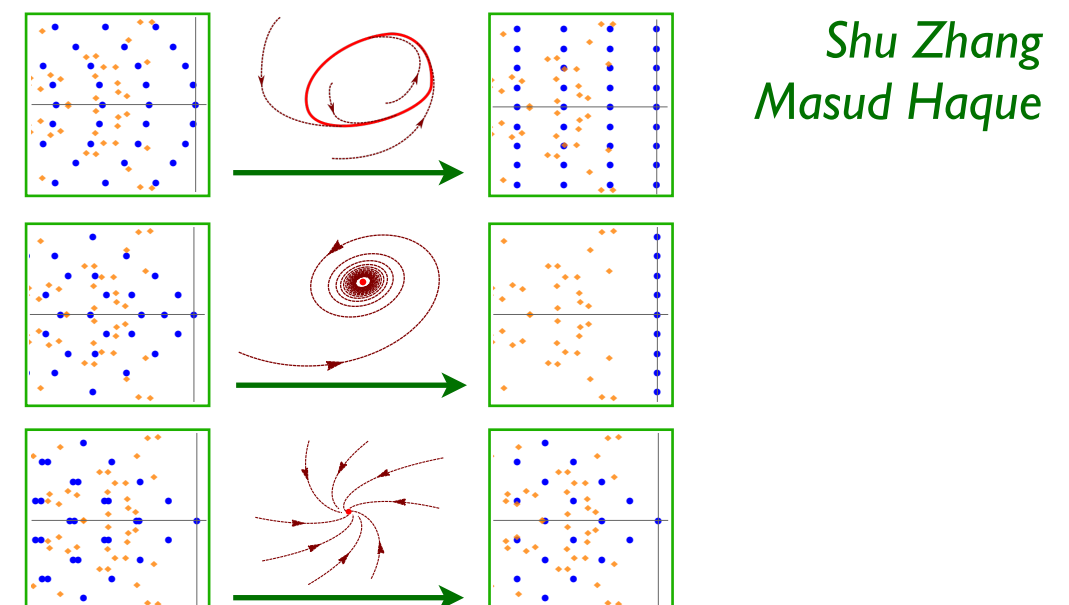
Long-range multipartite entanglement from local pump & static coupling



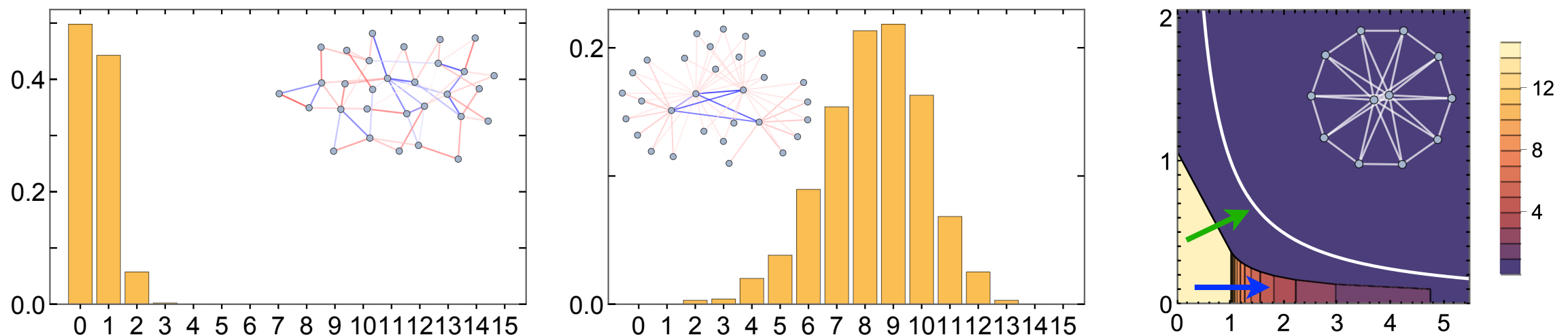
Global heating from local cooling (or vice versa)



Emergence of classical nonlinear phenomena



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