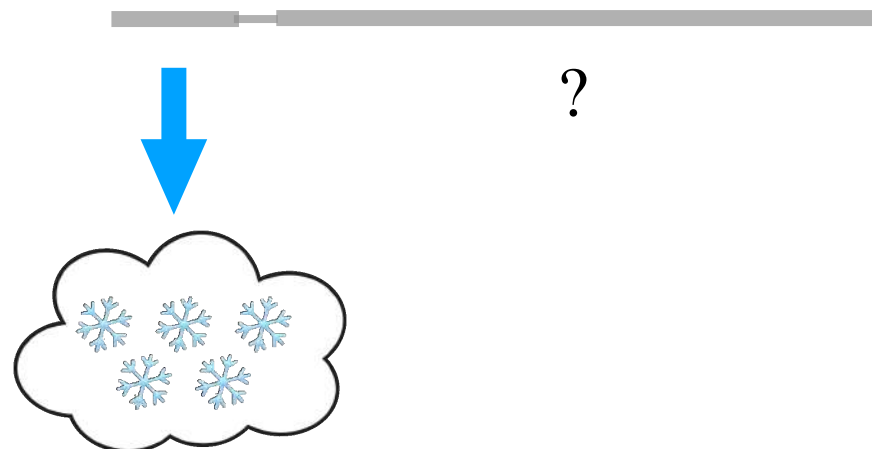


Anti-thermalization: Heating from cooling & vice versa

Shovan Dutta
Raman Research Institute

In collaboration with: Jaswanth Uppalapati (IISc)
Paul McClarty (CNRS)
Masud Haque (TU Dresden)



विज्ञान एवं प्रौद्योगिकी विभाग
DEPARTMENT OF
SCIENCE & TECHNOLOGY

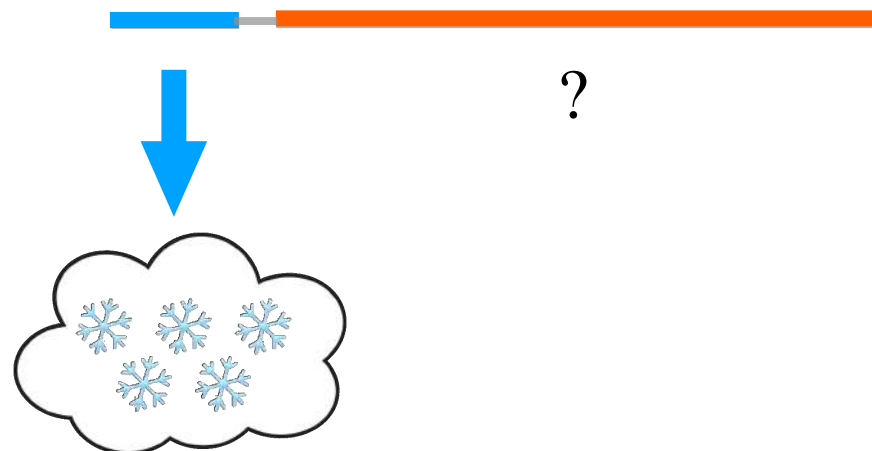
ICTS, November 2024



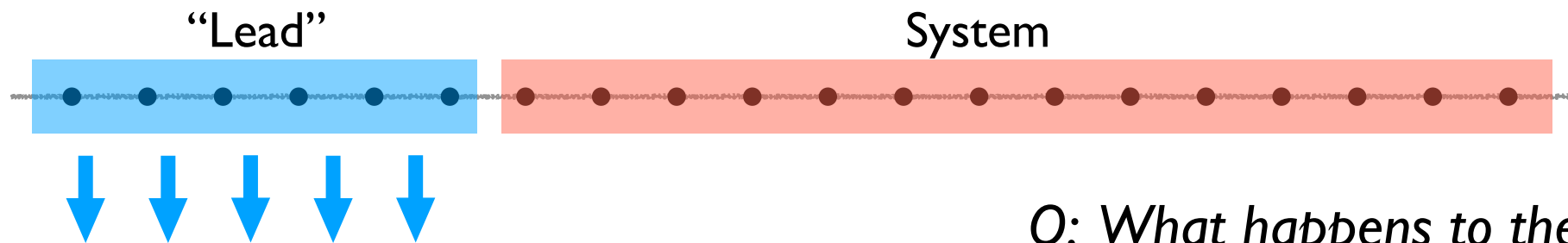
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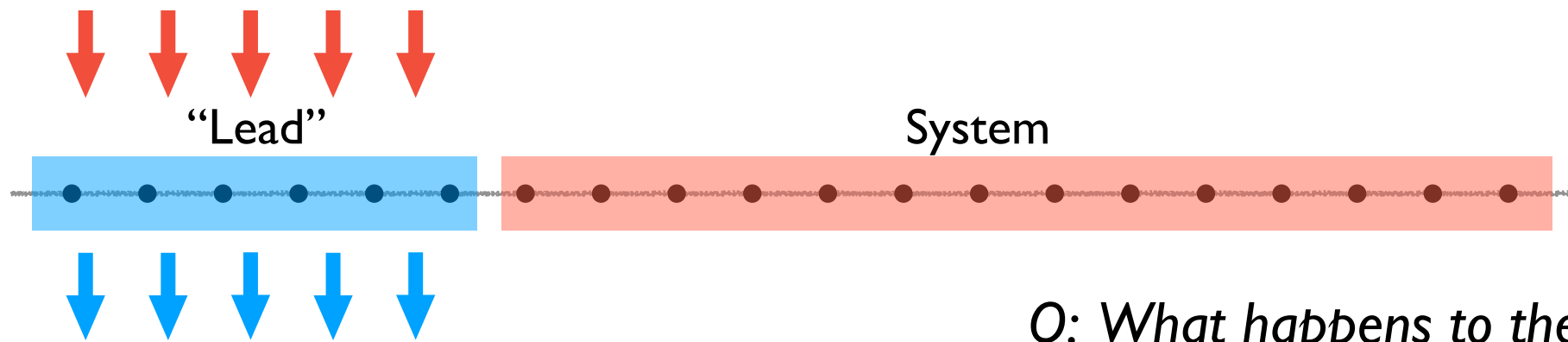


The setup

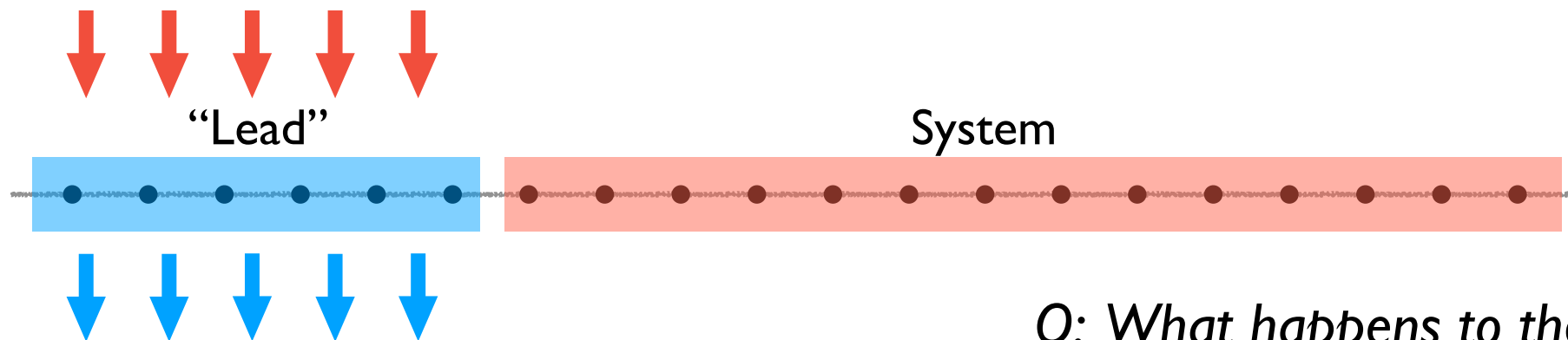


Q: What happens to the system?

The setup



The setup



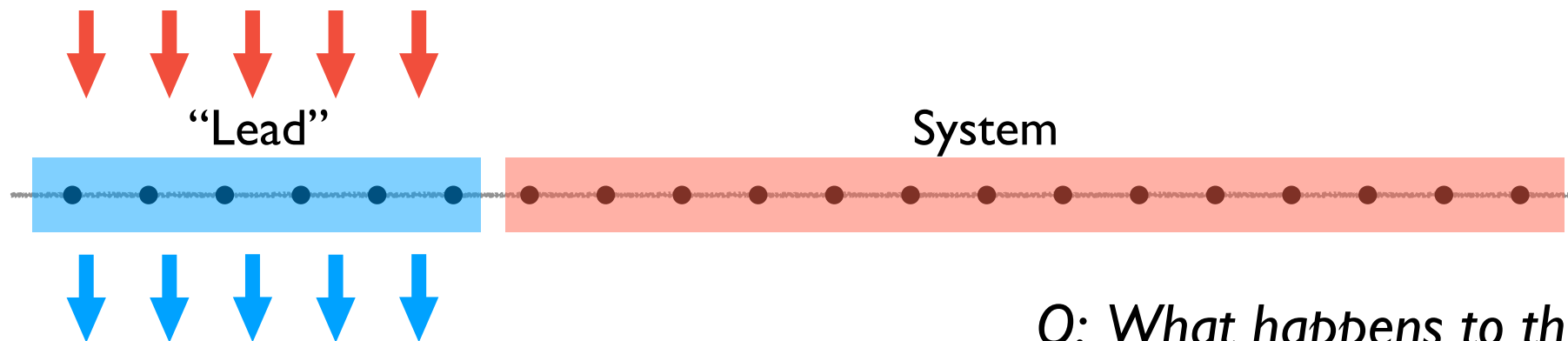
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Free-fermion lead + Markovian (Lindblad) dissipation — system reaches lead temperature for

- Infinite lead w/ bandwidth larger than system
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Finite-size simulations: approximate sympathetic cooling + works best in above limits

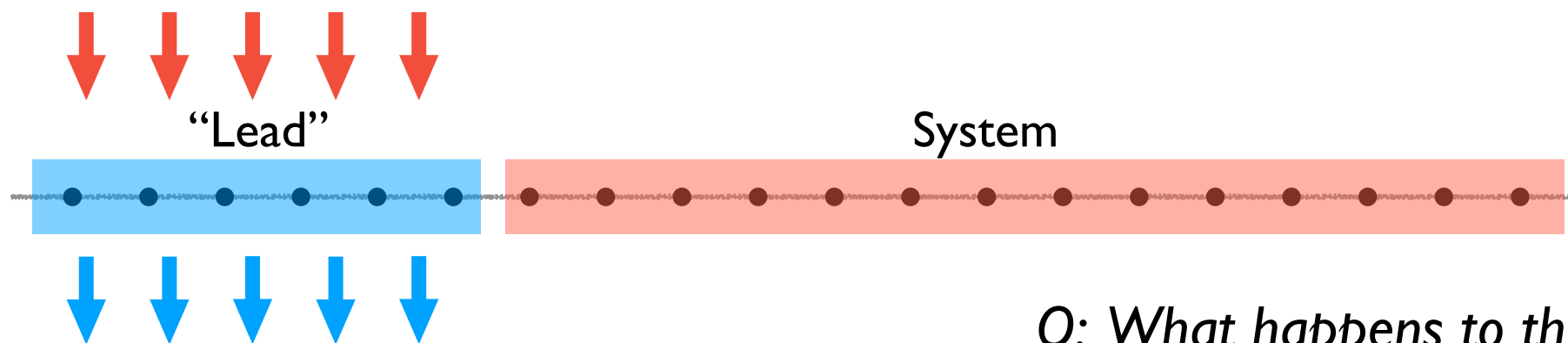
Raghunandan, Wolf, Ospelkaus, Schmidt, Weimer, Sci. Adv. 6, eaaw9268 (2020)

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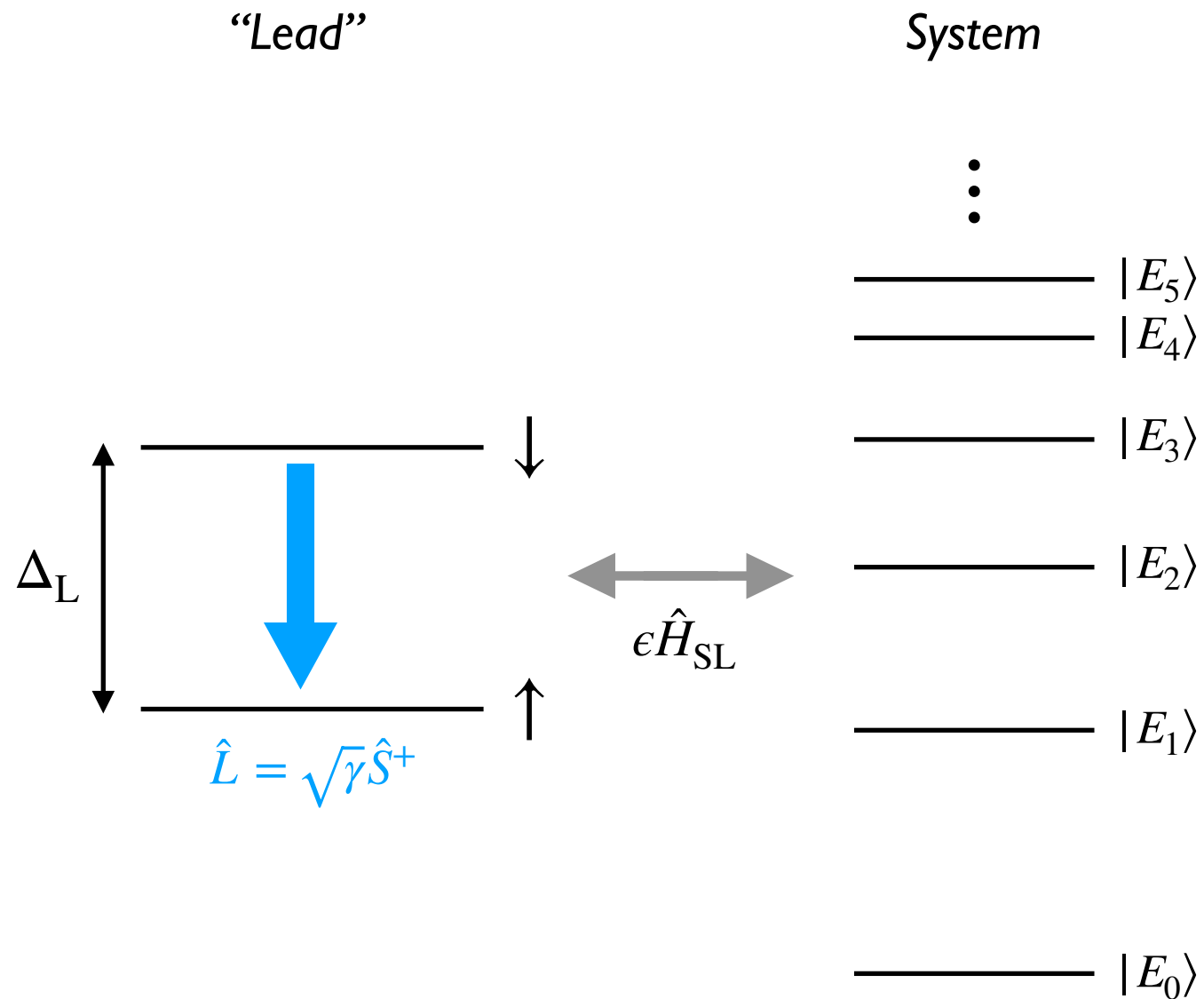
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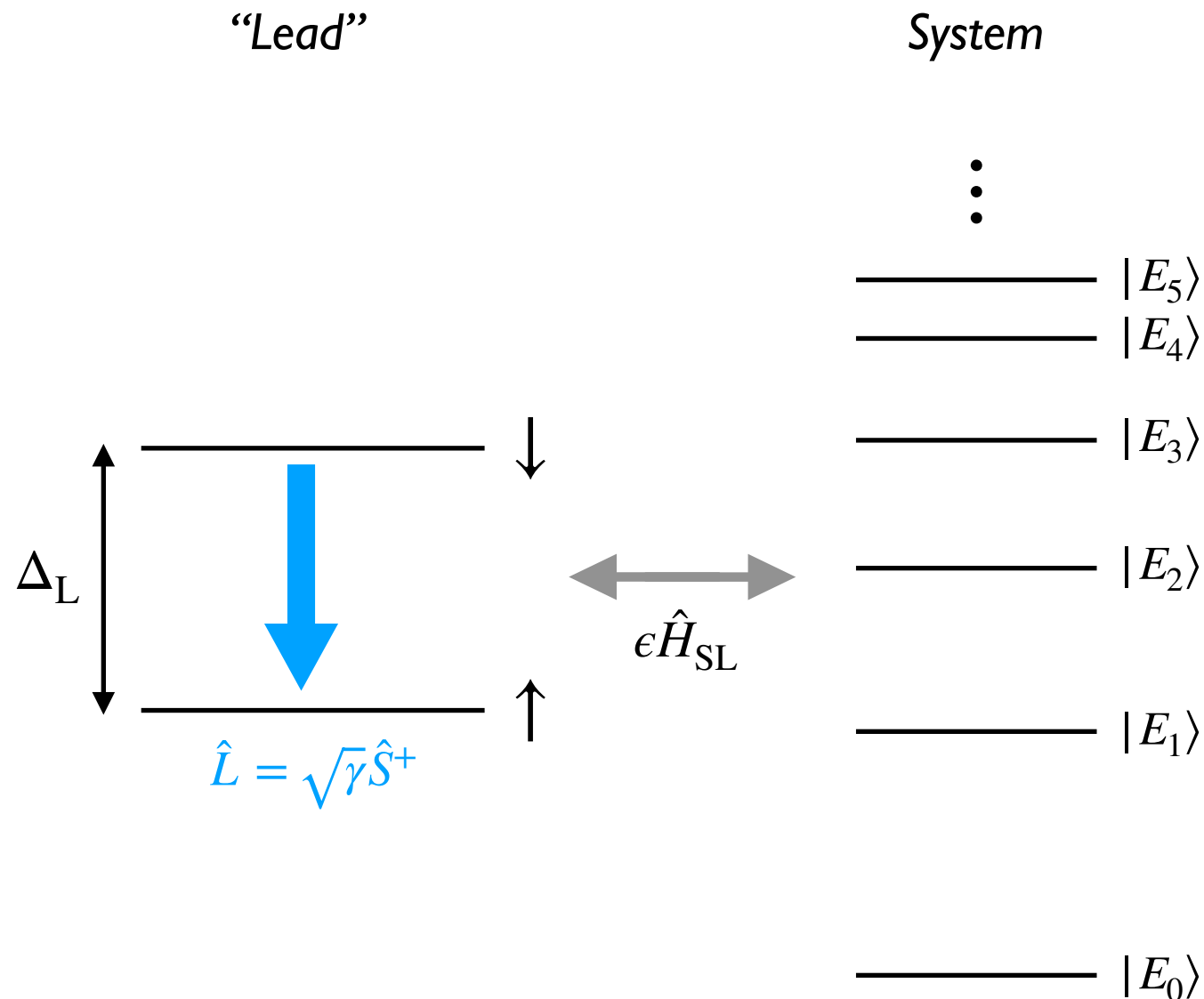
Palmero, Xu, Guo, Poletti, PRE 100, 022111 (2019)

Here: Spectacular failure in presence of symmetry — heating by cooling
(Even in the above ideal limits)

Why we expect sympathetic cooling



Why we expect sympathetic cooling

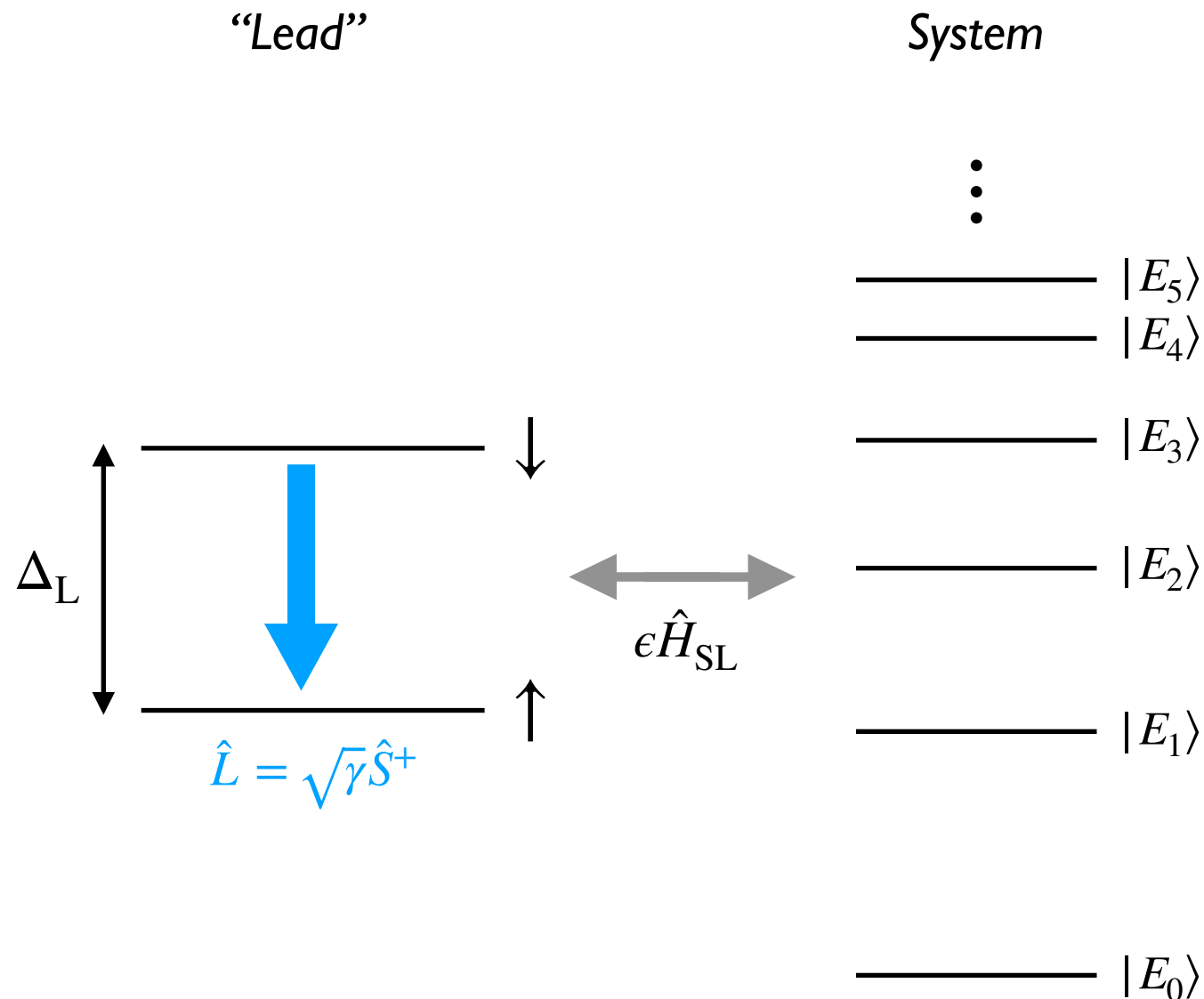


Steady state given by transitions between system eigenstates due to \hat{H}_{SL}

Perturbed eigenstates of \hat{H}_{total} :

$$|\uparrow \otimes E_i\rangle_p = |\uparrow \otimes E_i\rangle + \epsilon \sum_j c_{i,j} |\downarrow \otimes E_j\rangle + \epsilon \sum_{j \neq i} d_{i,j} |\uparrow \otimes E_j\rangle + O(\epsilon^2)$$

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Steady state given by transitions between system eigenstates due to \hat{H}_{SL}

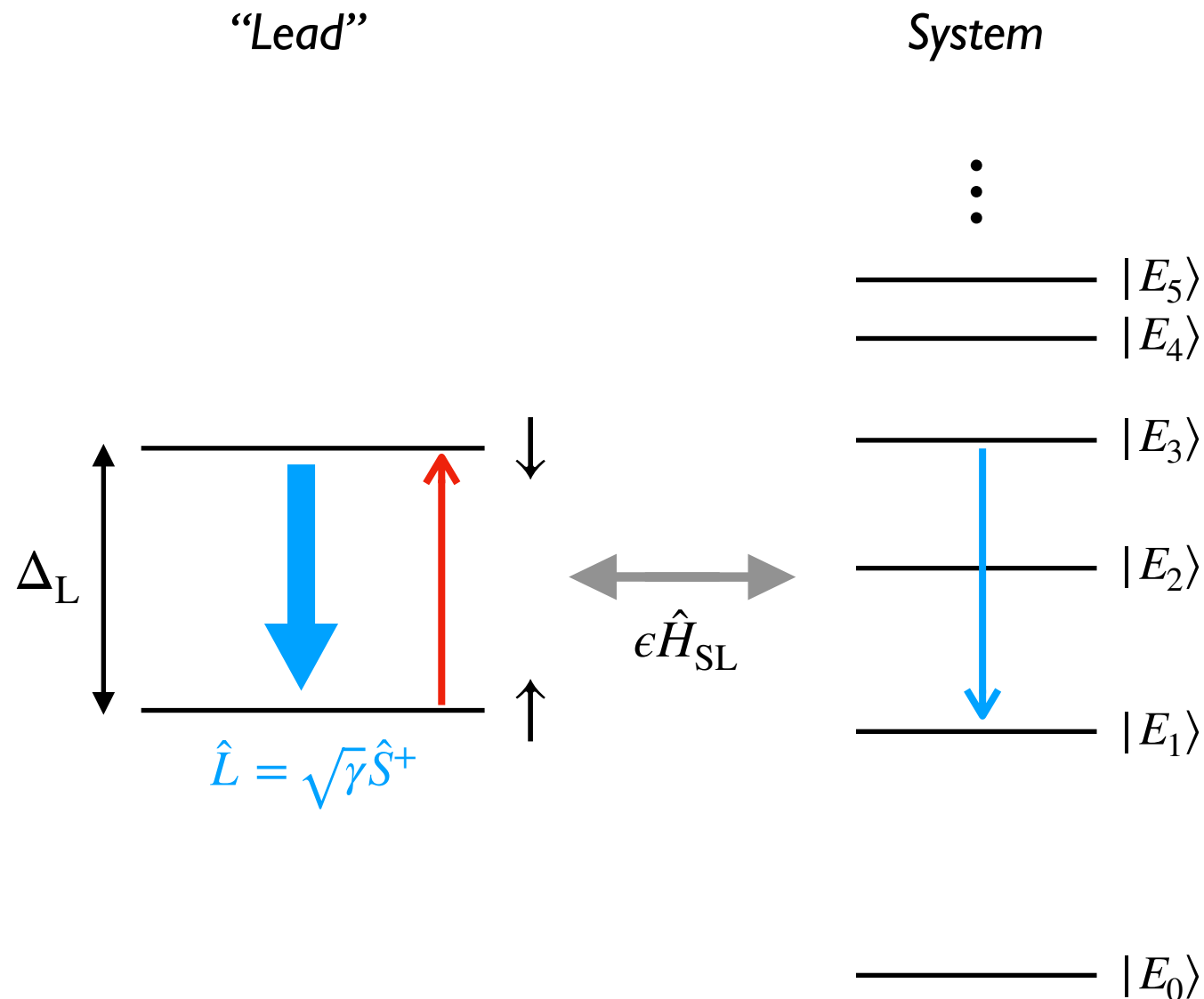
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\Rightarrow Transition rates

$$R_{i \rightarrow j} \approx \left| {}_p \langle \uparrow \otimes E_j | \hat{L} | \uparrow \otimes E_i \rangle_p \right|^2 \approx \gamma \epsilon^2 \frac{\left| \langle \downarrow \otimes E_j | \hat{H}_{SL} | \uparrow \otimes E_i \rangle \right|^2}{(E_i - E_j - \Delta_L)^2}$$

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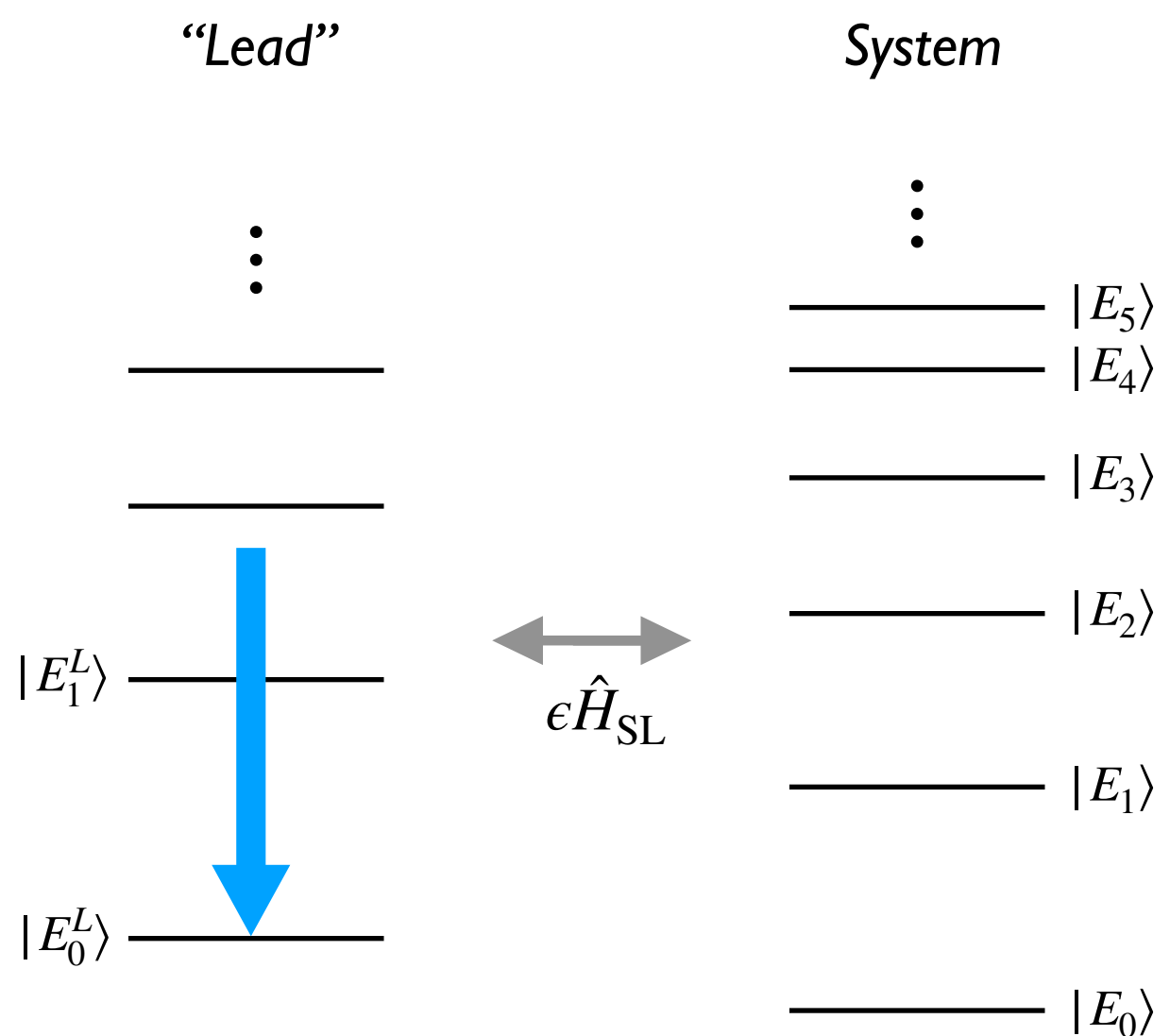
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Resonant cooling for $E_i - E_j = \Delta_L$

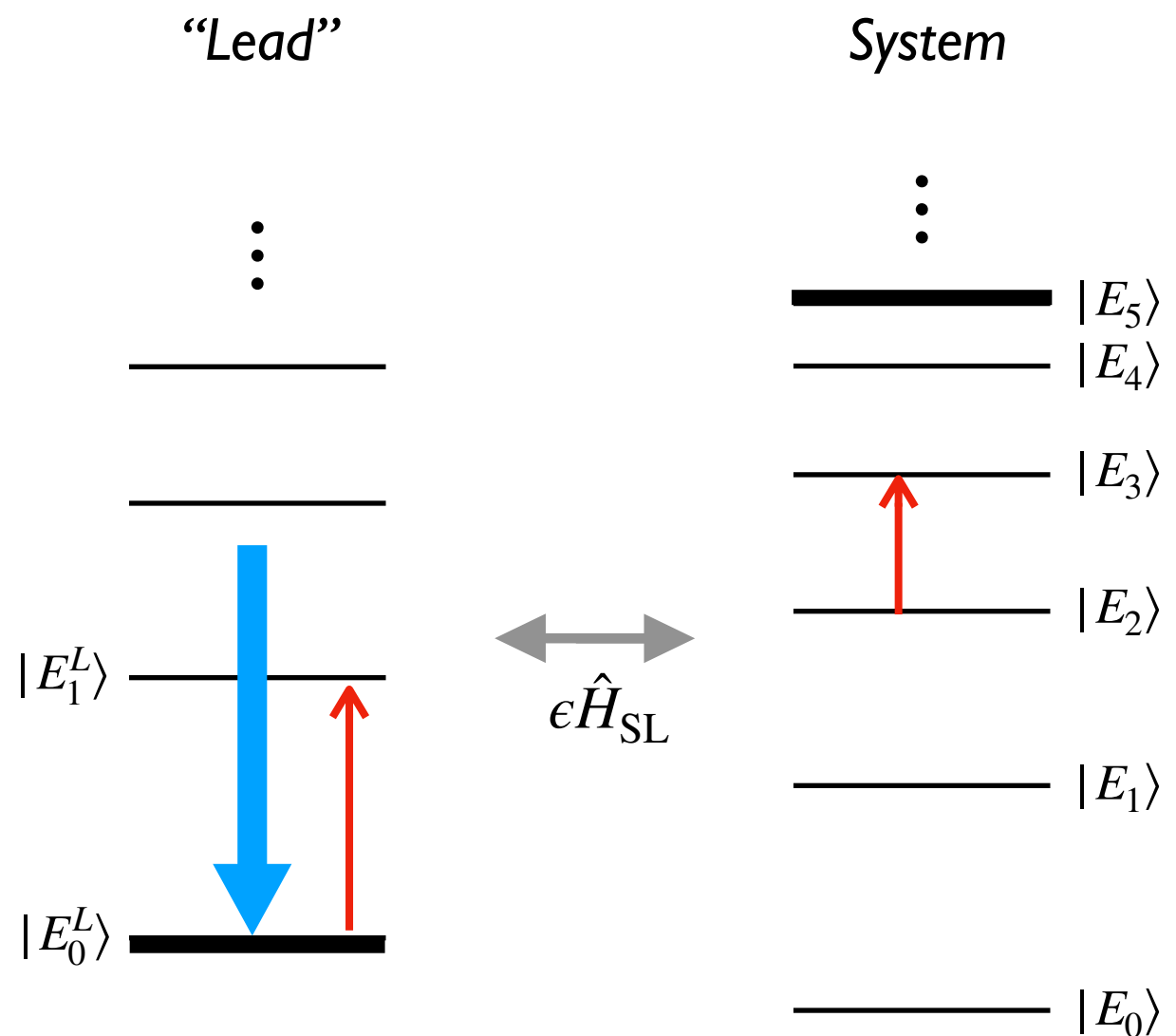
Heating by cooling — mechanism



\hat{H}_{SL} **either increases or decreases both E_S and E_L**

i.e. No exchange of energy

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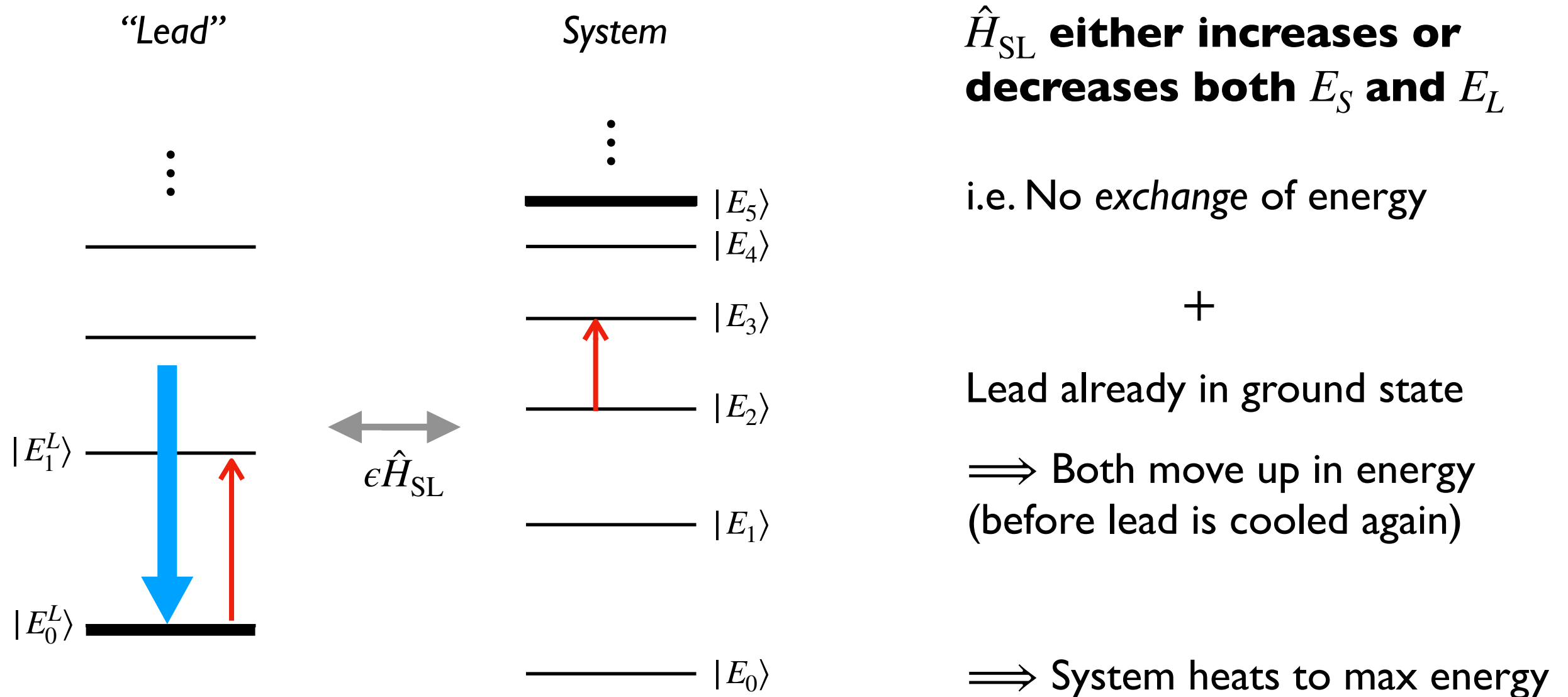
+

Lead already in ground state

\implies Both move up in energy
(before lead is cooled again)

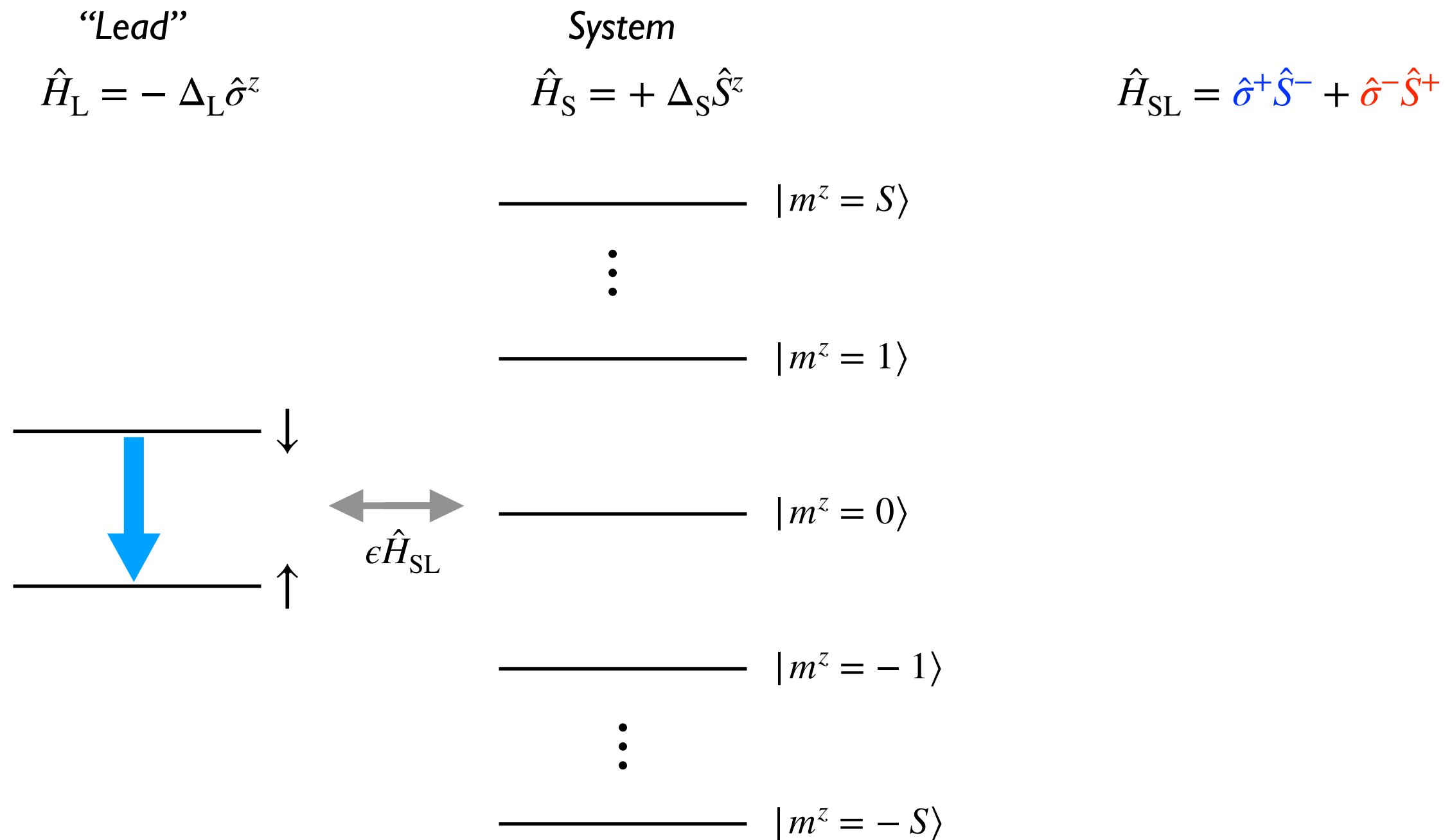
\implies System heats to max energy

Heating by cooling — mechanism

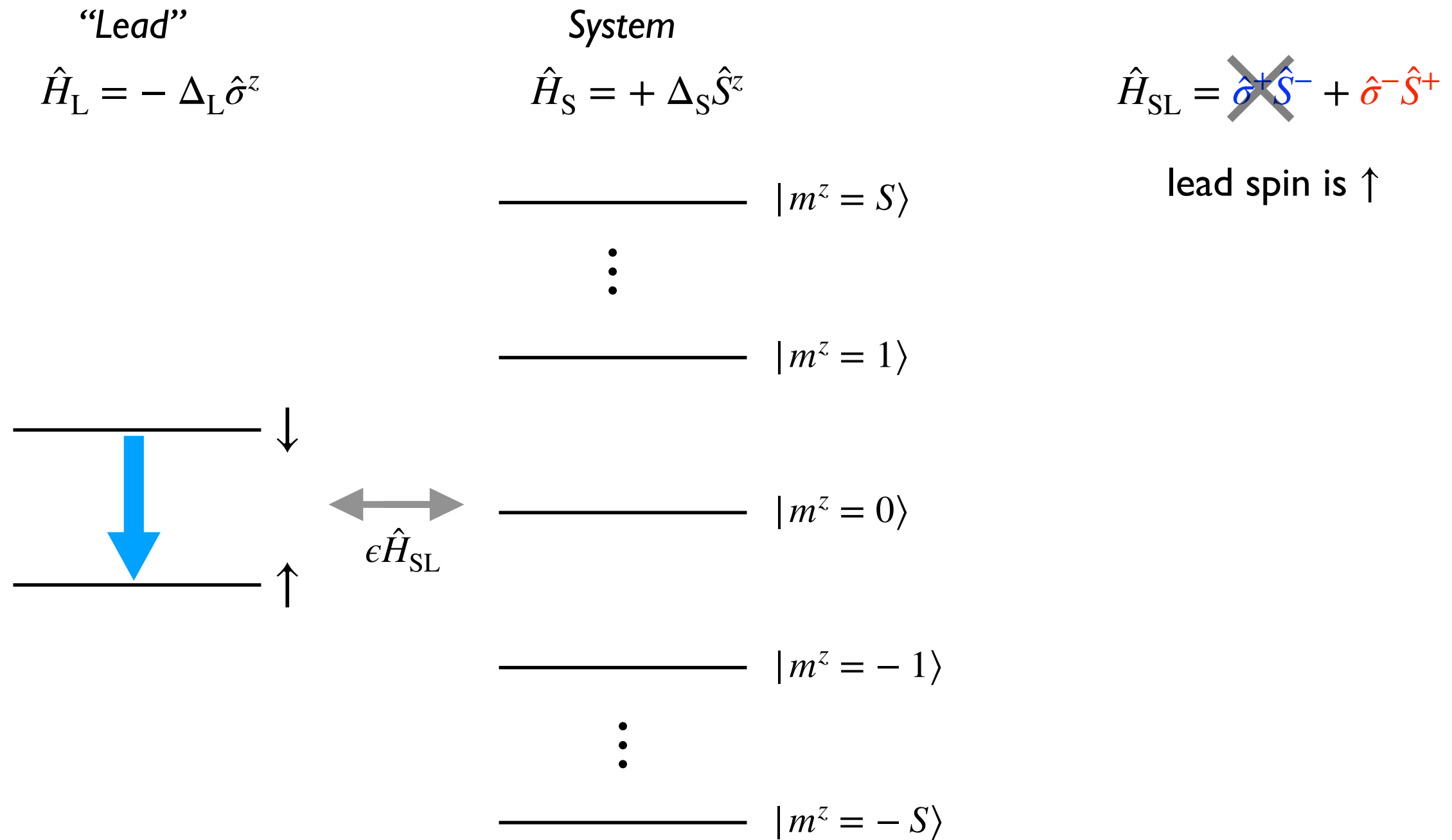


- No heating without cooling (i.e. no violation of 2nd law)
- Stop cooling \implies temperature gradient remains due to total energy conservation
- Such coupling arises in simple, realizable systems!

Simplest case



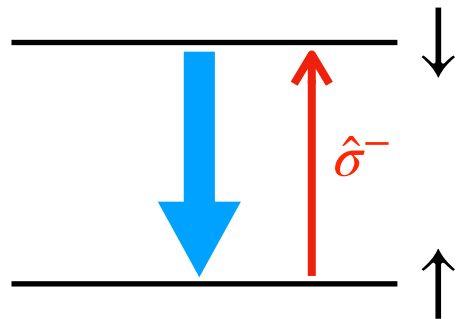
Simplest case



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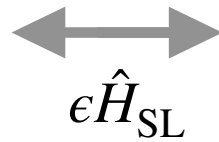
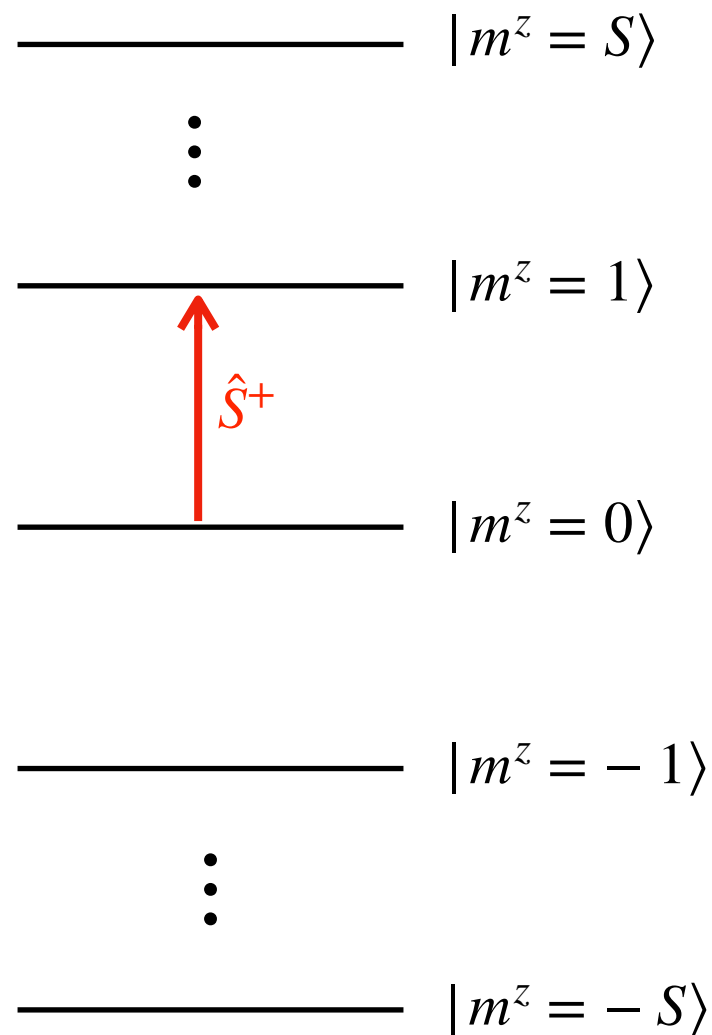
“Lead”

$$\hat{H}_L = -\Delta_L \hat{\sigma}^z$$



System

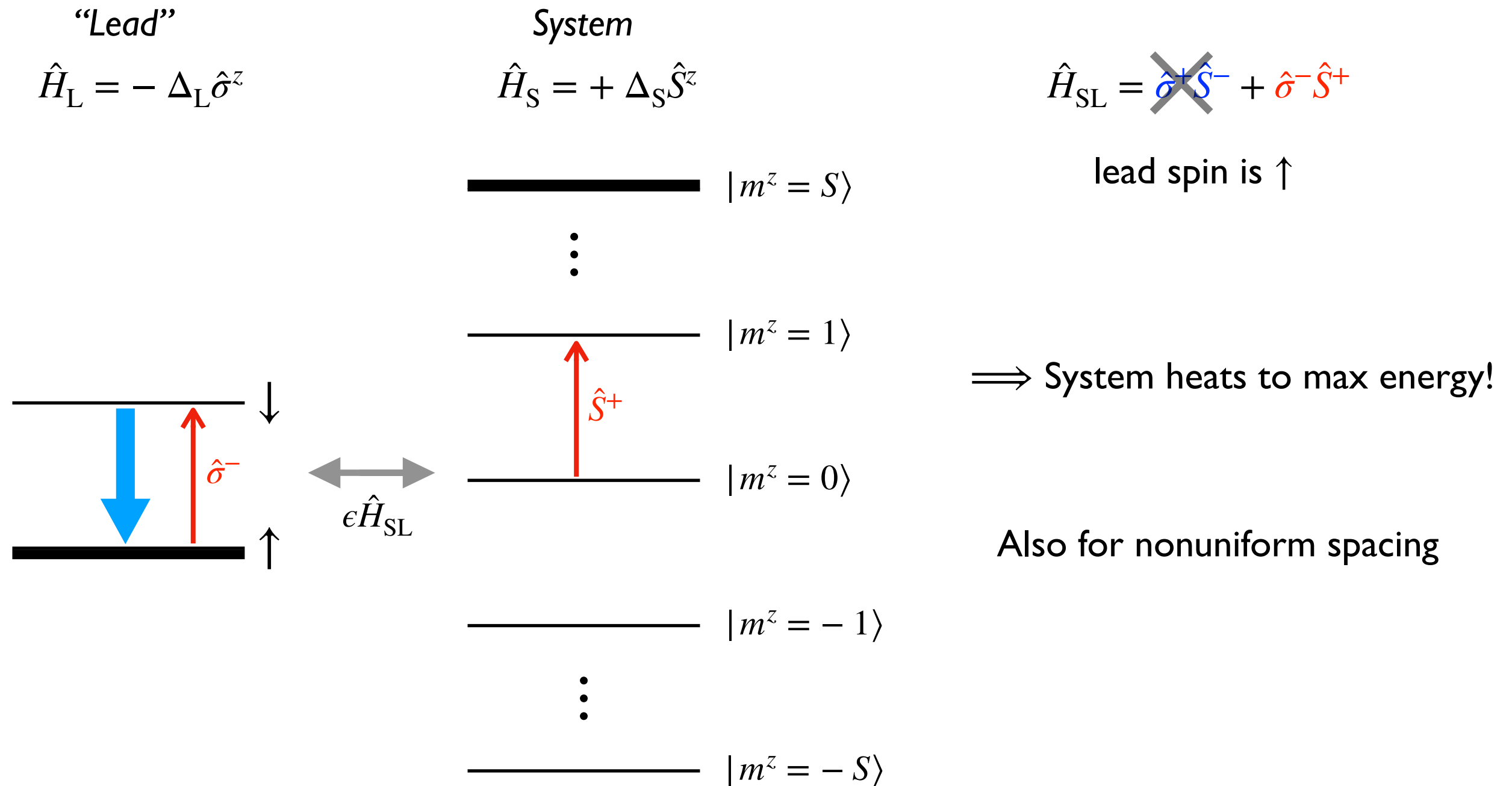
$$\hat{H}_S = +\Delta_S \hat{S}^z$$



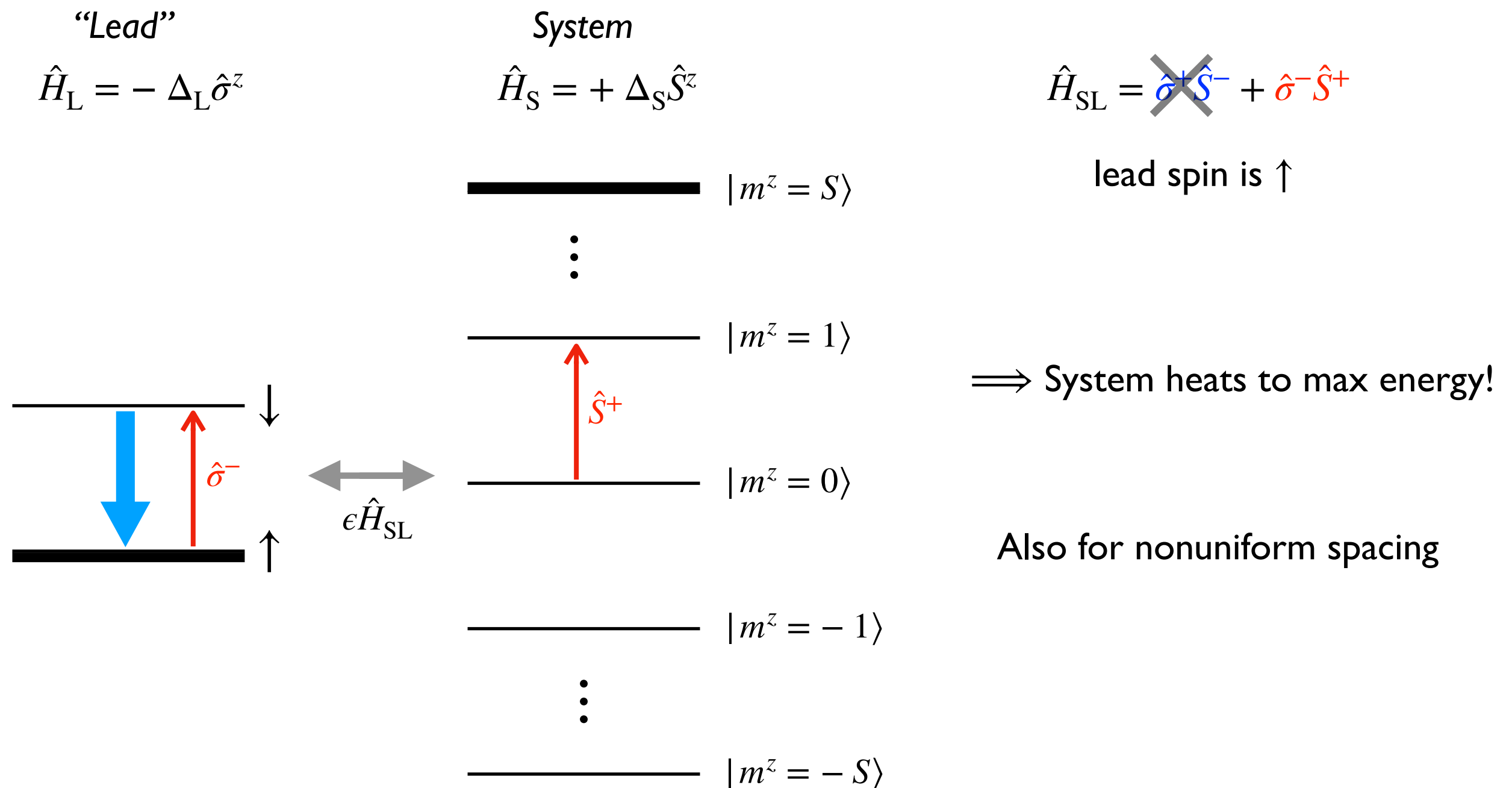
$$\hat{H}_{SL} = \cancel{\hat{\sigma}^+ \hat{S}^-} + \hat{\sigma}^- \hat{S}^+$$

lead spin is \uparrow

Simplest case

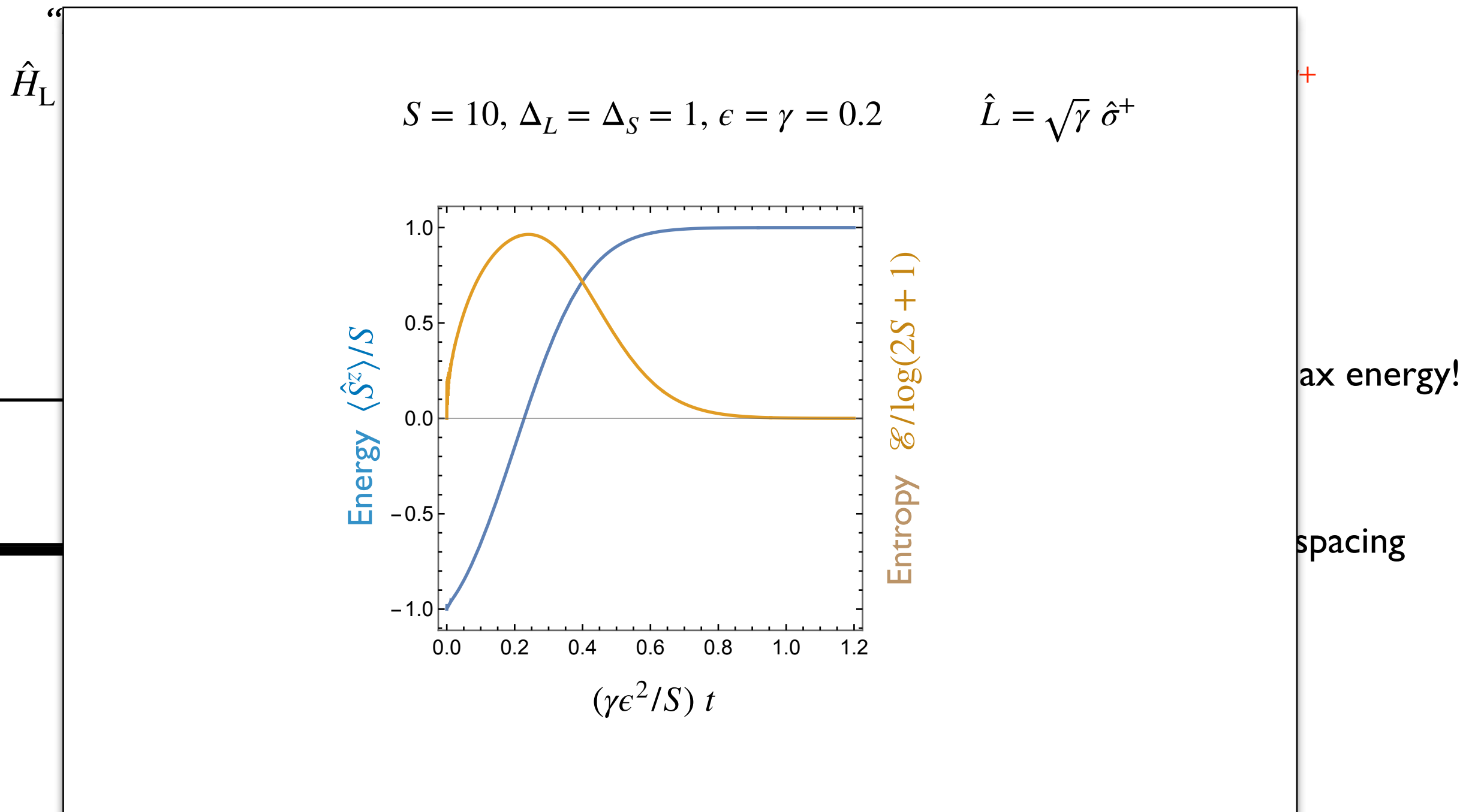


Simplest case



(1) E_L decreases w/ σ^z , (2) E_S increases w/ S^z , (3) Coupling conserves $\sigma^z + S^z$

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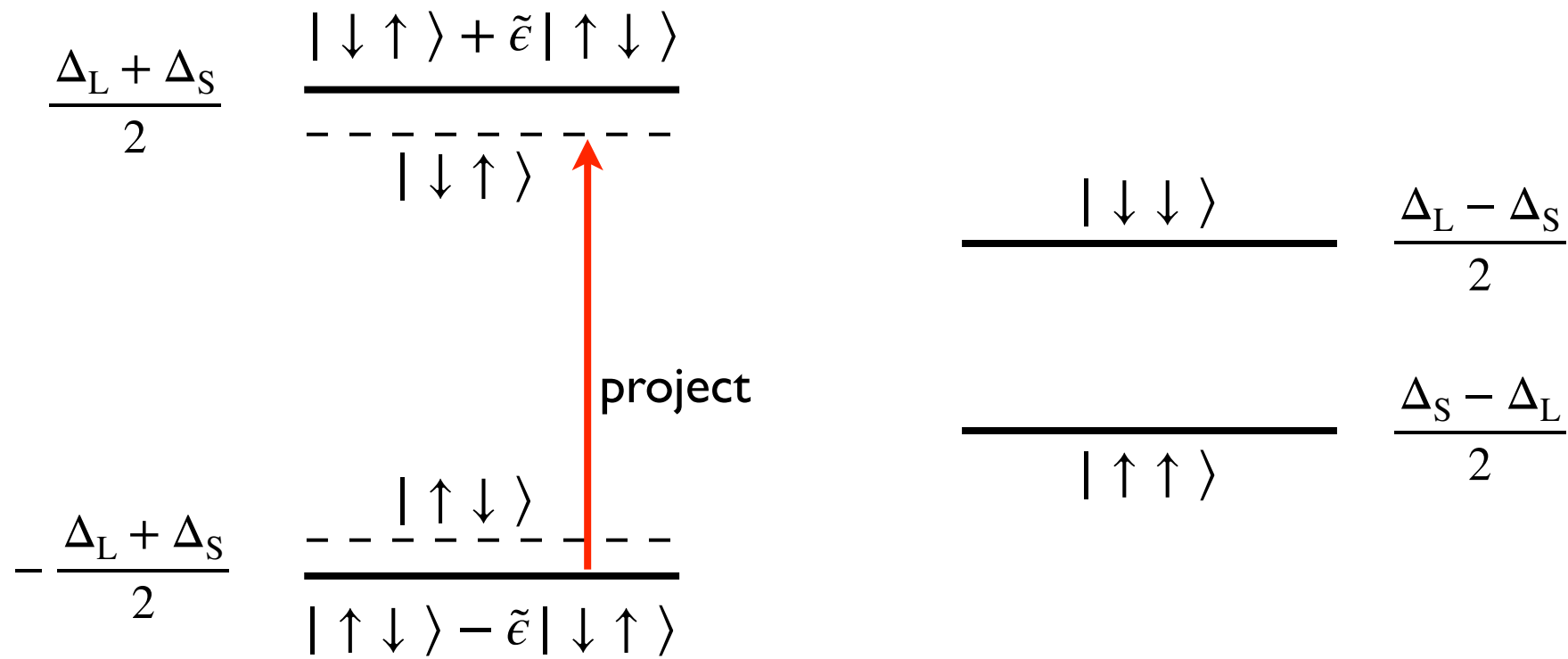
$$\frac{\Delta_L + \Delta_S}{2} \quad \frac{|\downarrow\uparrow\rangle + \tilde{\epsilon}|\uparrow\downarrow\rangle}{\text{---} \frac{|\downarrow\uparrow\rangle}{\text{---}} \text{---}}$$

$$-\frac{\Delta_L + \Delta_S}{2} \quad \frac{\text{---} \frac{|\uparrow\downarrow\rangle}{\text{---}} \text{---}}{|\uparrow\downarrow\rangle - \tilde{\epsilon}|\downarrow\uparrow\rangle}$$

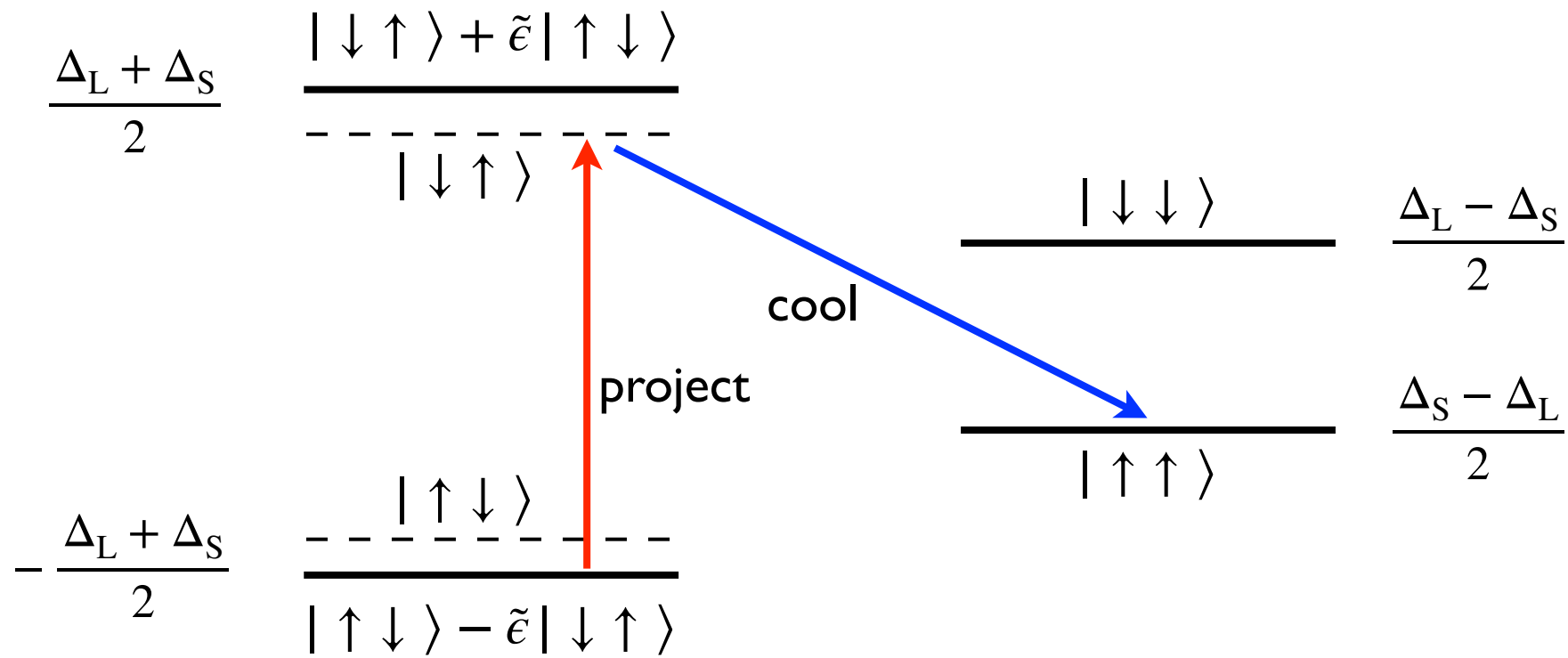
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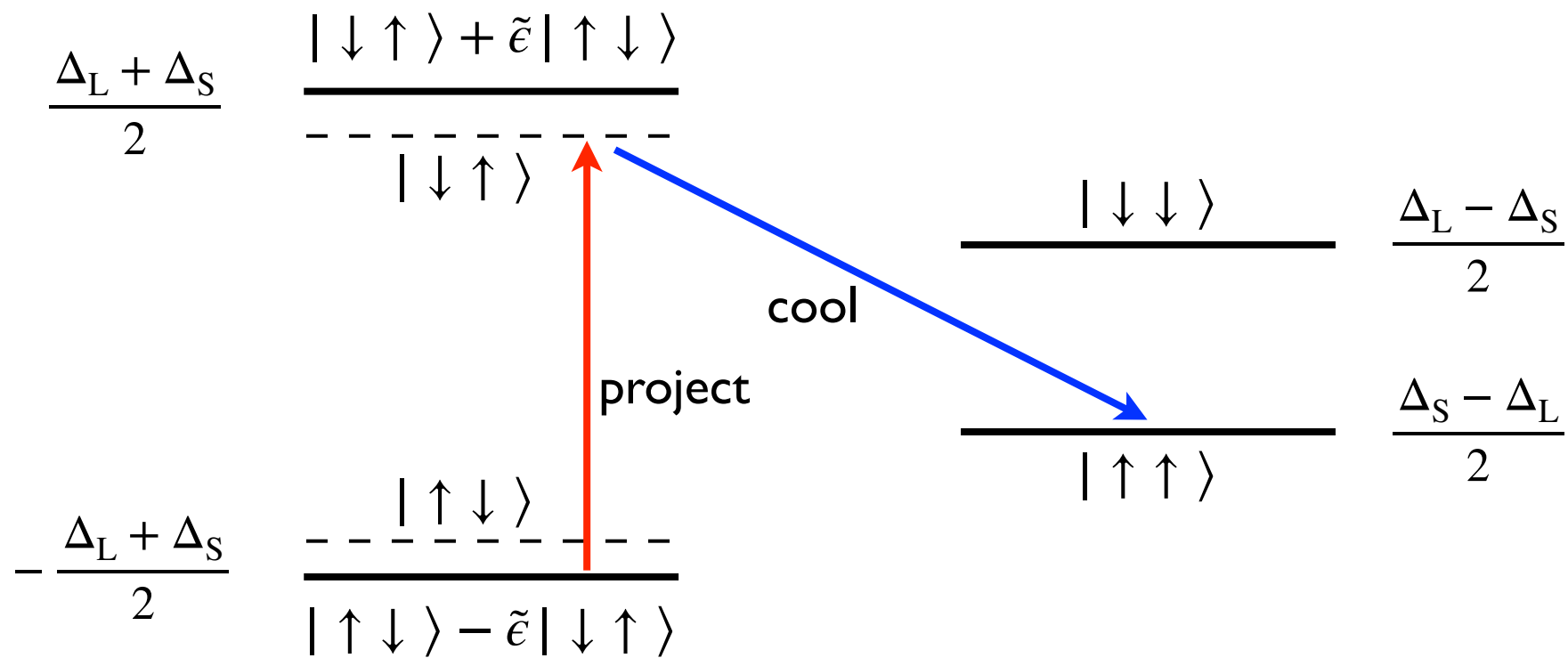
Where does the energy come from??



Where does the energy come from??



Where does the energy come from??



Energy comes from the measurement device

Finite temperature

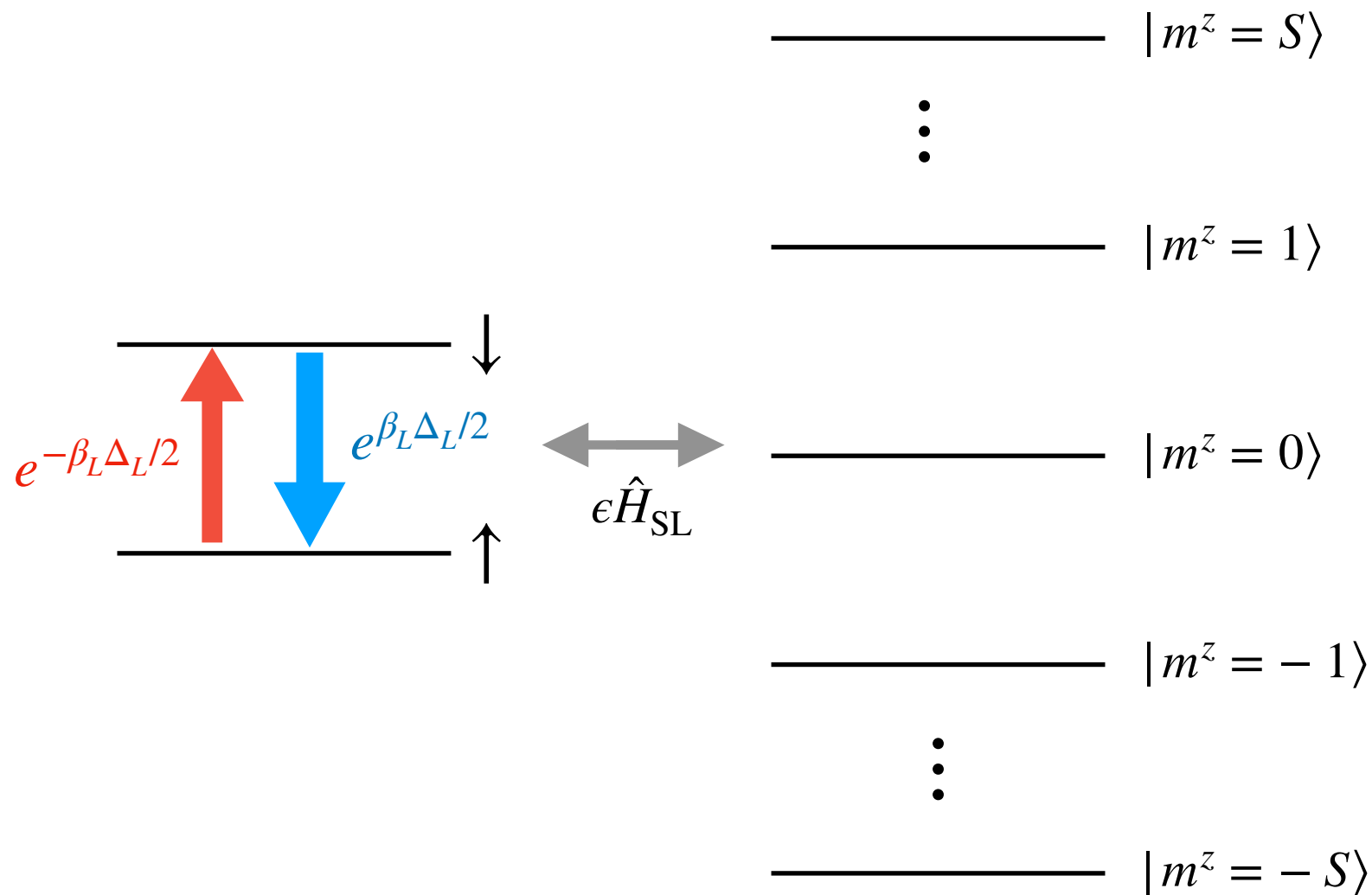
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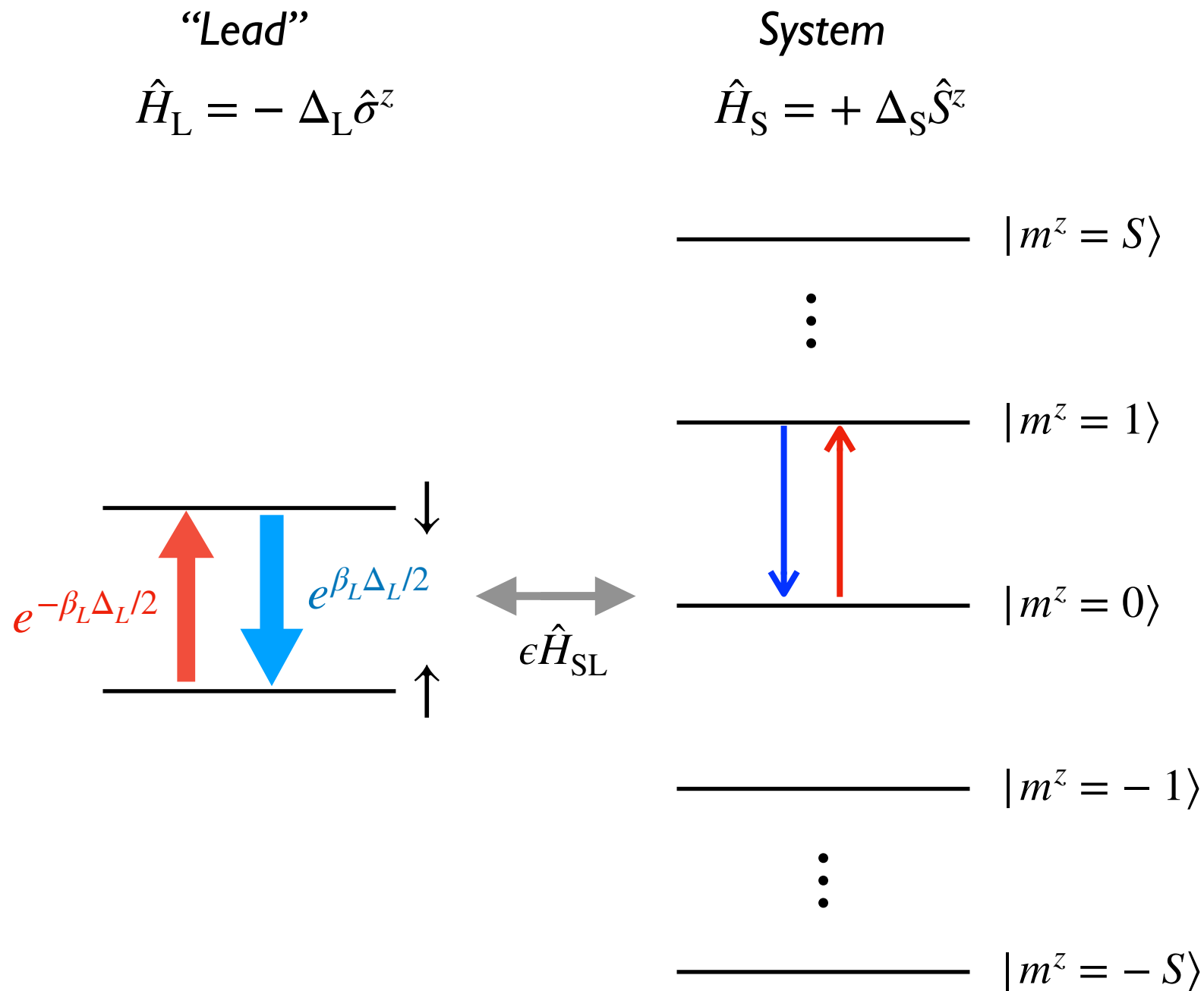
System

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$$\hat{H}_{SL} = \hat{\sigma}^+ \hat{S}^- + \hat{\sigma}^- \hat{S}^+$$



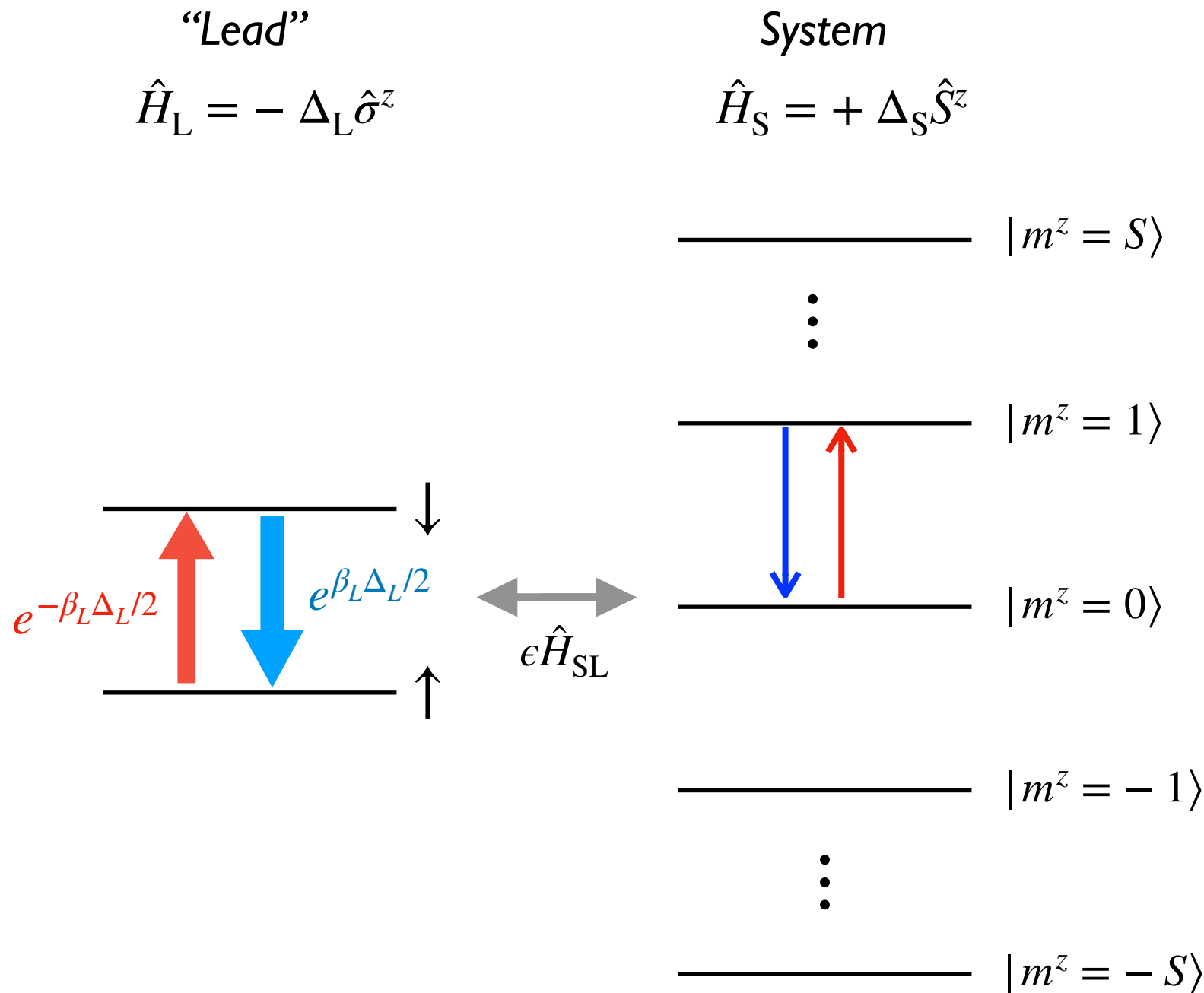
Finite temperature



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$R_{m^z+1 \rightarrow m^z}$	$R_{m^z \rightarrow m^z+1}$
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Finite temperature



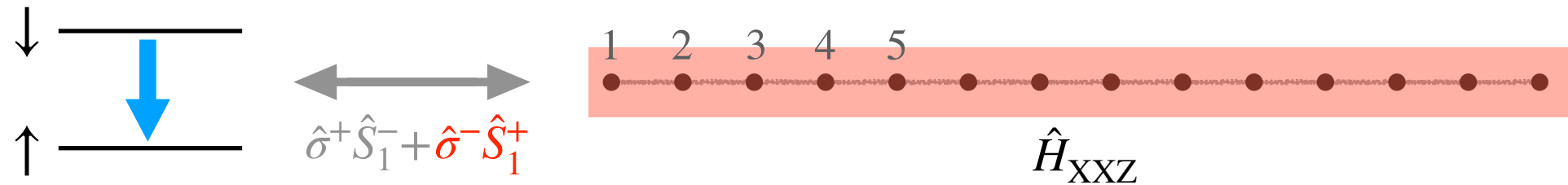
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$$\Rightarrow \beta_S = -\frac{\Delta_L}{\Delta_S} \beta_L$$

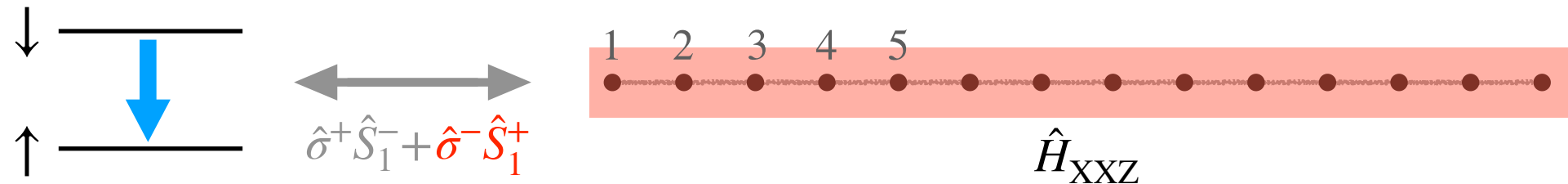
Extended system

Same argument for generic system that preserves total S^z (e.g. XXZ chain)



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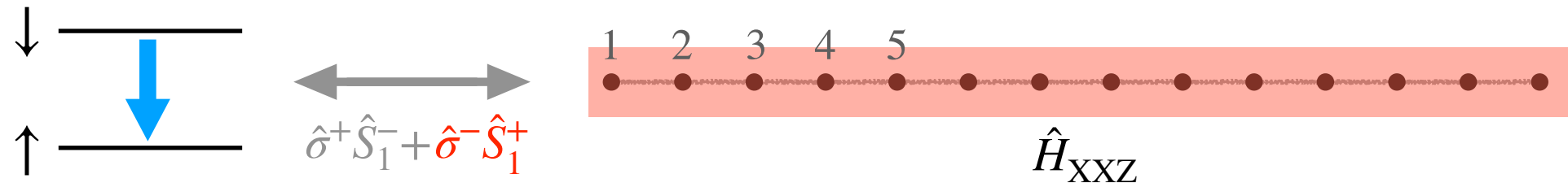


Not localized — excitation at site 1 is carried away into the bulk

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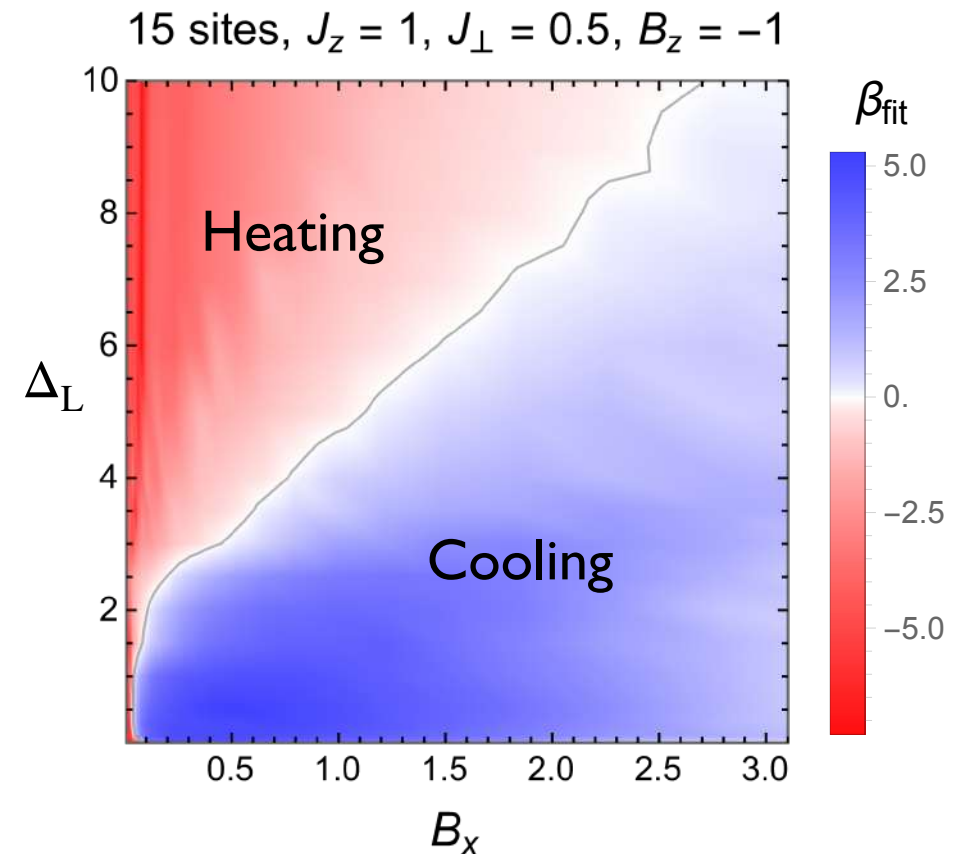


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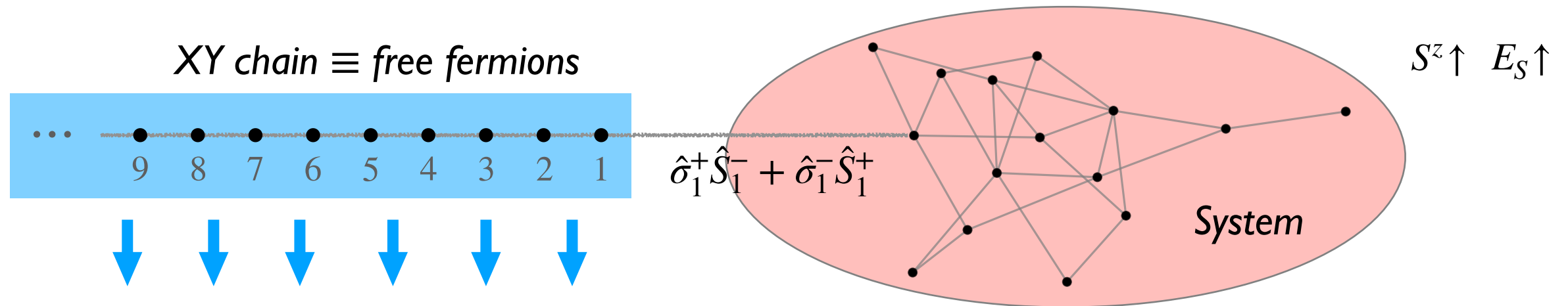
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$$\hat{H}_{\text{XXZ}} = \sum_i [J_z \hat{S}_i^z \hat{S}_{i+1}^z + J_\perp (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - B^z \hat{S}_i^z - \hat{B}^x \hat{S}_i^x]$$

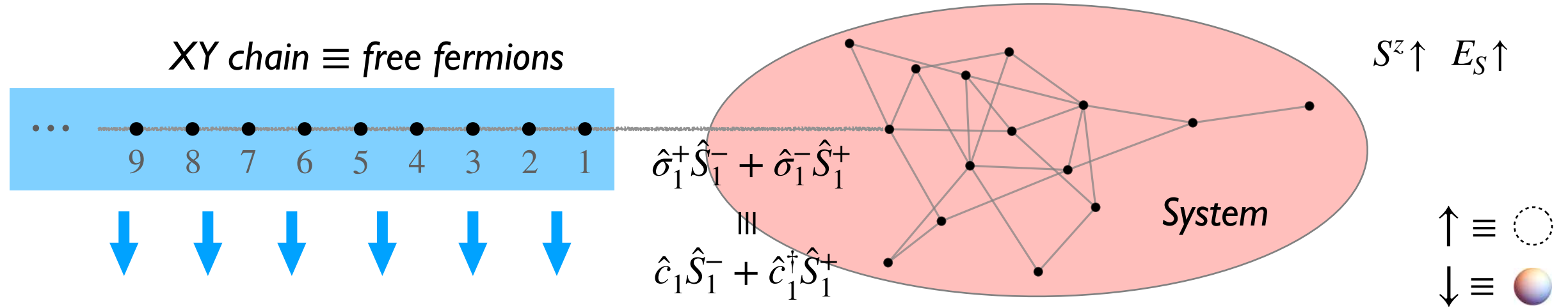
\downarrow
symmetry breaking



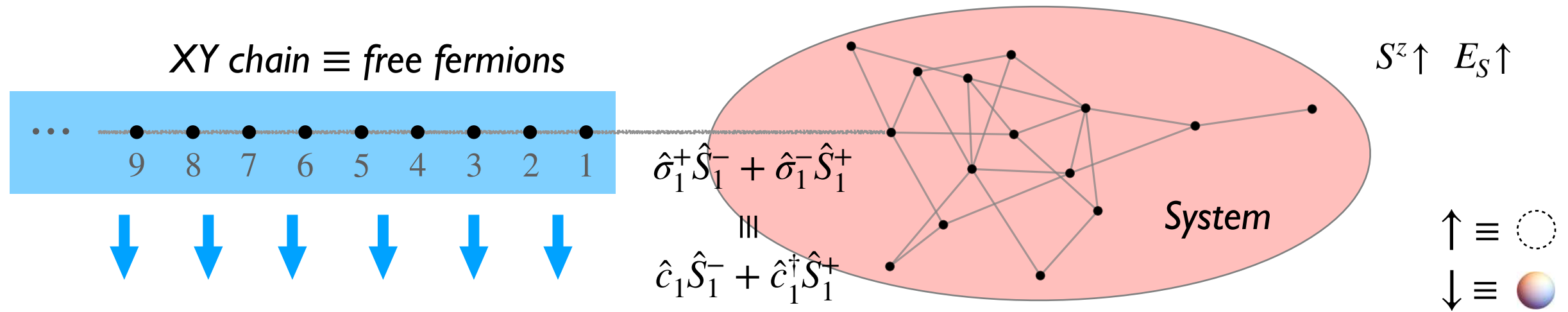
Infinite lead + generic system



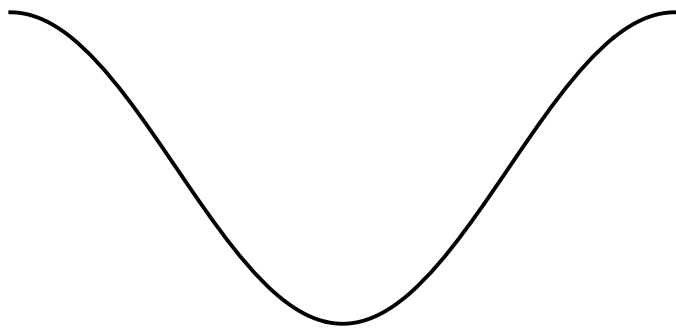
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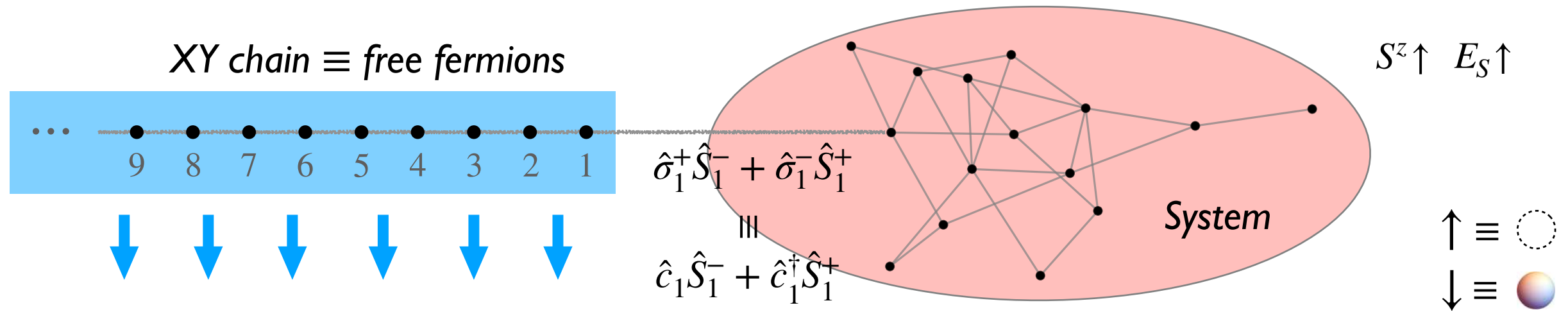
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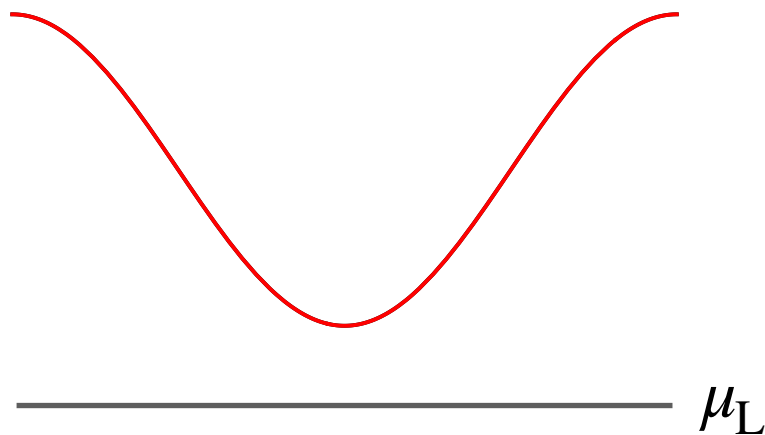
Lead spectrum for single-particle excitations: $\varepsilon_L(k) = -2J \cos k$



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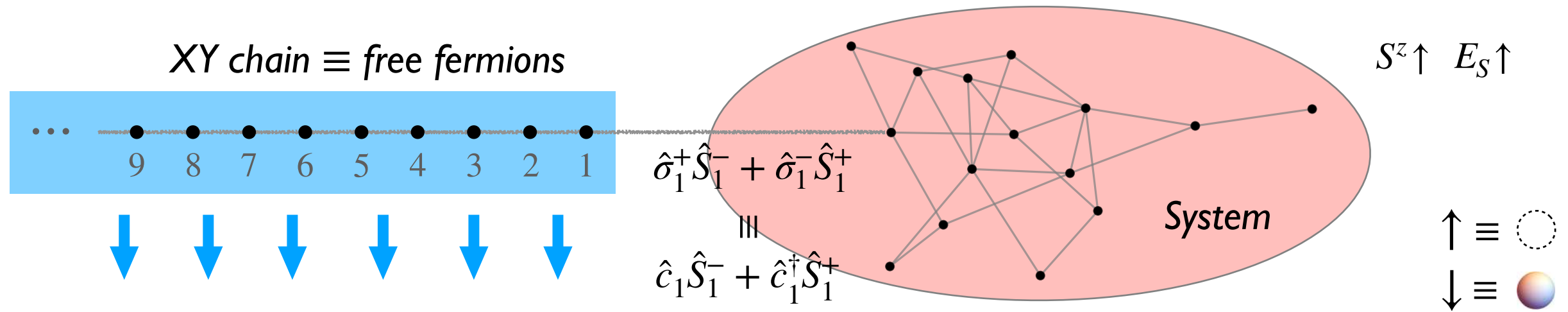
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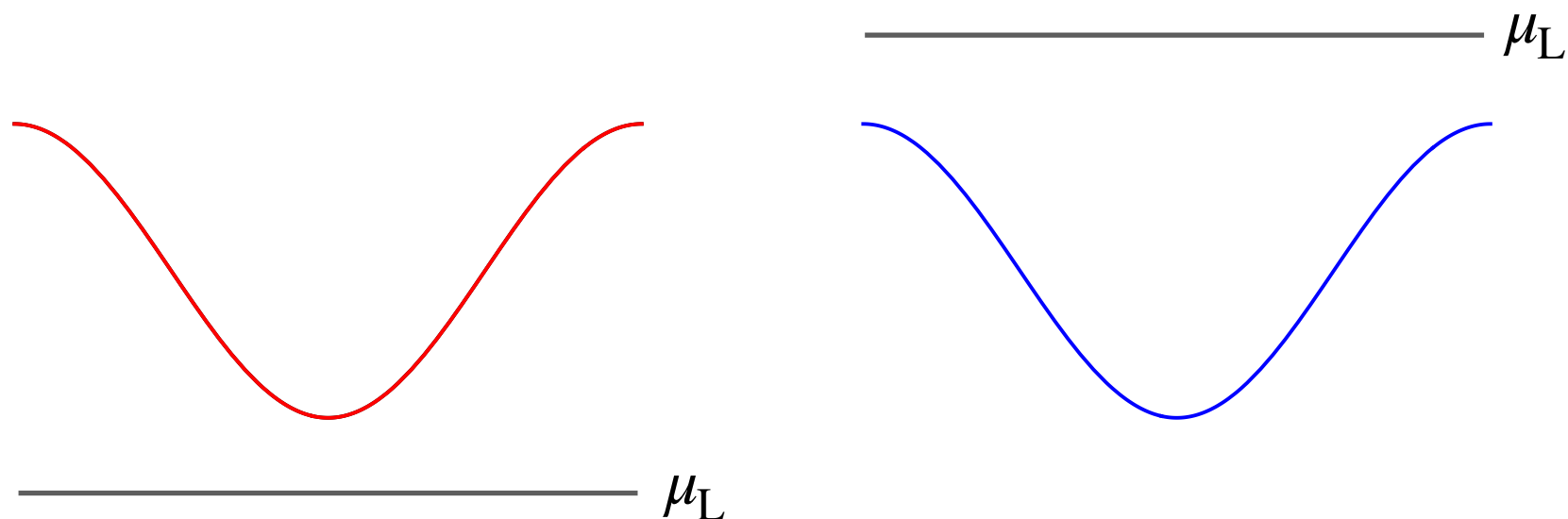
Only particle excitations: $\hat{c}_1^+ \hat{S}_1^- + \hat{c}_1^- \hat{S}_1^+$

\Rightarrow Heats to maximum S^z

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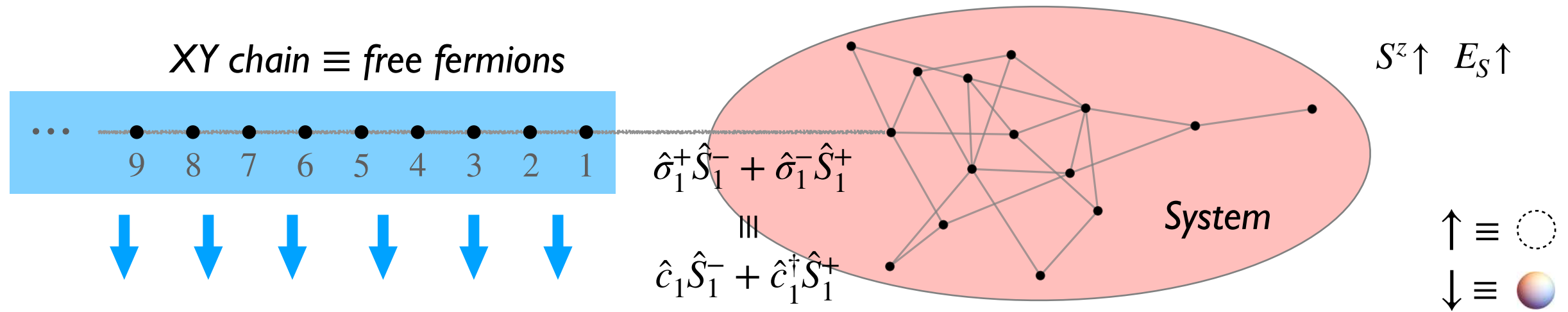
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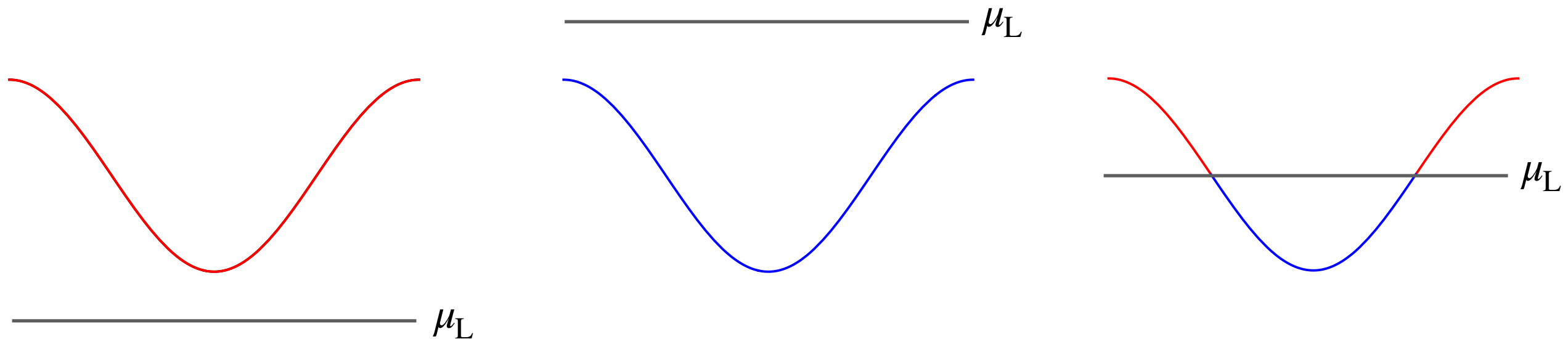
Only hole excitations: $\hat{c}_1 \hat{S}_1^- + \hat{c}_1^+ \hat{S}_1^+$

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Only hole excitations: $\hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$

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Both: $\hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$

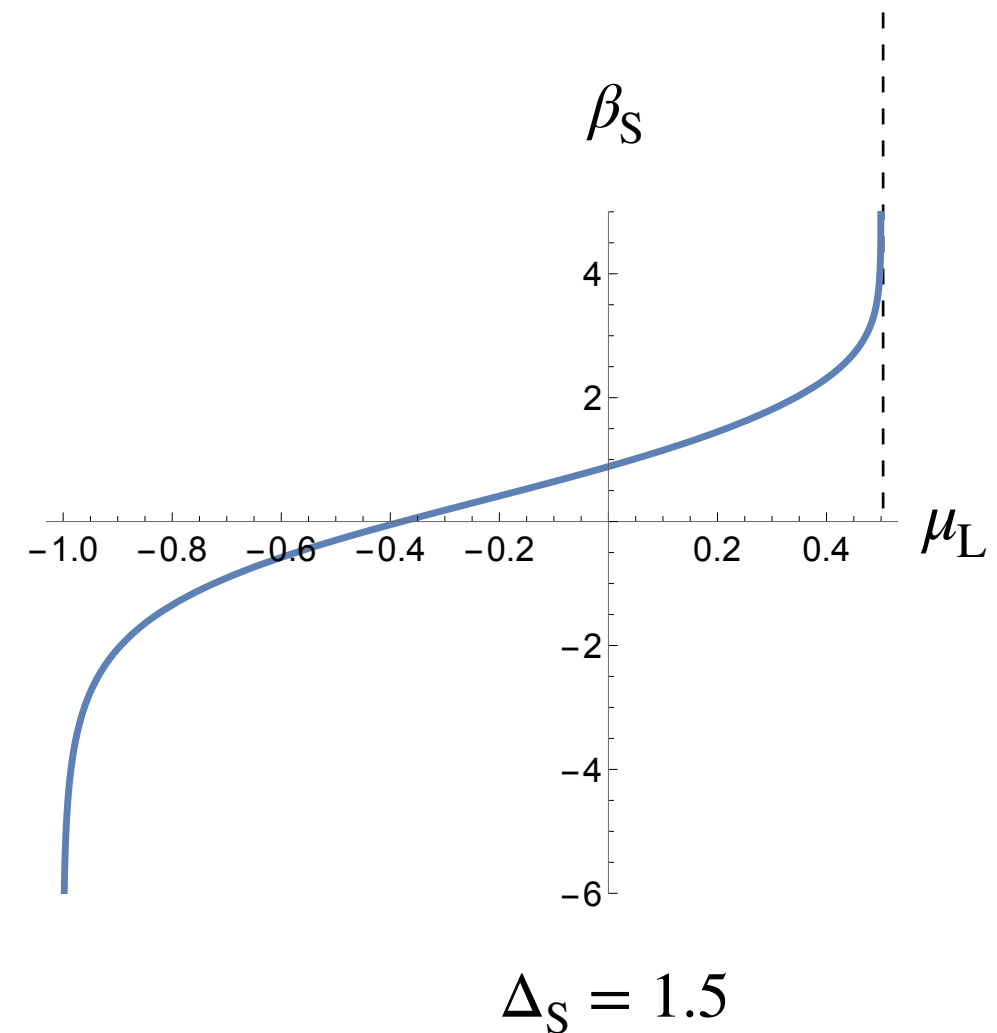
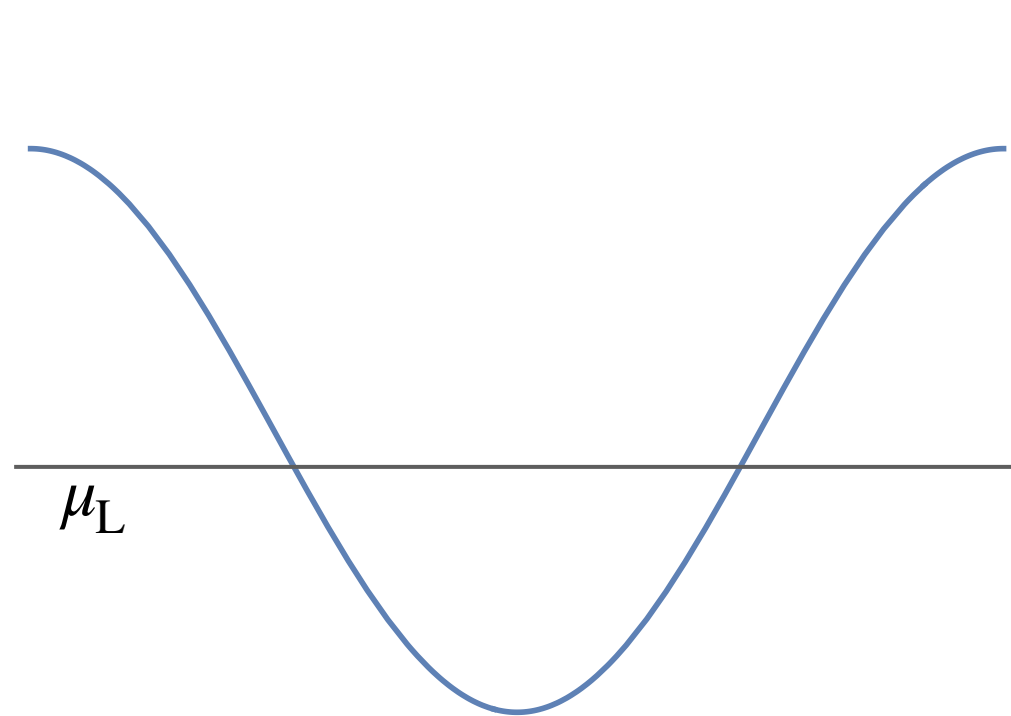
\Rightarrow Intermediate state

Free-fermion lead + N -level system

$$\hat{H}_S = \Delta_S \hat{S}^z$$

$$R_{m^z \rightarrow m^z+1} \propto \gamma \epsilon^2 \int_{\epsilon_L(k) > \mu_L} \frac{dk \sin^2 k}{(\Delta_S + |\epsilon_L(k) - \mu_L|)^2} \quad R_{m^z+1 \rightarrow m^z} \propto \gamma \epsilon^2 \int_{\epsilon_L(k) < \mu_L} \frac{dk \sin^2 k}{(\Delta_S - |\epsilon_L(k) - \mu_L|)^2}$$

$$\epsilon_L(k) = -\cos k$$



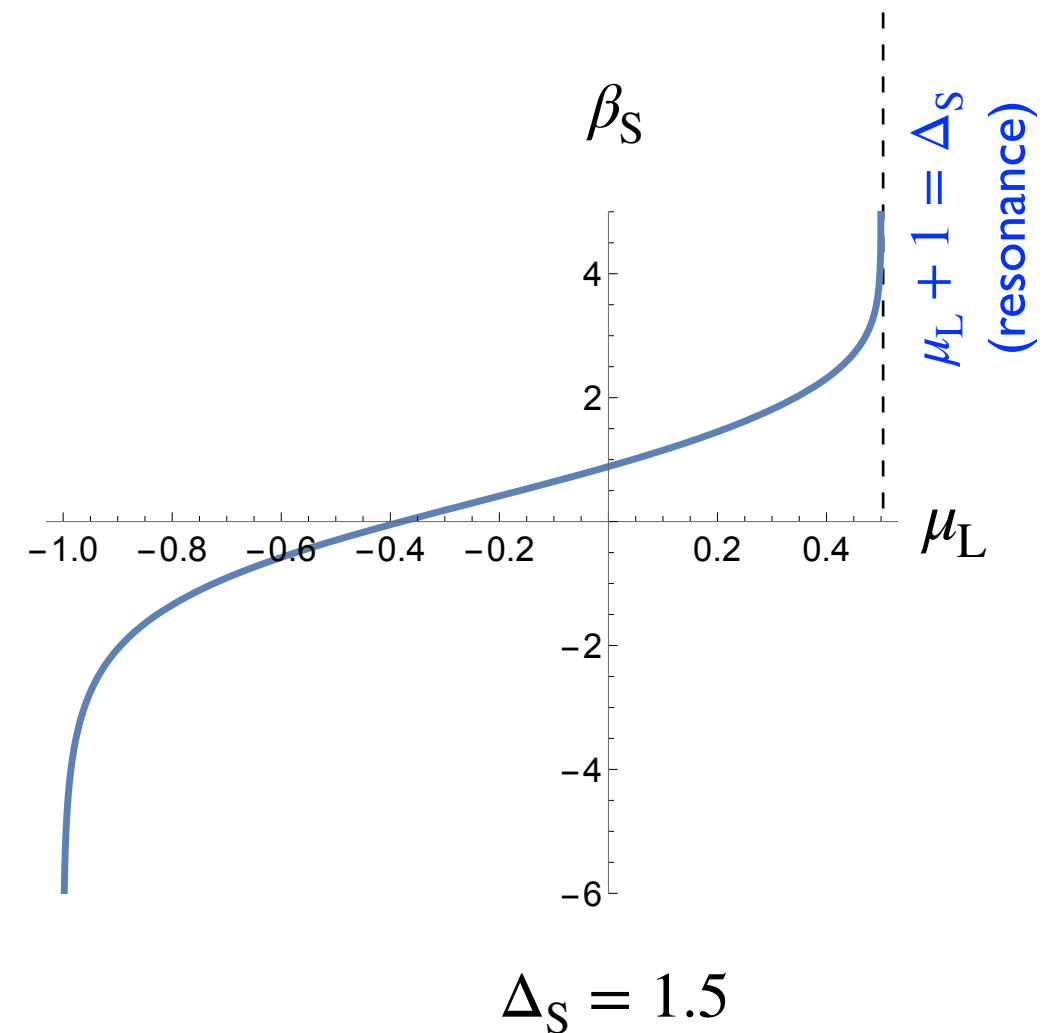
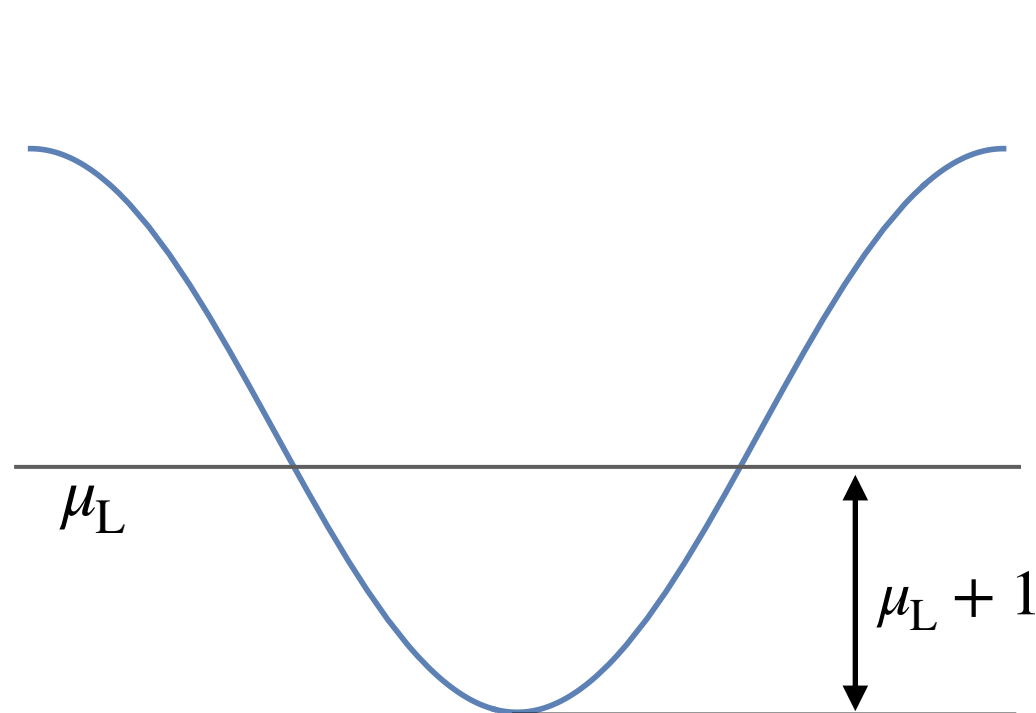
Analytic solution using detailed balance

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$$\Delta_S = 1.5$$

Analytic solution using detailed balance

Free-fermion lead + N -level system

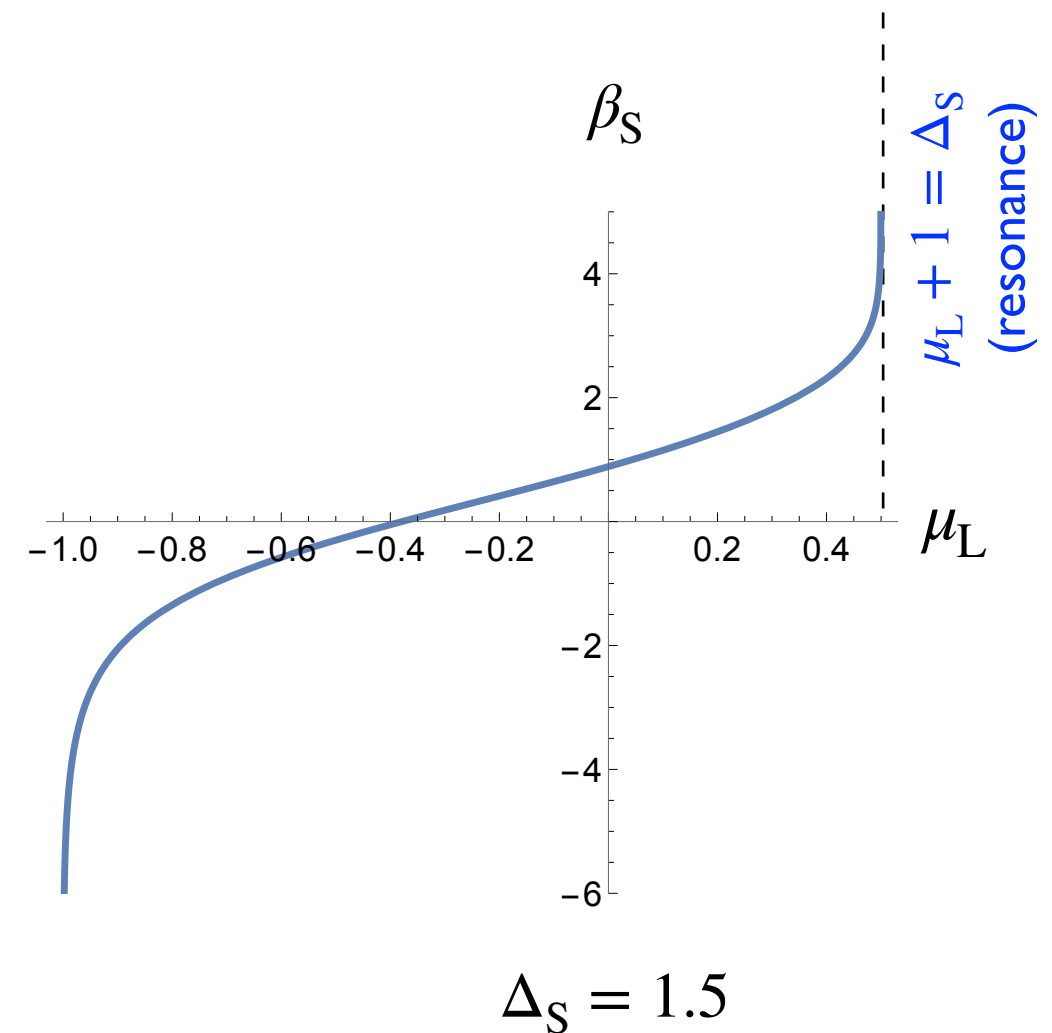
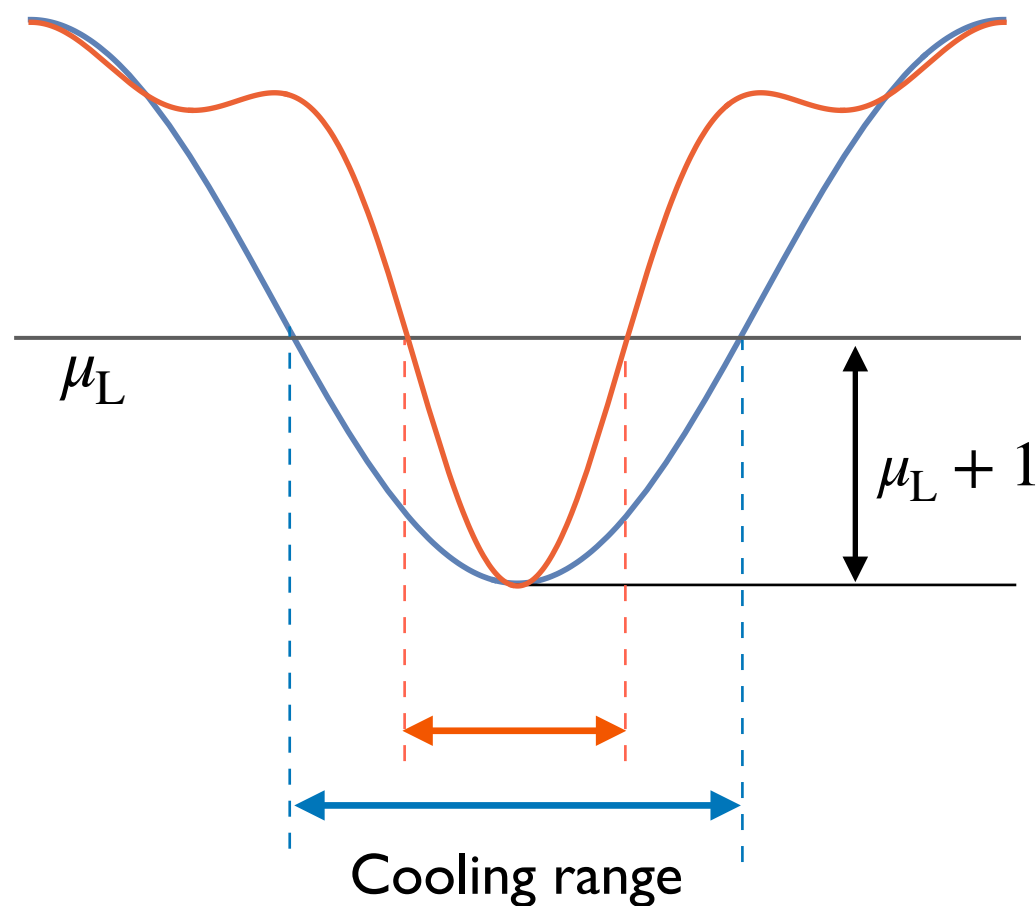
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$$\epsilon_L(k) = -\cos k$$

$$\epsilon_L(k) \sim -\sum_{n=1}^3 \frac{1}{n} \cos(nk) \quad \text{Fewer modes cool}$$



Analytic solution using detailed balance

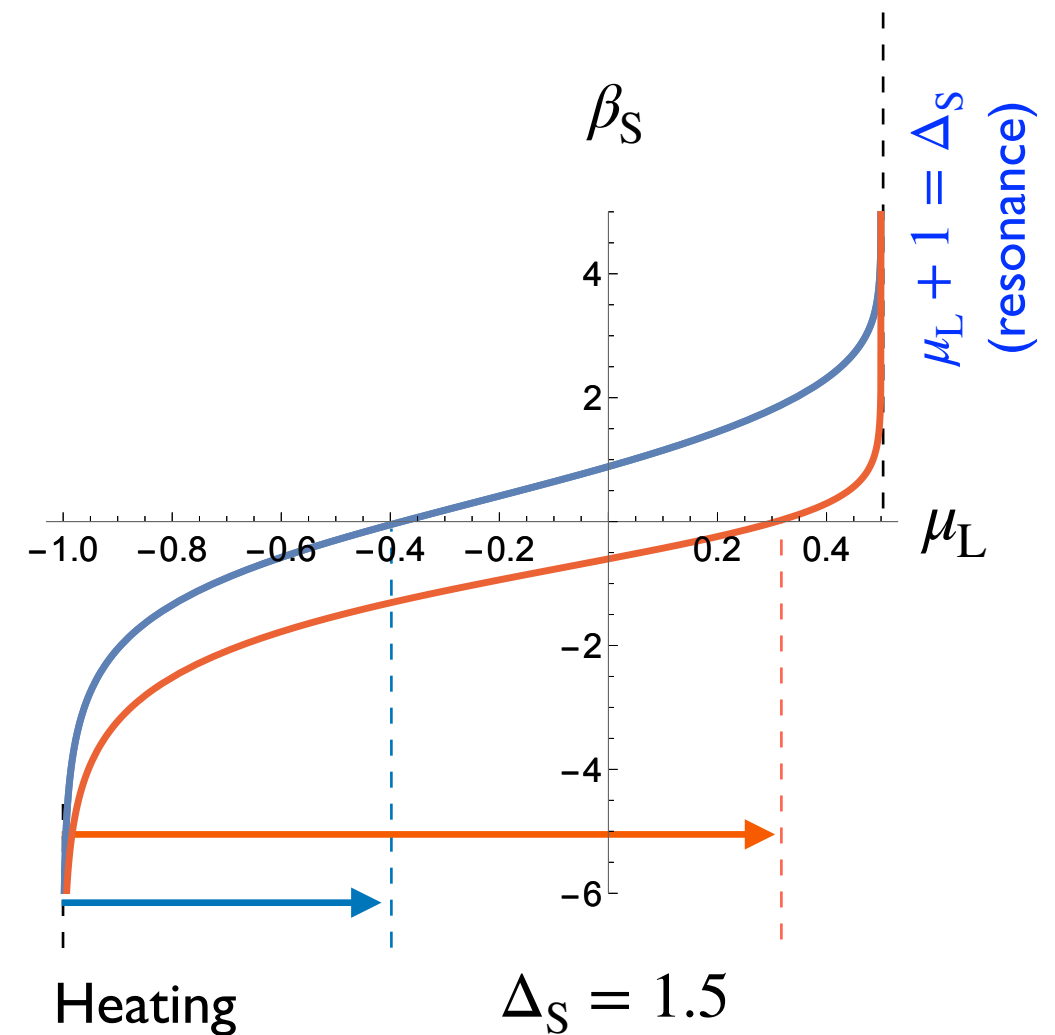
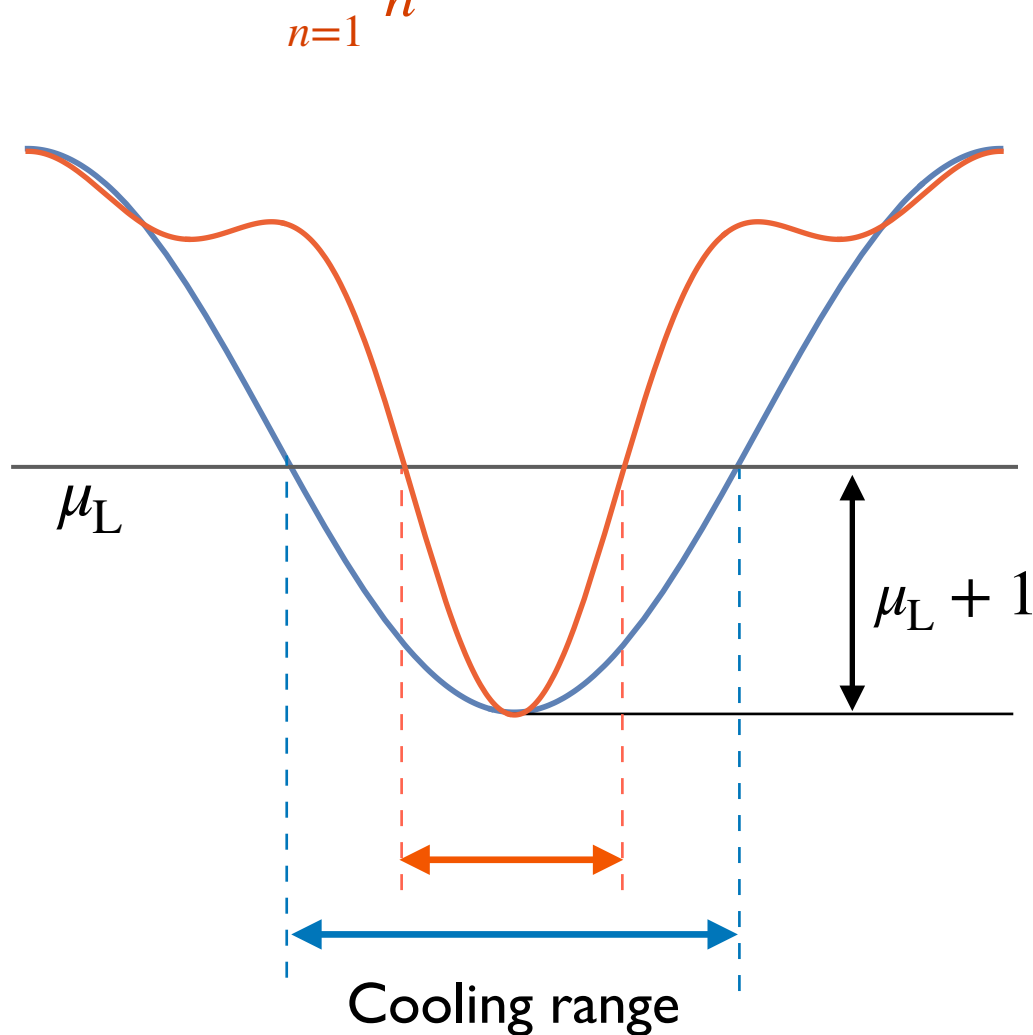
Free-fermion lead + N -level system

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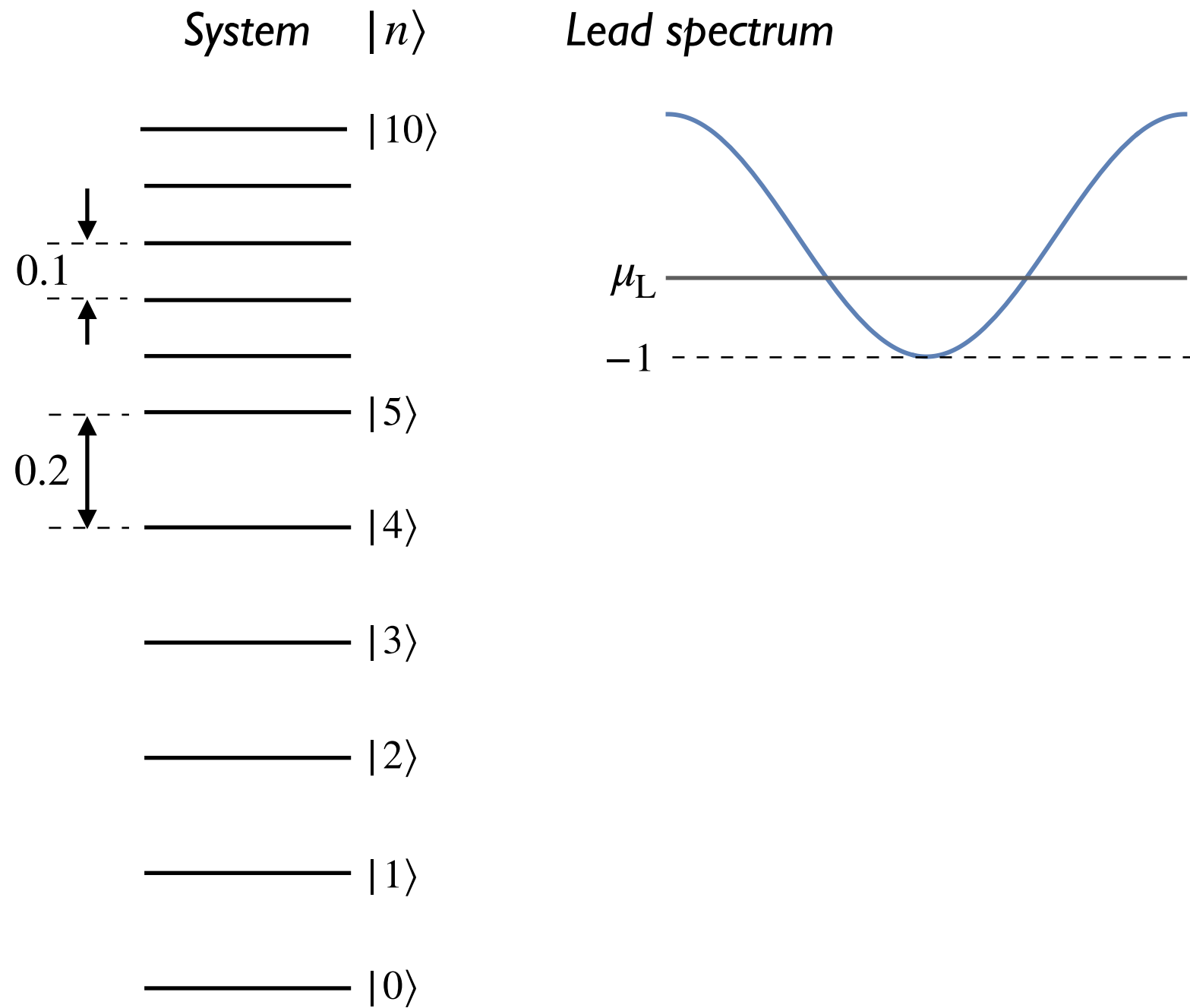
$$\epsilon_L(k) = -\cos k$$

$$\epsilon_L(k) \sim -\sum_{n=1}^3 \frac{1}{n} \cos(nk) \quad \text{Fewer modes cool} \implies \text{more heating}$$



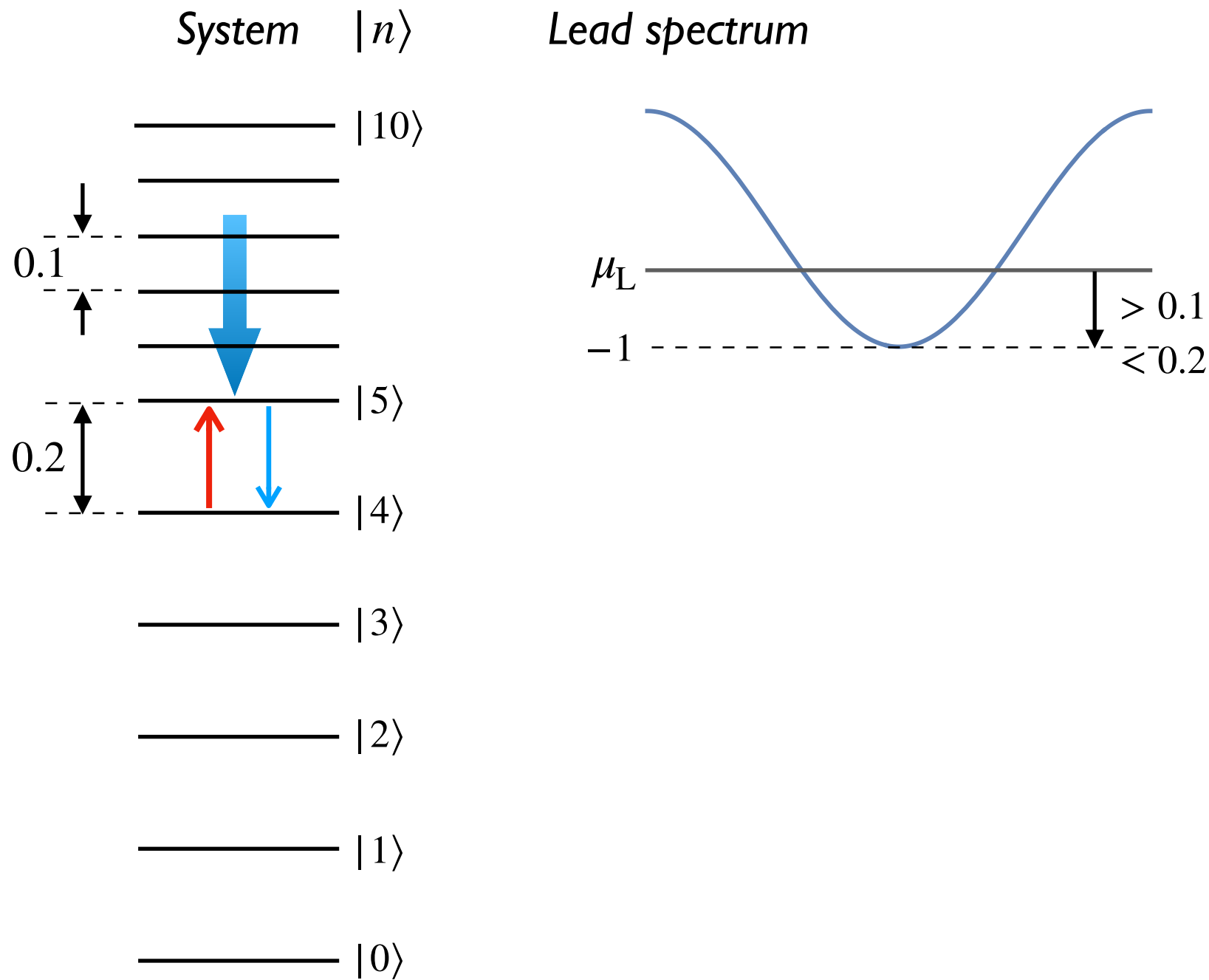
Analytic solution using detailed balance

Corollary I: Mid-spectrum nonclassical steady state



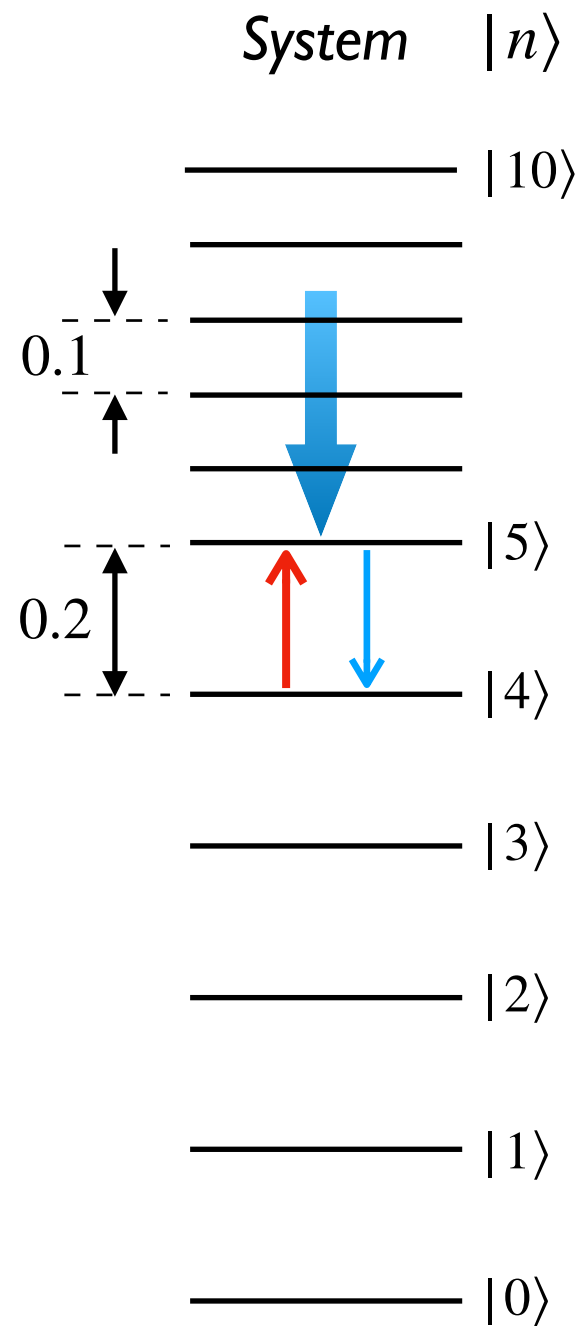
“Nonlinear cavity”

Corollary I: Mid-spectrum nonclassical steady state



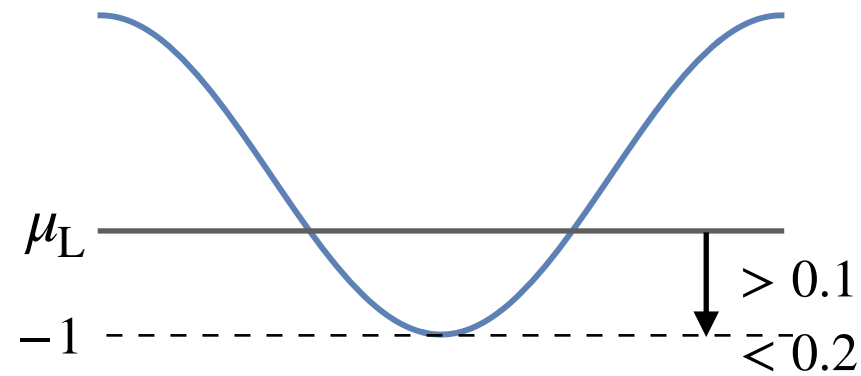
“Nonlinear cavity”

Corollary I: Mid-spectrum nonclassical steady state

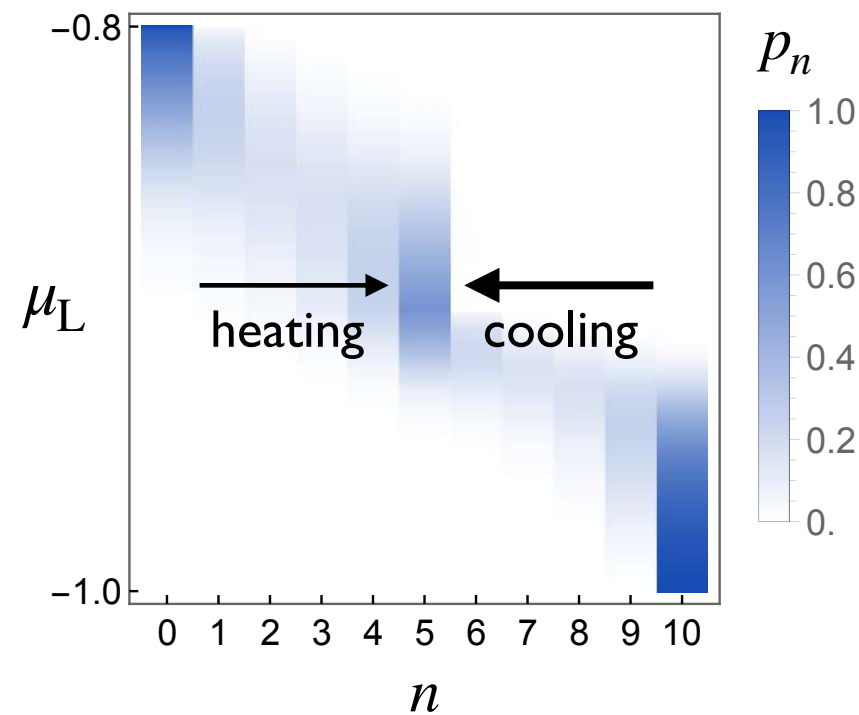


“Nonlinear cavity”

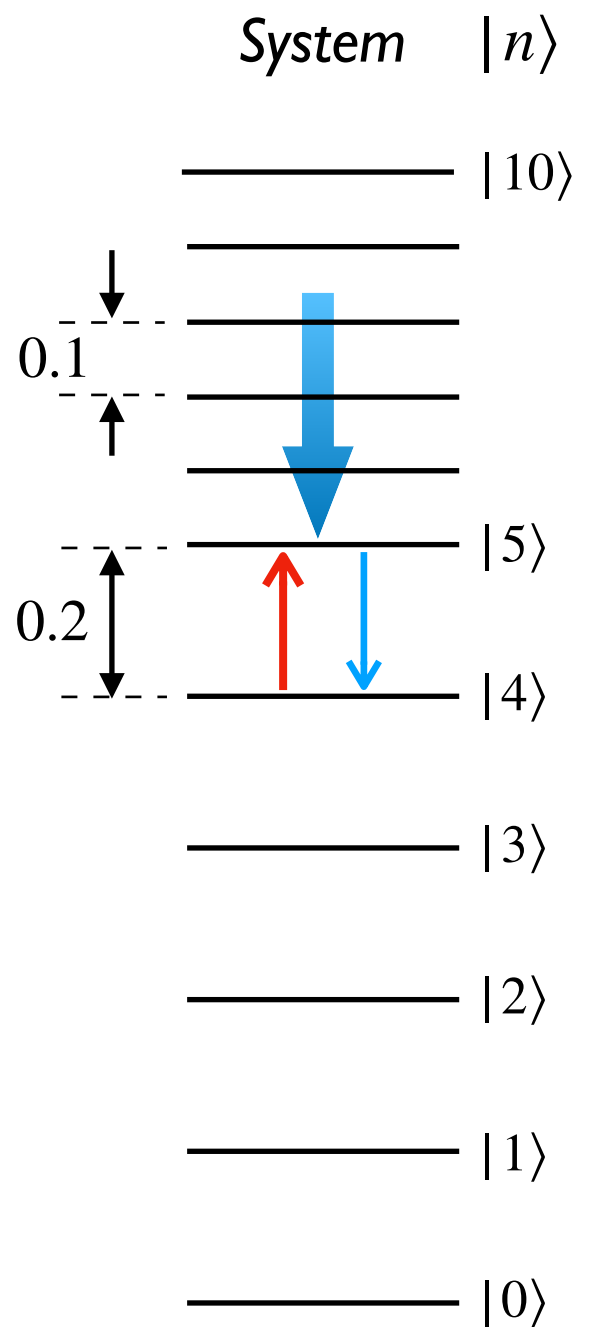
Lead spectrum



Steady-state occupations

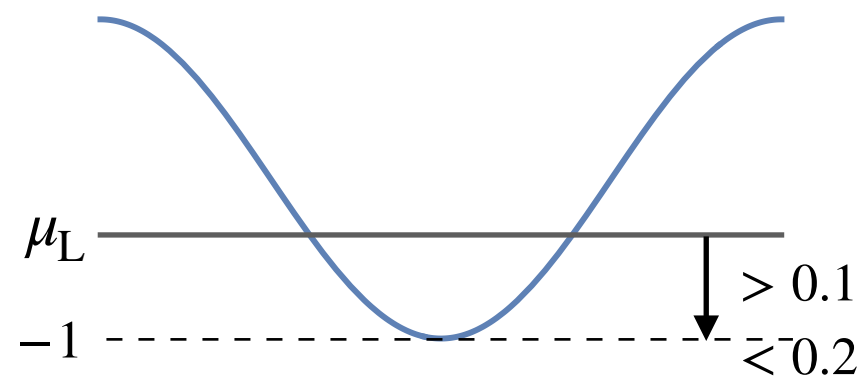


Corollary I: Mid-spectrum nonclassical steady state

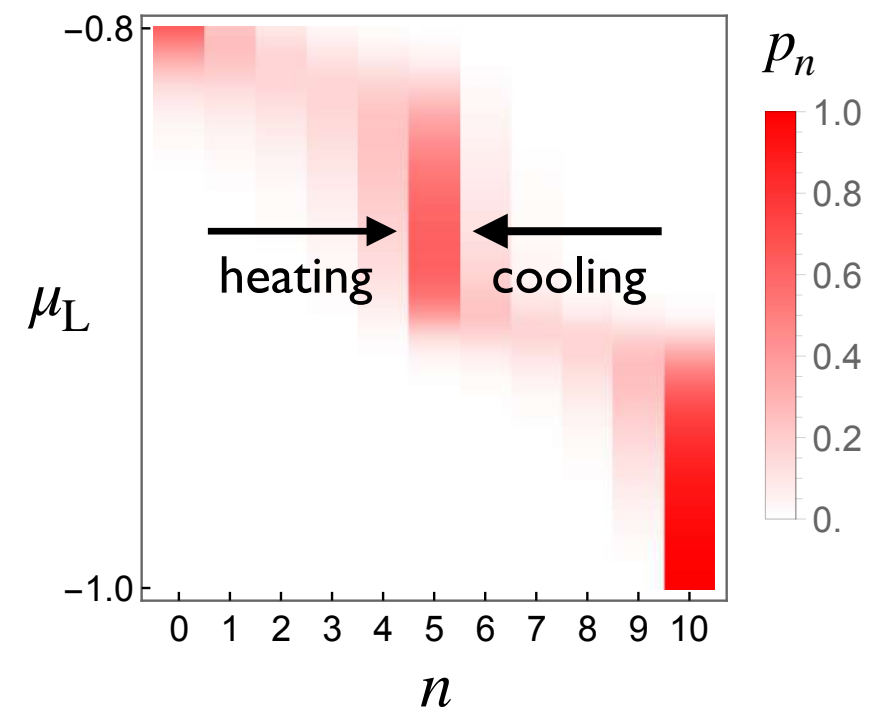
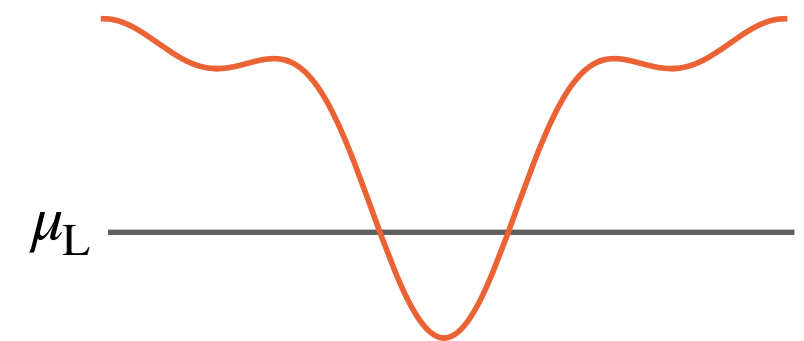
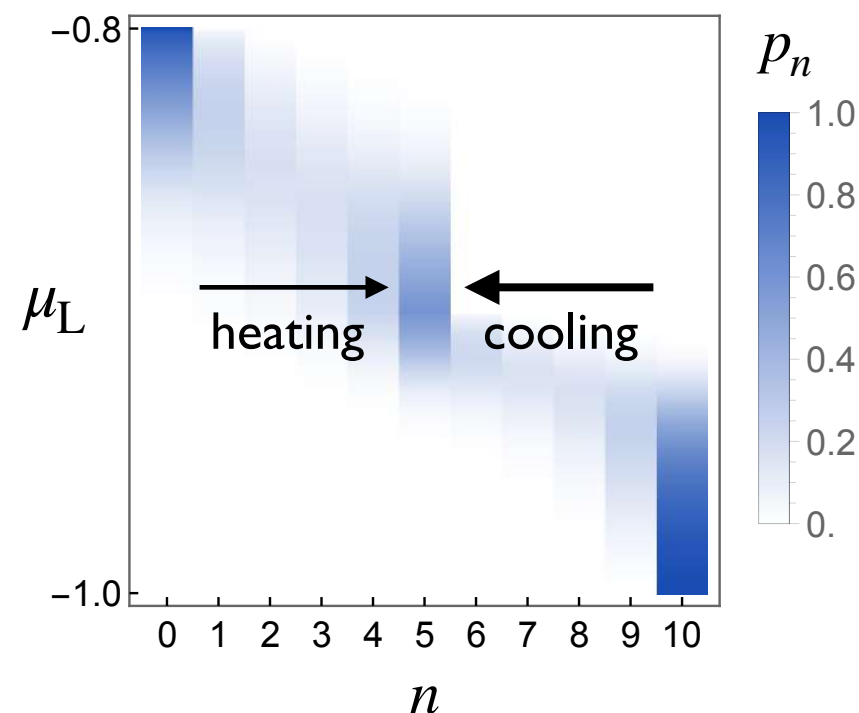


“Nonlinear cavity”

Lead spectrum

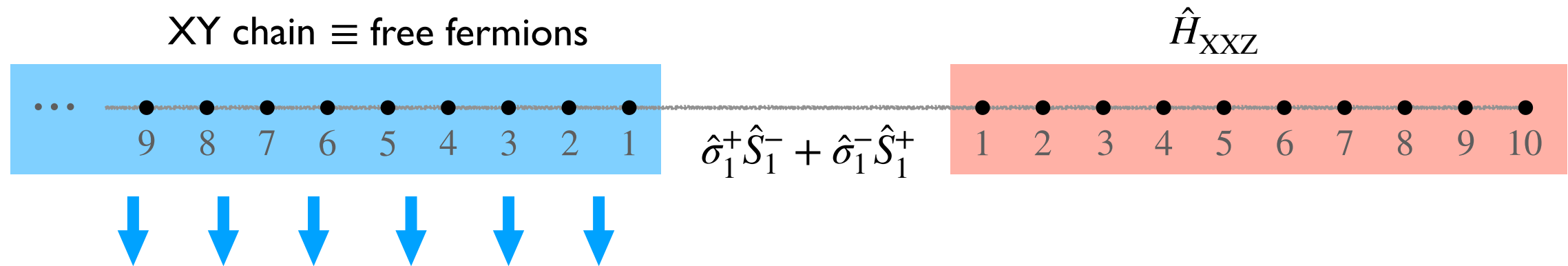


Steady-state occupations



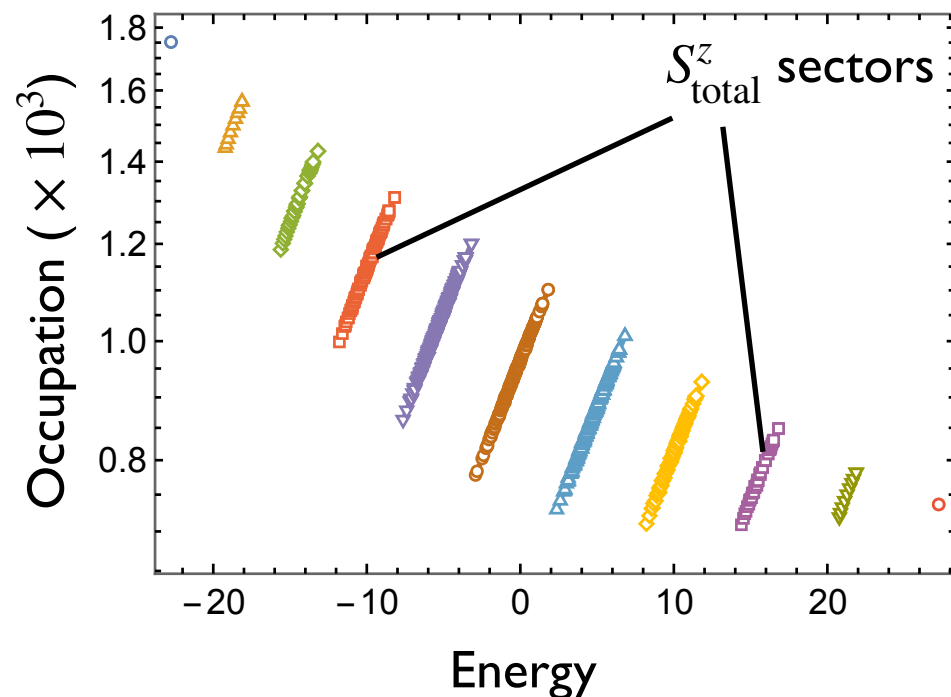
5-photon Fock state

Corollary 2: Global cooling + subsector heating

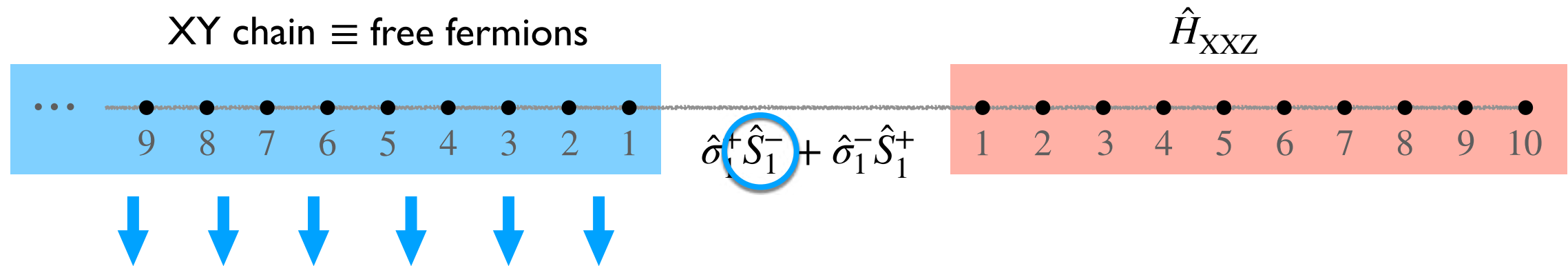


$$\hat{H}_{\text{XXZ}} = \sum_i \left[J_z \hat{S}_i^z \hat{S}_{i+1}^z + J_{\perp} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - B_z \hat{S}_i^z \right]$$

$$J_z = 1, J_{\perp} = 0.5, B_z = -5, \mu_L + 1 = 0.9$$

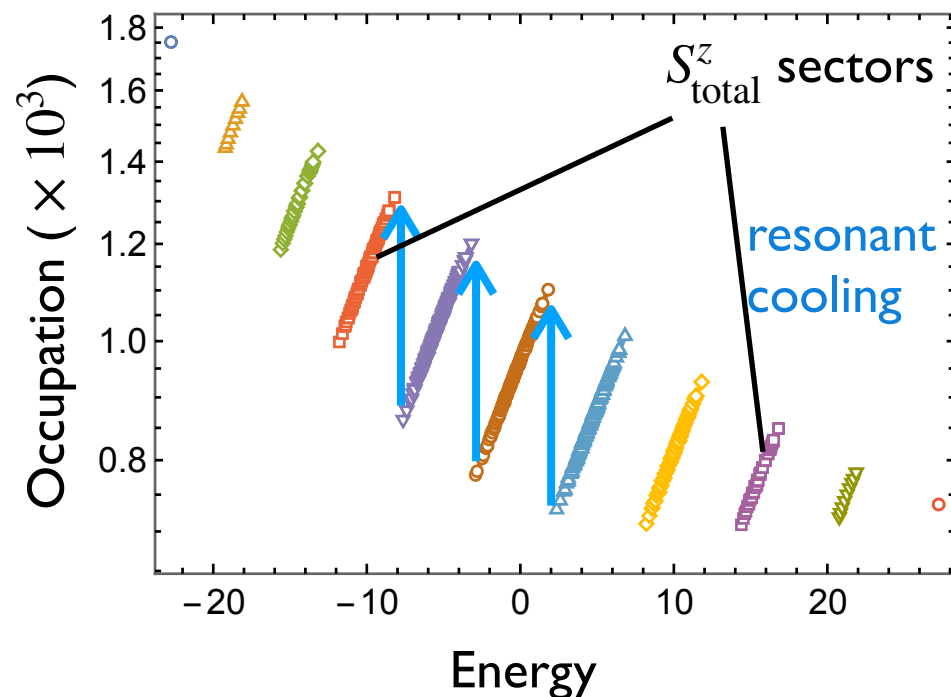


Corollary 2: Global cooling + subsector heating

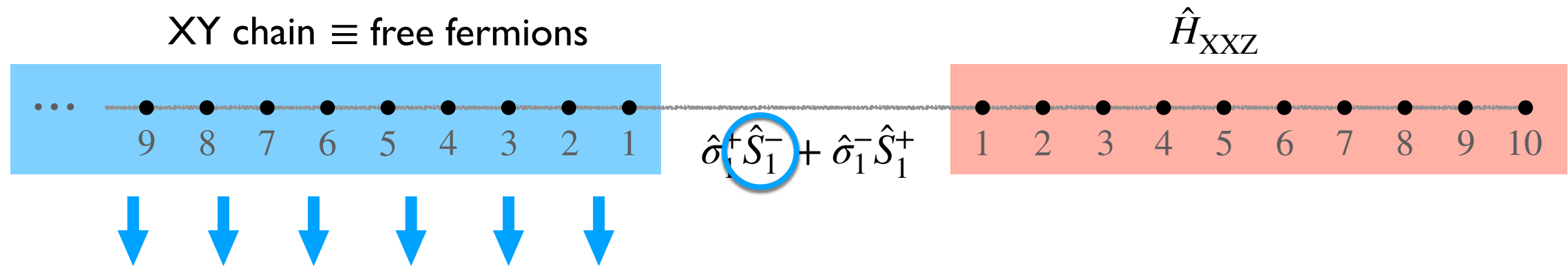


$$\hat{H}_{\text{XXZ}} = \sum_i \left[J_z \hat{S}_i^z \hat{S}_{i+1}^z + J_{\perp} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - B_z \hat{S}_i^z \right]$$

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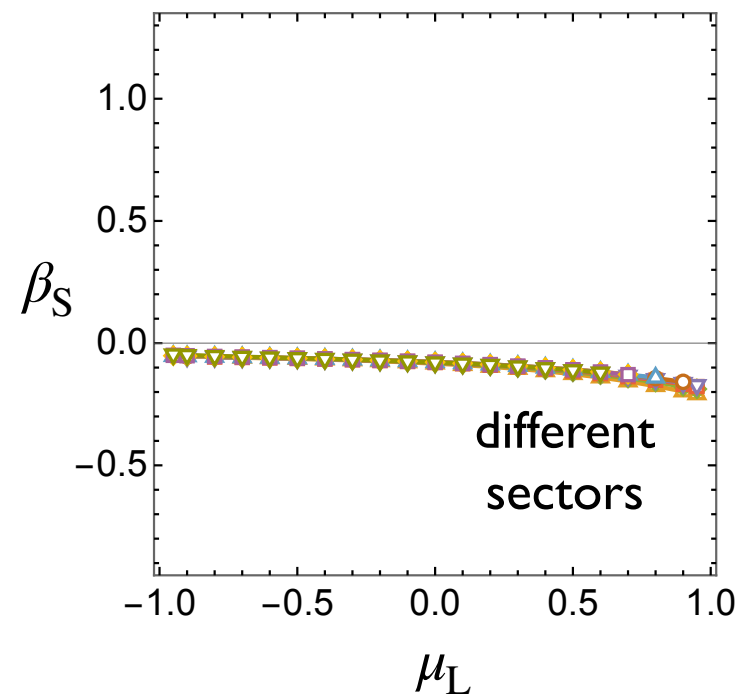
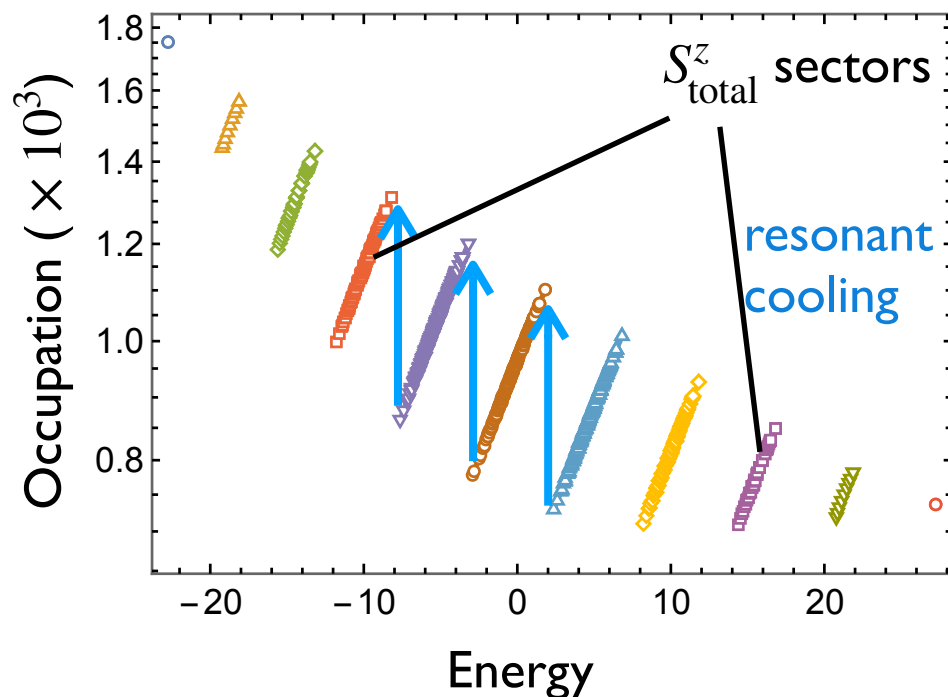


Corollary 2: Global cooling + subsector heating

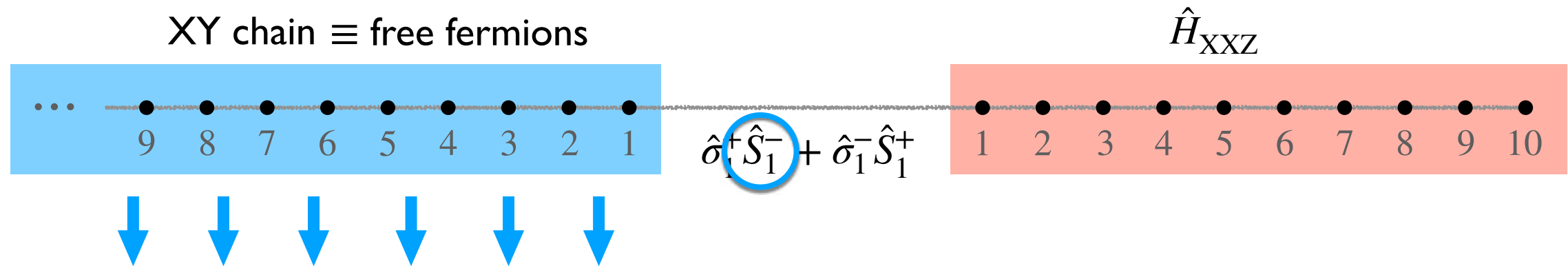


$$\hat{H}_{\text{XXZ}} = \sum_i [J_z \hat{S}_i^z \hat{S}_{i+1}^z + J_{\perp} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - B_z \hat{S}_i^z]$$

$$J_z = 1, J_{\perp} = 0.5, B_z = -5, \mu_L + 1 = 0.9$$

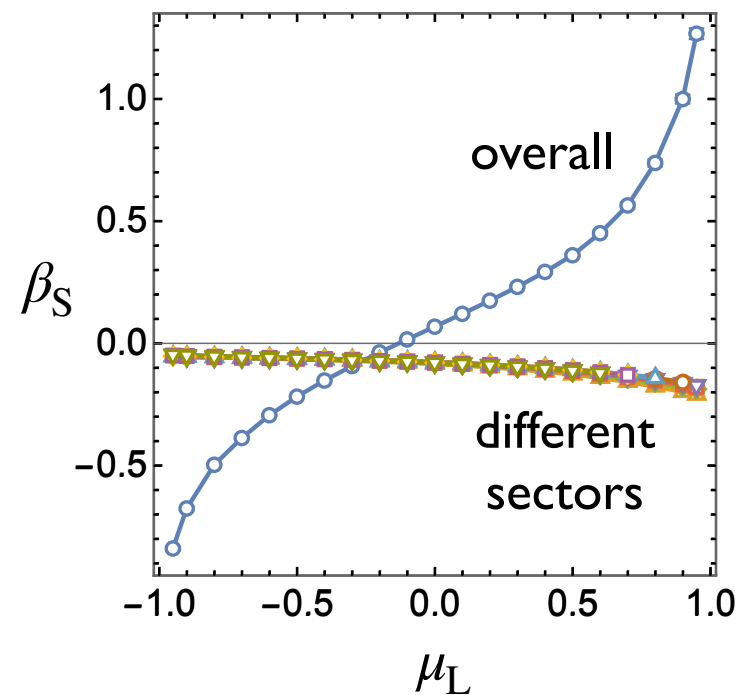
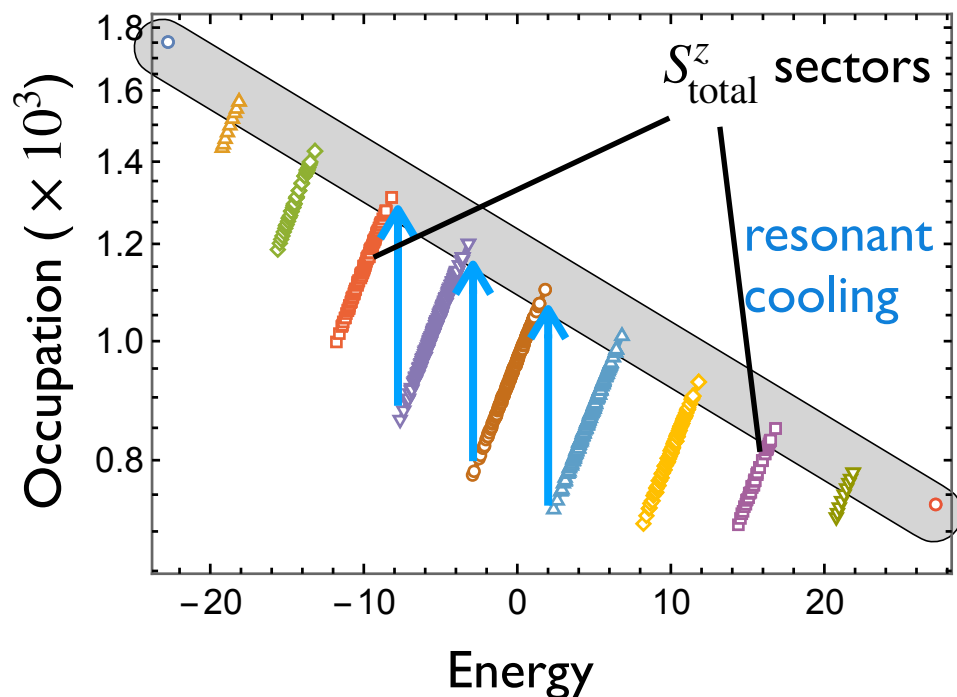


Corollary 2: Global cooling + subsector heating

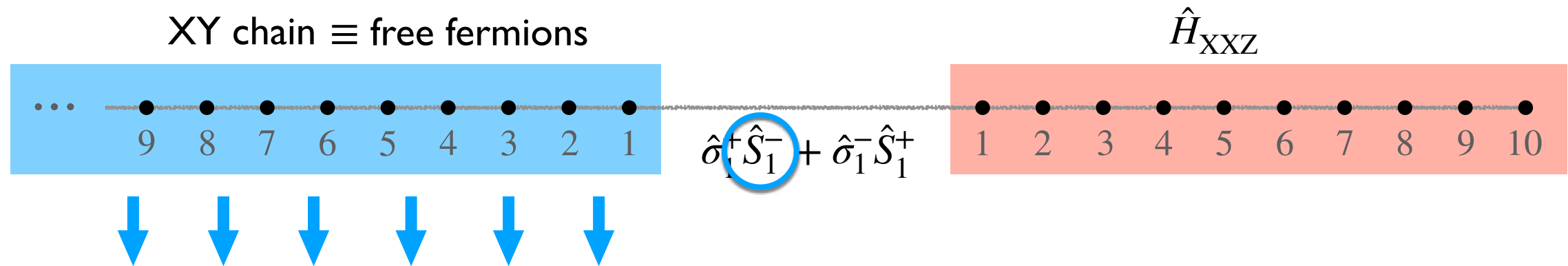


$$\hat{H}_{\text{XXZ}} = \sum_i [J_z \hat{S}_i^z \hat{S}_{i+1}^z + J_{\perp} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - B_z \hat{S}_i^z]$$

$$J_z = 1, J_{\perp} = 0.5, B_z = -5, \mu_L + 1 = 0.9$$

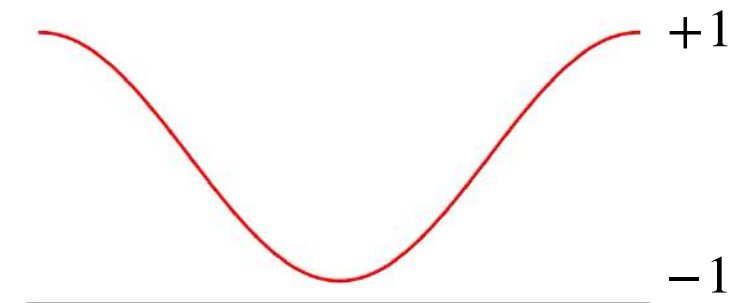
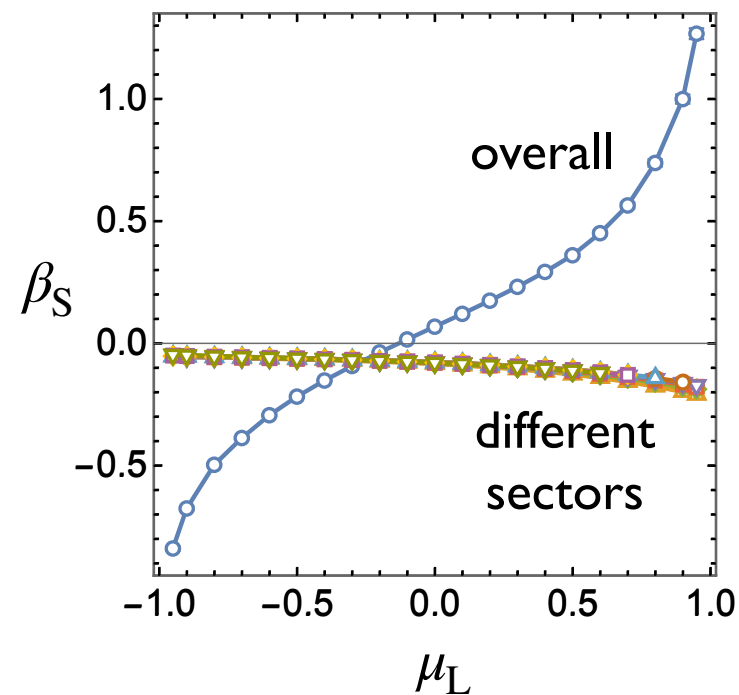
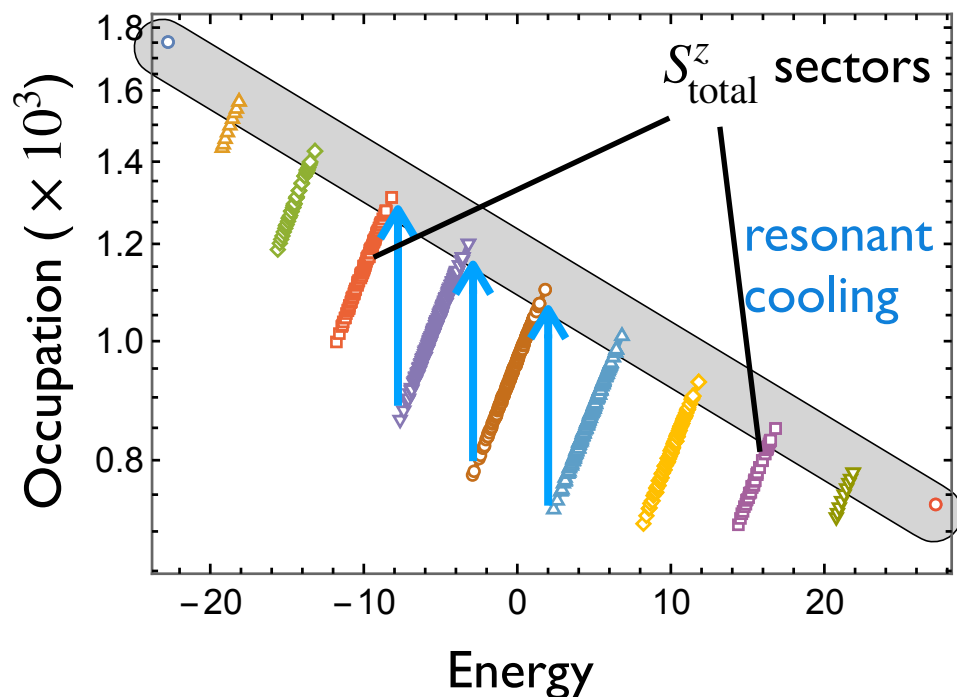


Corollary 2: Global cooling + subsector heating

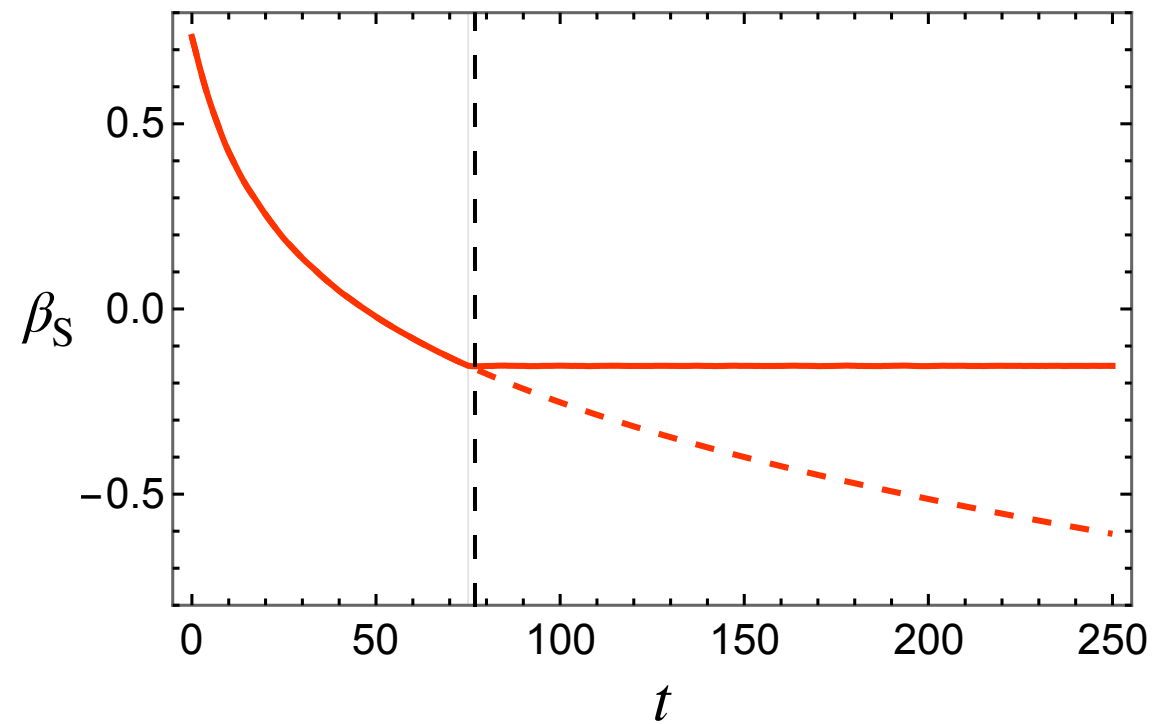
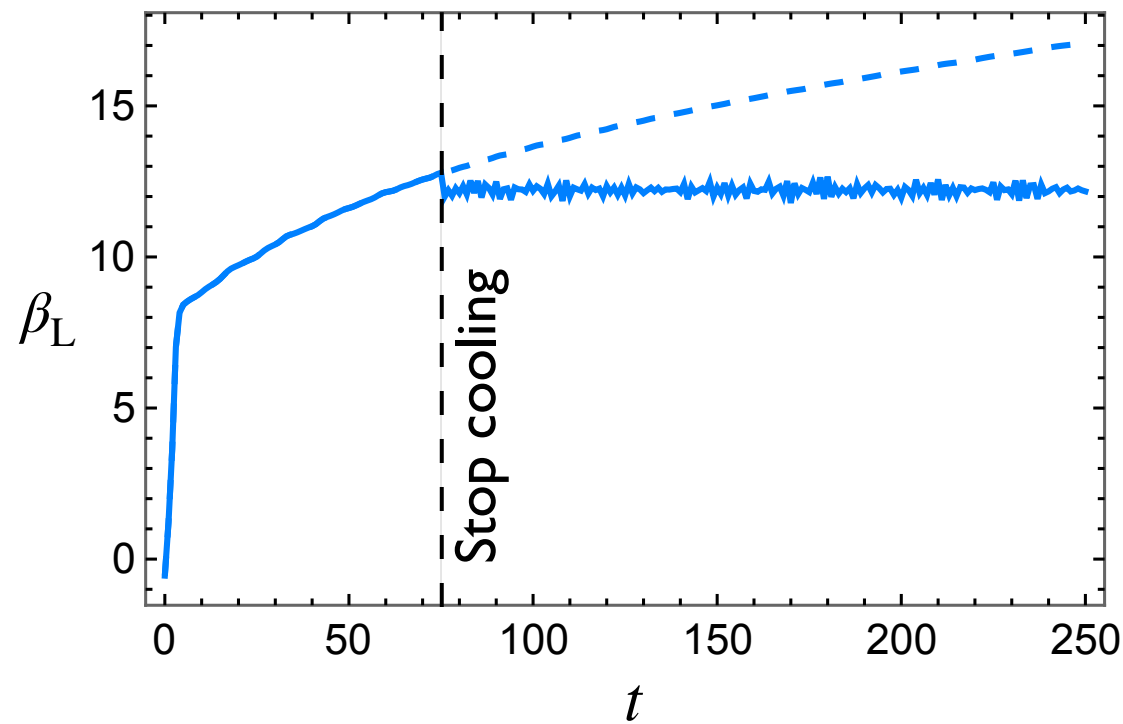
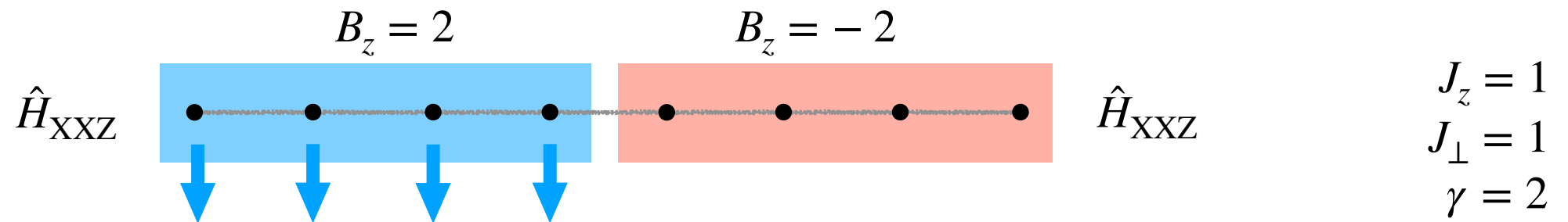


$$\hat{H}_{\text{XXZ}} = \sum_i [J_z \hat{S}_i^z \hat{S}_{i+1}^z + J_{\perp} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - B_z \hat{S}_i^z]$$

$$J_z = 1, J_{\perp} = 0.5, B_z = -5, \mu_L + 1 = 0.9$$



Corollary 3: Stable temperature gradient



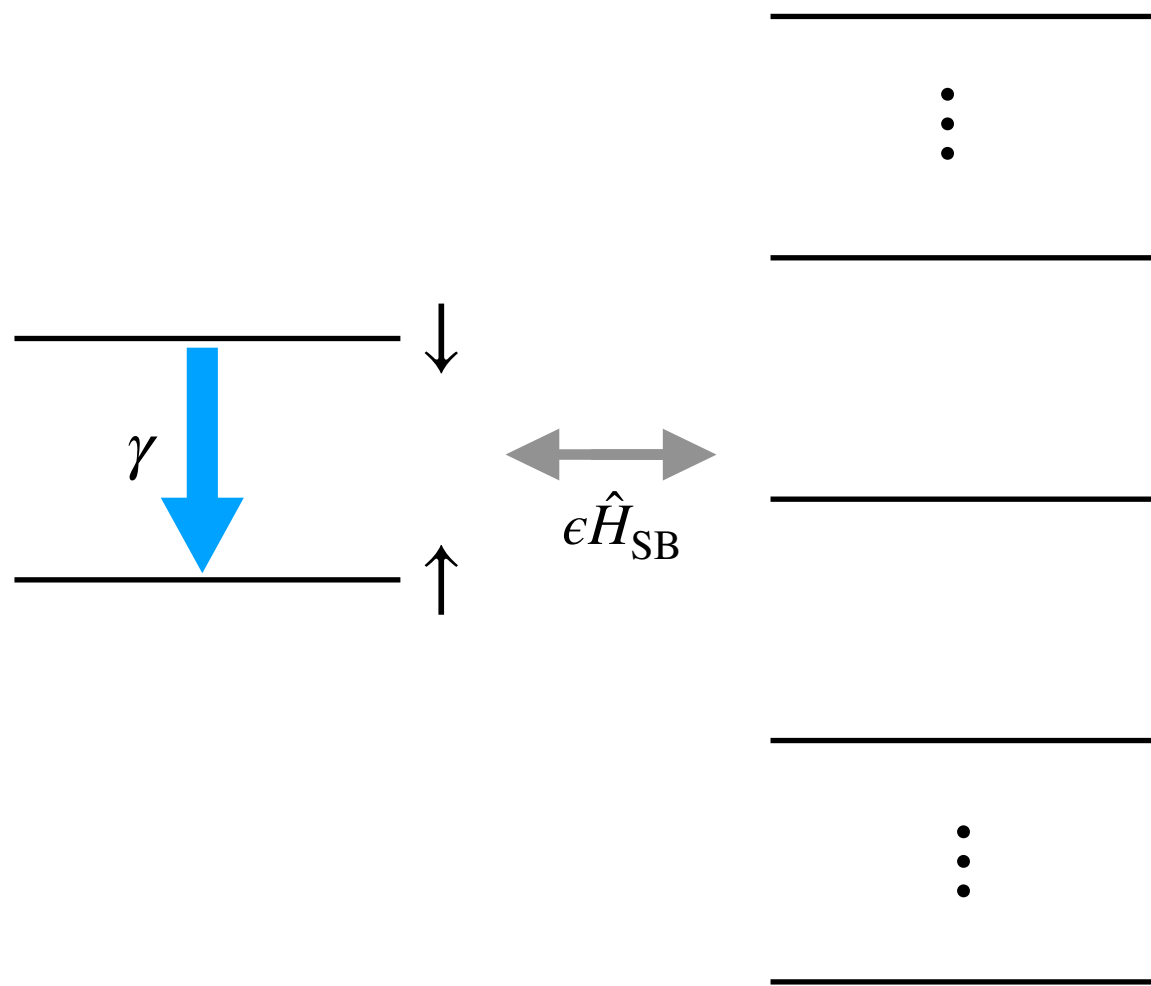
No energy exchange \implies temperature difference remains

Symmetry breaking in the coupling

$$\hat{H}_L = -\hat{\sigma}^z$$

$$\hat{H}_S = \Delta_S \hat{S}^z$$

$$\hat{H}_{SL} = \hat{\sigma}^x \hat{S}^x + \overset{\downarrow}{\eta} \hat{\sigma}^y \hat{S}^y$$

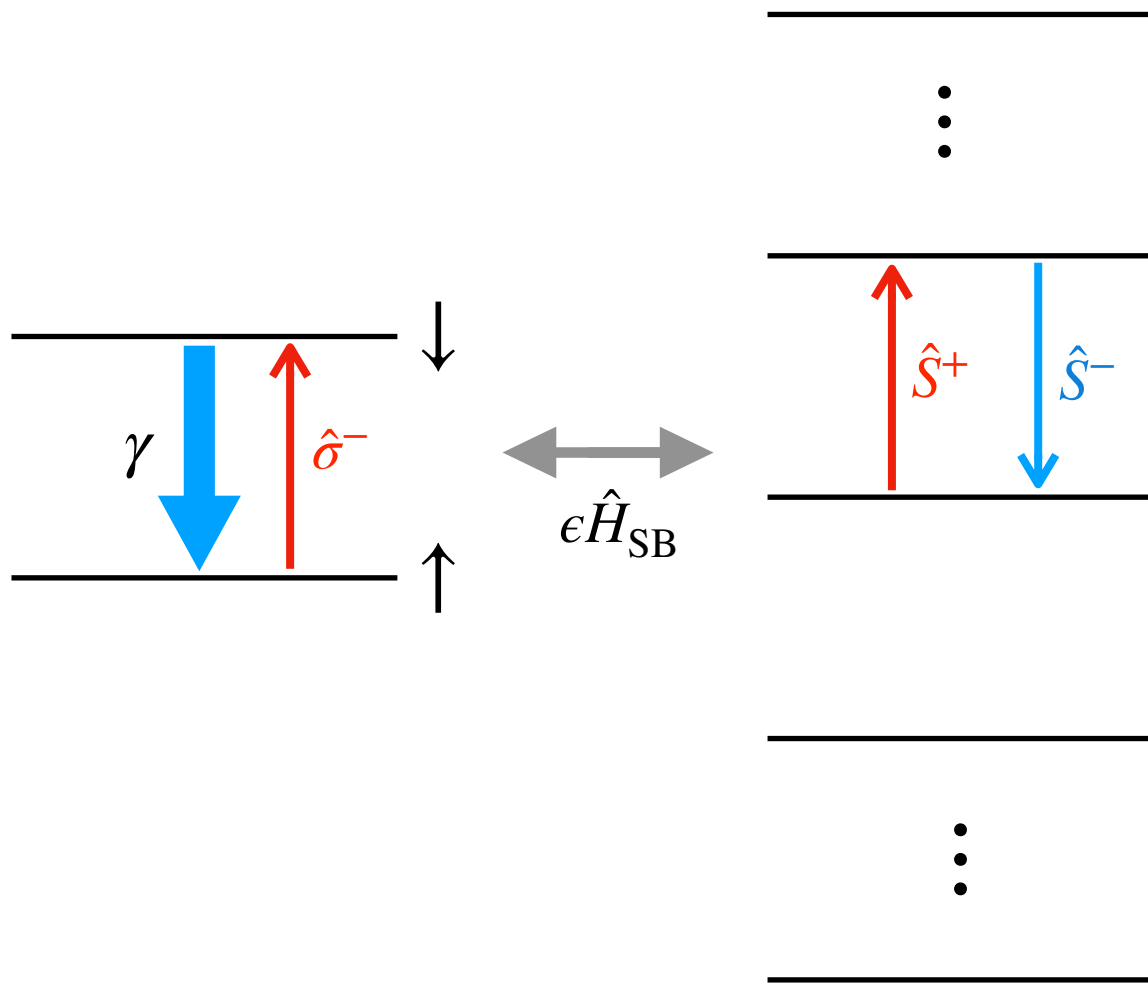


Symmetry breaking in the coupling

$$\hat{H}_L = -\hat{\sigma}^z$$

$$\hat{H}_S = \Delta_S \hat{S}^z$$

$$\begin{aligned} \hat{H}_{SL} &= \hat{\sigma}^x \hat{S}^x + \eta \hat{\sigma}^y \hat{S}^y \\ &= \frac{1+\eta}{2} (\hat{\sigma}^+ \hat{S}^- + \hat{\sigma}^- \hat{S}^+) + \frac{1-\eta}{2} (\hat{\sigma}^+ \hat{S}^+ + \hat{\sigma}^- \hat{S}^-) \end{aligned}$$

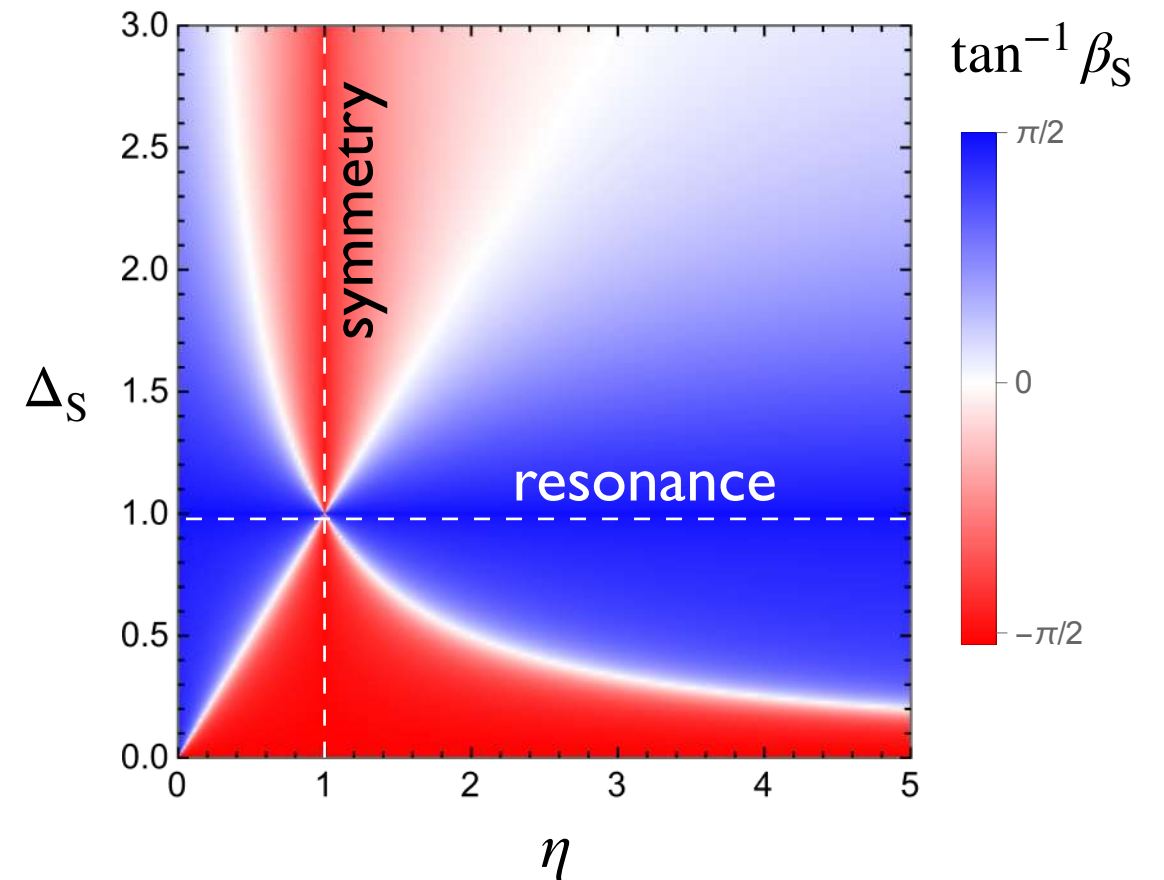
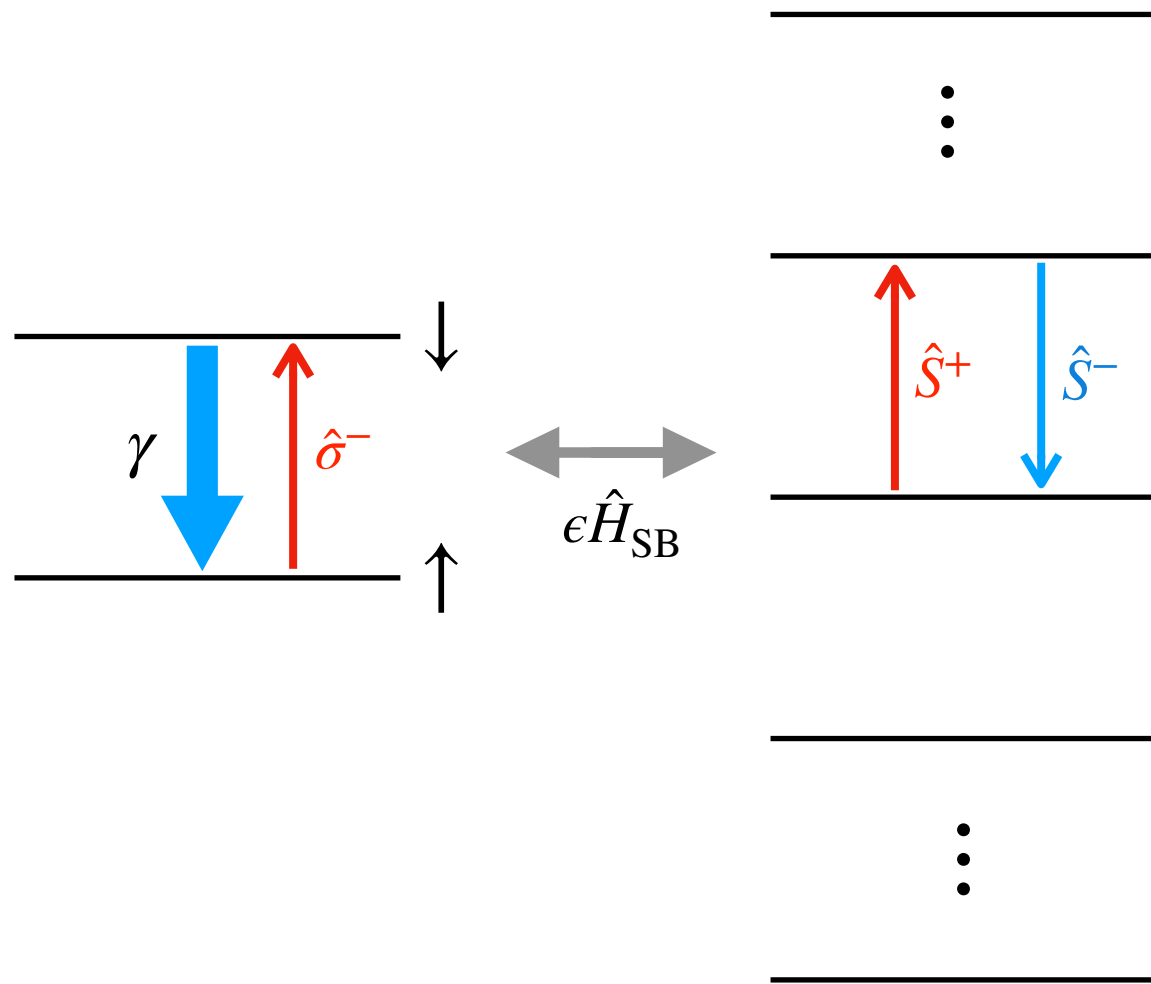


Symmetry breaking in the coupling

$$\hat{H}_L = -\hat{\sigma}^z$$

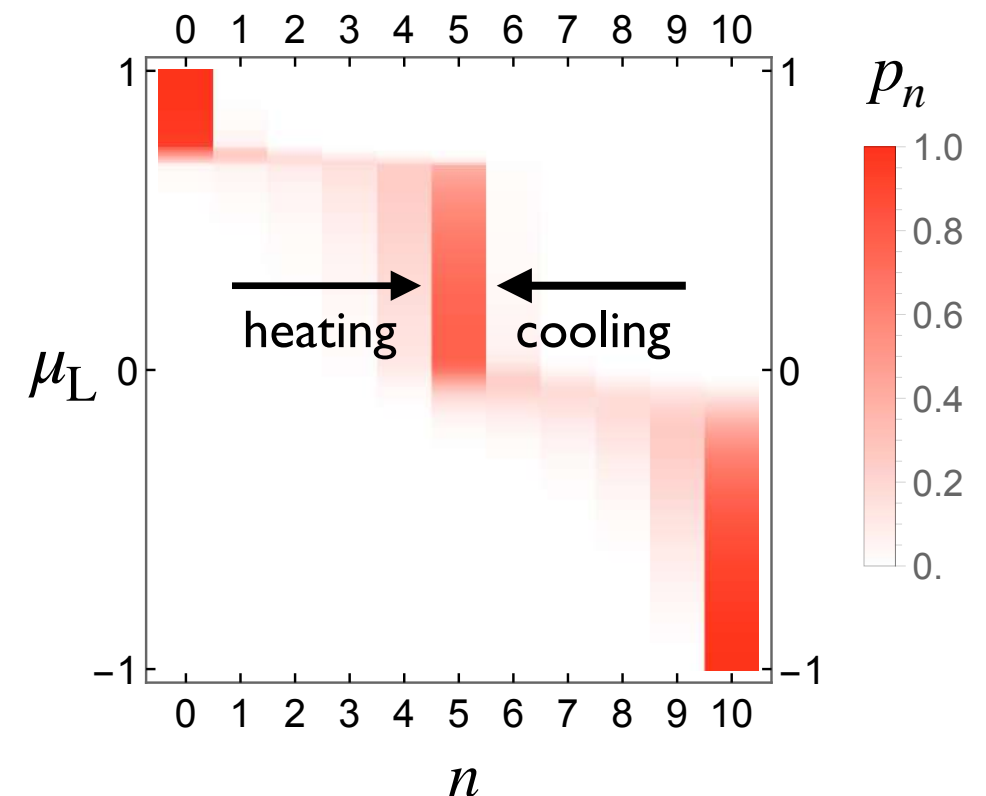
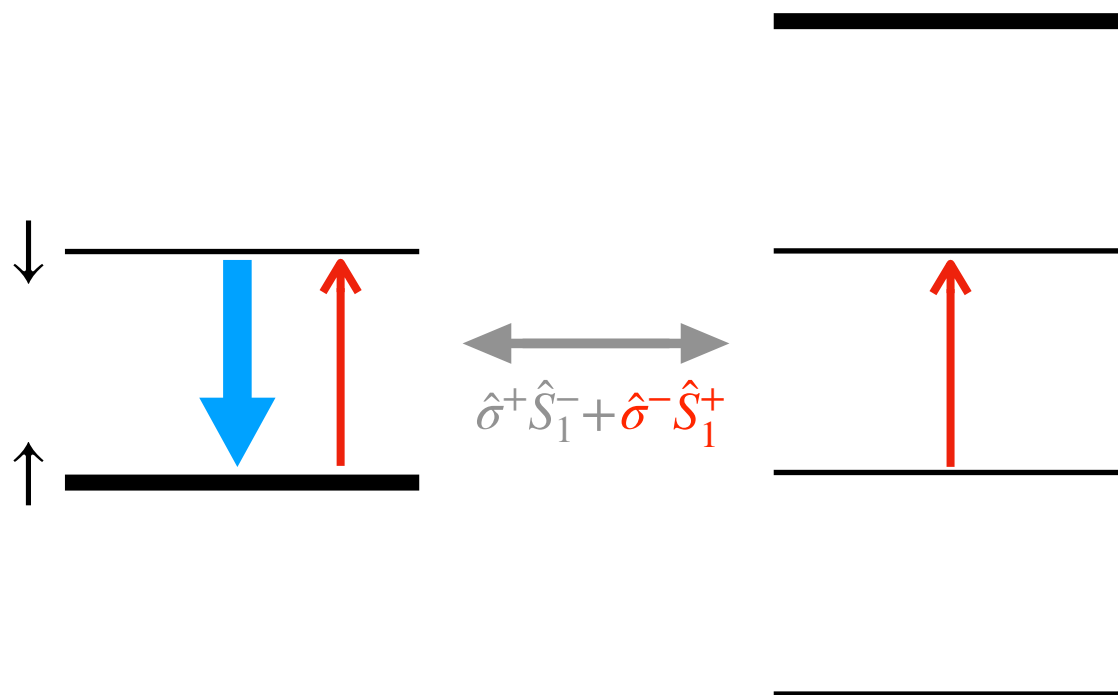
$$\hat{H}_S = \Delta_S \hat{S}^z$$

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Steady state has detailed balance:
$$\beta_S = \frac{2}{\Delta_S} \ln \left| \frac{1-\eta}{1+\eta} \cdot \frac{1+\Delta_S}{1-\Delta_S} \right|$$

- “Anti-thermalization” due to conserved quantum number related to energy
- N -photon Fock state & other non-thermal steady states
- Realize in circuit/cavity QED, Rydberg array, cold atoms/ions...



J Uppalapati, M Haque, P McClarty, SD, [arXiv:2412.07630](https://arxiv.org/abs/2412.07630)