

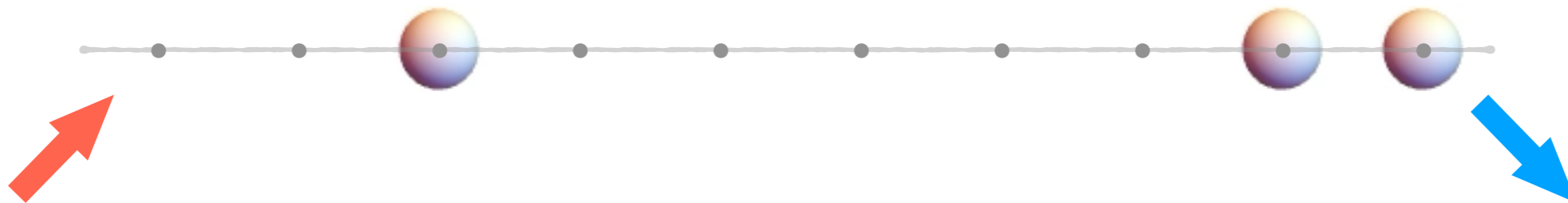
Ergodicity breaking in kinetically constrained open quantum systems

Shovan Dutta

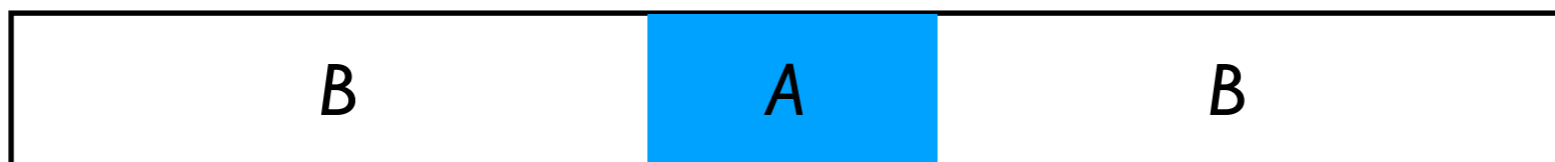
Raman Research Institute



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IIST



Thermalisation in isolated quantum systems



- Subsystem evolves to thermal state: $\lim_{t \rightarrow \infty} \hat{\rho}_A(t) = \text{Tr}_B(\hat{\rho}^{\text{eq}}) \sim e^{-\beta \hat{H}_A}$
- B acts as thermal bath for A

Ergodicity: Long-time average = microcanonical ensemble avg
(for any initial state in a small energy window)

⇒ **Eigenstate Thermalisation Hypothesis (ETH)**

- Local operators are smooth functions of energy $\langle E_\alpha | \hat{O} | E_\beta \rangle \sim \delta_{\alpha,\beta} f_O(E_\alpha)$
- Level repulsion (Wigner-Dyson statistics)
- Expected to hold for generic nonintegrable systems

D'Alessio, Kafri, Polkovnikov, Rigol, *Adv. Phys.* 2016
Gogolin, Eisert, *Rep. Prog. Phys.* 2016

Ergodicity breaking in isolated quantum systems

Taxonomy:

Moudgalya, Bernevig, Regnault, Rep. Prog. Phys. 2022

Papic, arXiv 2021

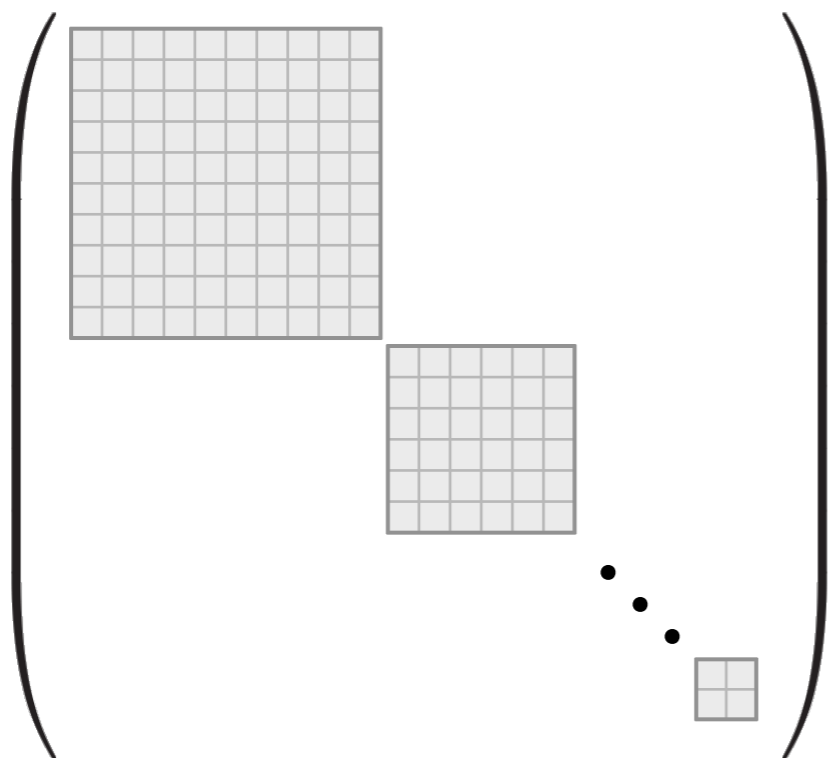
	Strong/weak ETH	Entanglement	Level statistics
Ergodic	Yes/yes	Volume	Wigner–Dyson
Integrable	No/no	Volume/sub-volume	Poisson
MBL	No/no	Area	Poisson
Quantum scarred weakly fragmented	No/yes	Volume/sub-volume	Wigner–Dyson
Strongly fragmented	No/no	Volume/sub-volume	Poisson

- *Integrability*
 - extensive number of conserved quantities; No ETH (e.g. transverse Ising)
- *Many-body localisation*
 - eigenstates localised due to disorder; No ETH (e.g. disordered XXZ)
- *Quantum scars*
 - small set of initial states don't thermalise; Weak ETH (e.g. PXP)
- ***Hilbert-space fragmentation***
 - different levels of ergodicity breaking in the same system

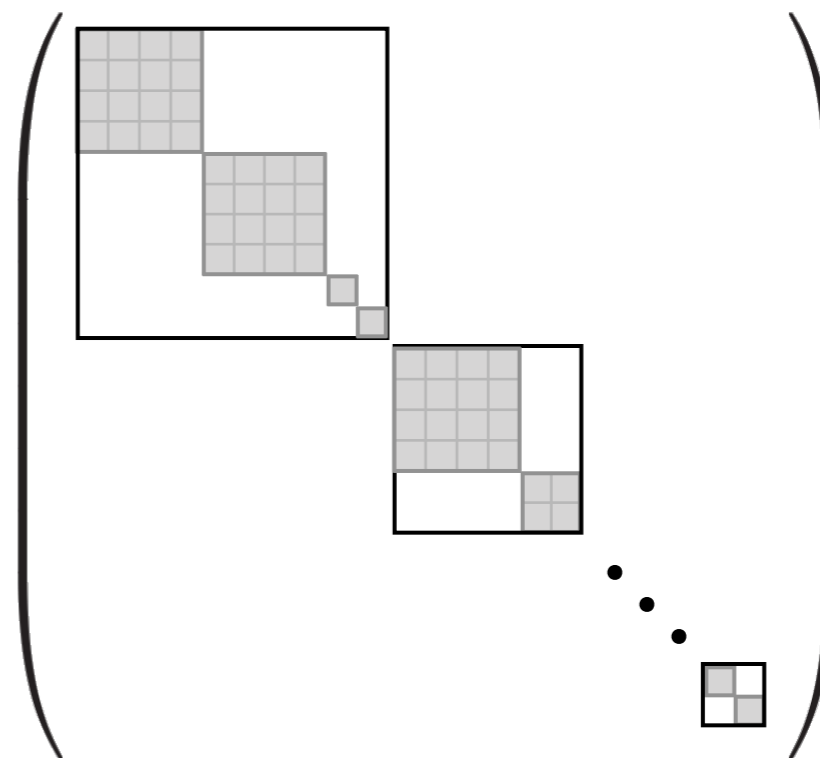
Hilbert-space fragmentation

Kinetic constraints fragment Hilbert space into exponentially many subspaces

Conventional symmetry



Fragmented Krylov subspaces

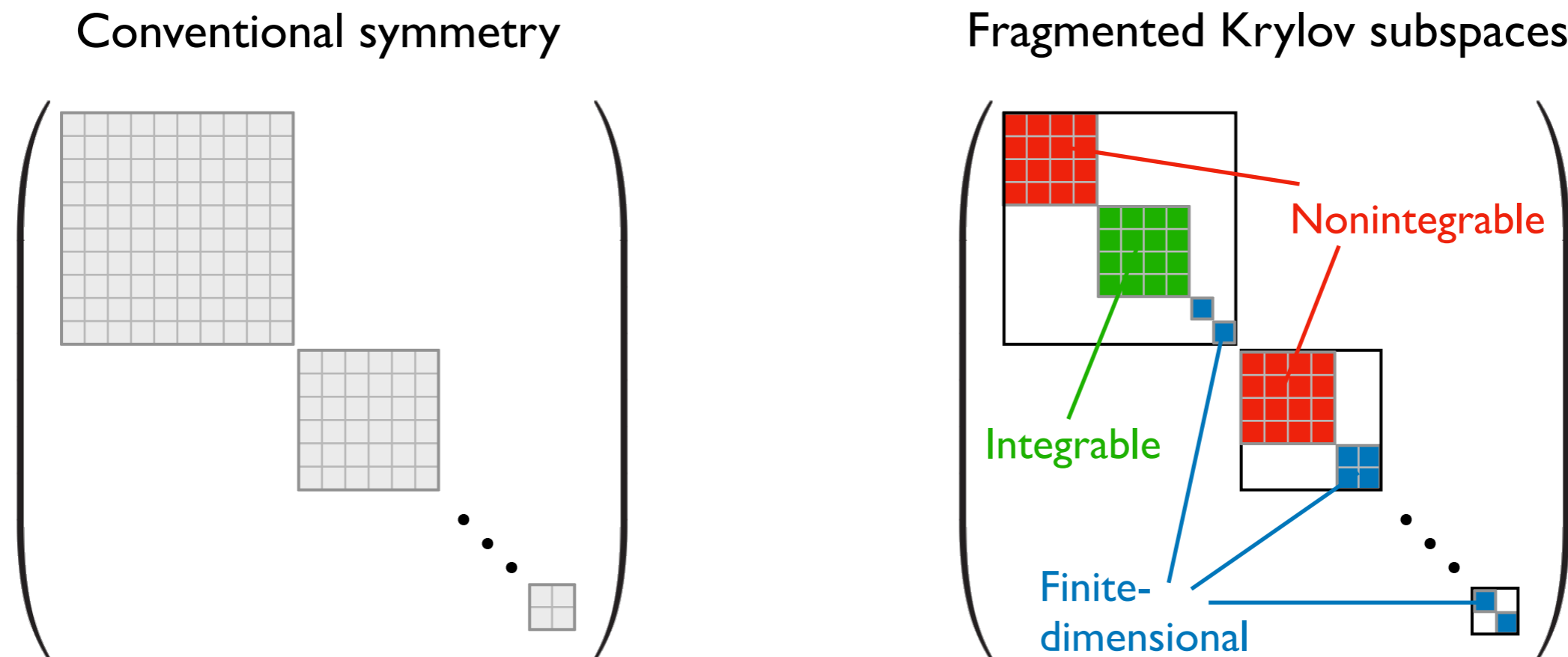


Moudgalya, Prem, Nankishore, Regnault, Bernevig, arXiv 2019

Hahn, McClarty, Luitz, SciPost 2021

Hilbert-space fragmentation

Kinetic constraints fragment Hilbert space into exponentially many subspaces



Dynamics within different Krylov subspaces can satisfy full / weak / no ETH!

Moudgalya, Prem, Nandkishore, Regnault, Bernevig, arXiv 2019

Hahn, McClarty, Luitz, SciPost 2021

Fragmentation in a dipole-conserving chain

\hat{H} that conserves a $U(1)$ charge and its dipole moment

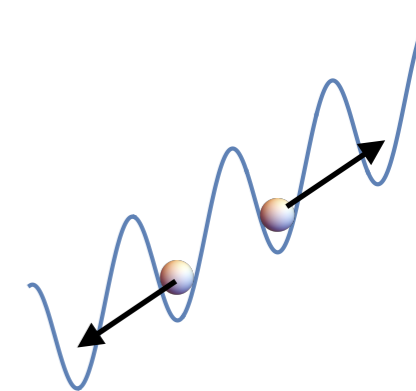
$$\hat{H} = \sum_{i=2}^{L-2} (\hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_{i+1} \hat{c}_{i+2}^\dagger + \text{h.c.}) + V(\{\hat{n}_i\})$$

Pair-hopping model
Moudgalya *et al* '19, '22

— conserves particle number N & dipole moment $D = \sum_i i n_i$ (centre of mass)

— models (i) interacting fermions on a strongly tilted lattice

(ii) quantum Hall physics on a thin cylinder



of (N, D) sectors $\sim L^3/6$ — polynomially many

Frozen states: $\underbrace{1\ 1\ 1}_{\geq 3}\ \underbrace{0\ 0\ 0}_{\geq 3}\ 1\ 1\ 1\dots 0\ 0\ 0$ — exponentially many

2-dimensional subspaces: $1\ 1\ 1\ 0\ 0\ 0\ \boxed{1\ 0\ 0\ 1}\ 0\ 0\ 0\dots 1\ 1\ 1$ — exponentially many

Exponentially large integrable subspace: $\underbrace{1\ 0\ 0}_{\uparrow}\ \underbrace{1\ 0\ 1}_{\downarrow}\ \underbrace{1\ 0\ 0}_{\downarrow}\ \underbrace{1\ 0\ 0}_{\uparrow}\ \underbrace{1\dots 1\ 0}_{\downarrow}$ — \hat{H}_{XX}

Interaction range controls fragmentation

$$\hat{H} = \sum_i \sum_{j=i+1}^{i+r} (\hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_j \hat{c}_{j+1}^\dagger + \mathbf{h.c.}) + V(\{\hat{n}_i\})$$

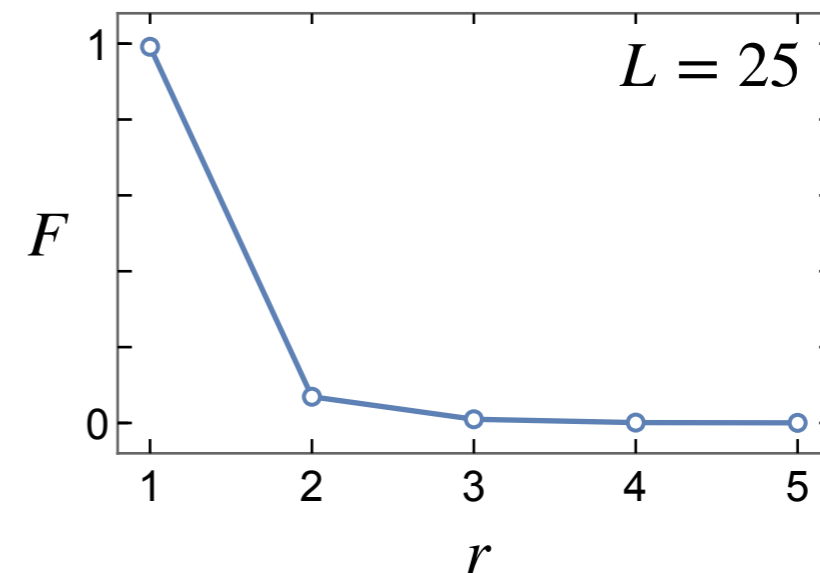
- Fragmented for any finite r

Exponentially many frozen states

$$\underbrace{1 \ 1 \dots 1}_{\geq r+2} \ 0 \ 0 \dots 0 \ 1 \ 1 \dots 1 \dots$$
$$\underbrace{\hspace{10em}}_{\geq r+2}$$

- Degree of fragmentation falls with r

$$F = 1 - \frac{\text{max subspace dimension}}{\text{size of symmetry sector}}$$



Sala, Rakovszky, Verresen, Knap, Pollmann, PRX 2020

Khemani, Hermele, Nandkishore, PRB 2020

Interaction range controls transport

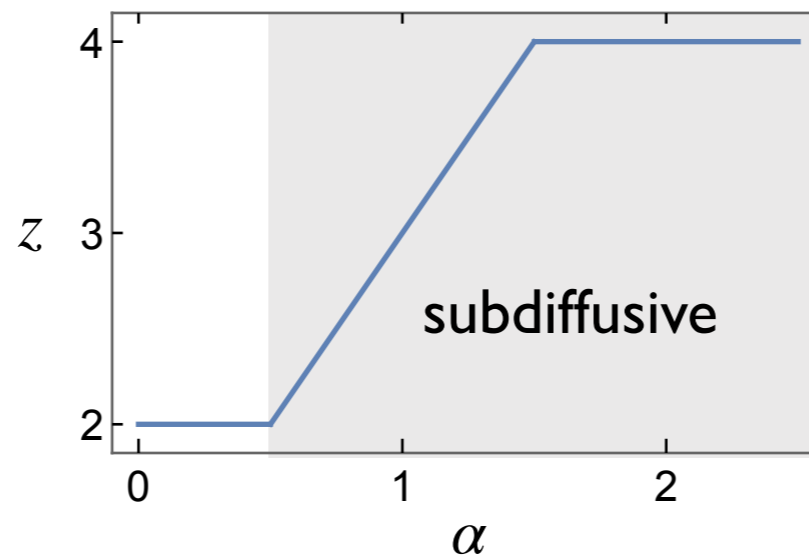
$$\hat{H} = \sum_i \sum_j \frac{J}{|i-j|^\alpha} (\hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_j \hat{c}_{j+1}^\dagger + \text{h.c.}) + V(\{\hat{n}_i\})$$

How long-wavelength excitations relax (to uniform infinite-temperature state)

$$e^{-\gamma_k t}, \quad \gamma_k \sim k^z \quad \text{for small } k \quad z : \text{dynamical exponent}$$

$\alpha \rightarrow \infty$: charge & dipole locally conserved \implies subdiffusion w/ $z = 4$

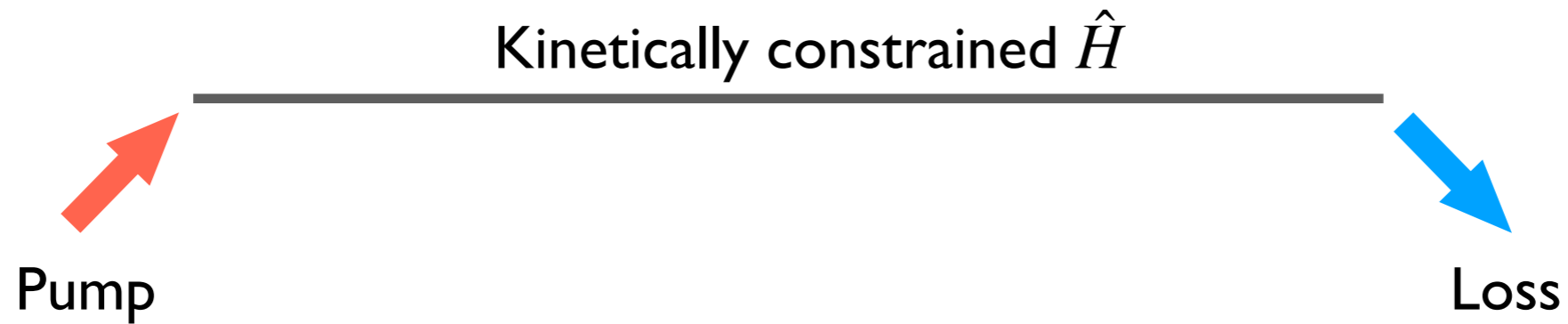
$\alpha \rightarrow 0$: only charge locally conserved \implies diffusion ($z = 2$)



Morningstar, O'Dea, Richter, PRB 2023

Gliozzi, May-Mann, Hughes, De Tomasi, arXiv 2023

Our setup

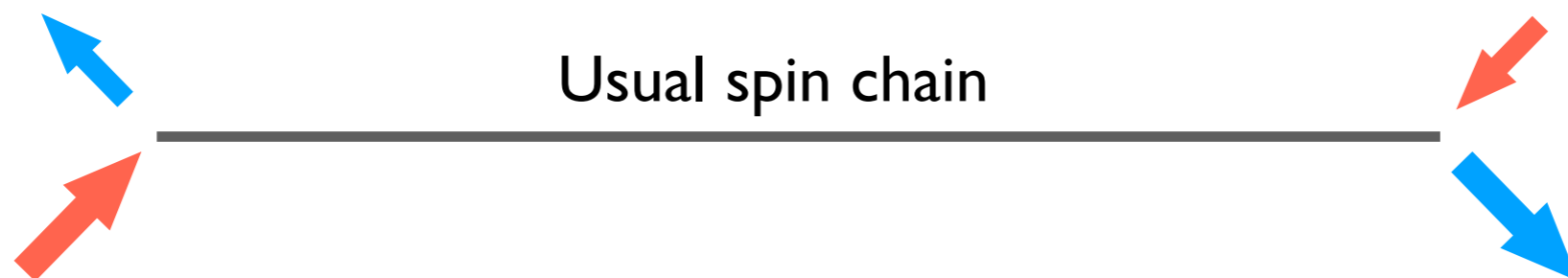


Can we drive a current?

What is the structure of the steady states?

Does fragmentation survive?

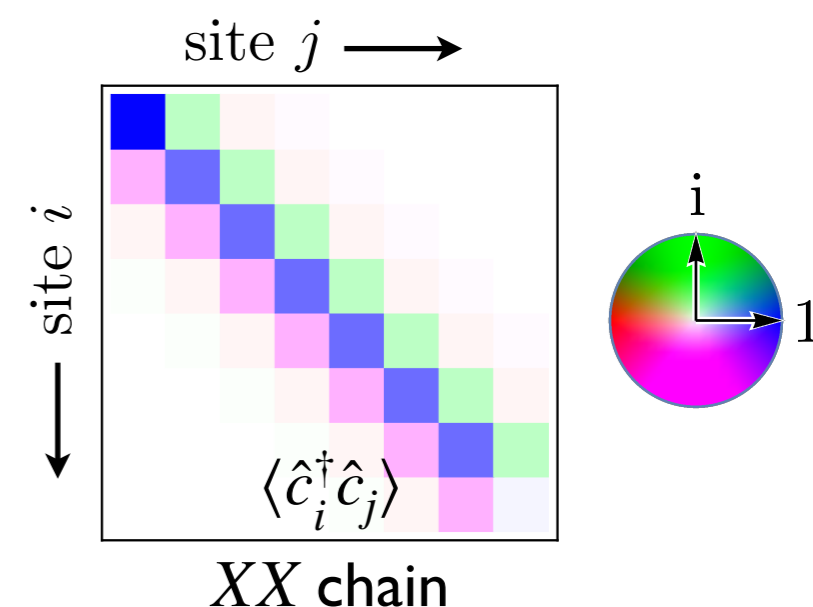
Boundary-driven spin chains



Typically: Unique current-carrying steady state

Large body of literature:

- Dissipative phase transitions
- Ballistic, diffusive, anomalous transport
- Exact matrix-product state solutions
- \vdots



Landi, Poletti, Schaller, RMP 2022

Prosen, Pižorn, PRL 2008

Žnidarič, PRL 2011

Prosen, PRL 2011

Purkayastha, Dhar, Kulkarni, PRB 2017

Steady state(s) of (Markovian) dissipative systems

- Generically, steady state is unique

$\{\hat{H}, \hat{L}_1, \hat{L}_2, \dots\}$ generate the entire algebra of operators (“ergodic”) Evans, 1977

- *Strong symmetry* \implies Multiple mixed steady states

$[\hat{S}, \hat{H}] = [\hat{S}, \hat{L}_j] = 0$ — each symmetry sector has at least 1 steady state

Buča, Prosen, NJP 2012

Albert, Jiang, PRA 2014

- *Decoherence-free subspace (DFS)*

Eigenstates of \hat{H} that are unaffected by dissipation: $\hat{L}_j |E_k\rangle = 0 \quad \forall j$

DFS: $\{|E_k\rangle, k = 1, 2, \dots, \Lambda\}$ — evolves unitarily Λ : DFS dimension

Lidar, Chuang, Whaley, PRL 1998

- *Noiseless subsystem*

Eigenstates of the form $|\alpha_i\rangle_A \otimes |\beta_j\rangle_B$ where dissipation acts only on A

\implies steady states: $\hat{\rho}_A \otimes |\beta_j\rangle\langle\beta_k|$

Knill, Laflamme, Viola, PRL 2000

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- **Memory of initial state, quantum memory, passive error correction** Lidar, Brun, 2013

Back to our setup

$$\hat{H} = \sum_{i < j} J_{i,j} (\hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_j \hat{c}_{j+1}^\dagger + \text{h.c.}) + V(\{\hat{n}_i\})$$

Two limits:

- All $J_{i,j}$ are nonzero \implies No fragmentation

All Fock states with a given (N, D) are connected by \hat{H}

- $J_{i,j} \neq 0$ only if $j = i + 1$ (pair-hopping model) \implies Strongly fragmented

Two types of boundary drive:

- Pump & loss at opposite ends
- Pump & loss at both ends

Note: pump/loss does NOT preserve N or D

Case I A.

Unfragmented (but dipole-conserving) \hat{H}

+

Pump-loss at *opposite* ends

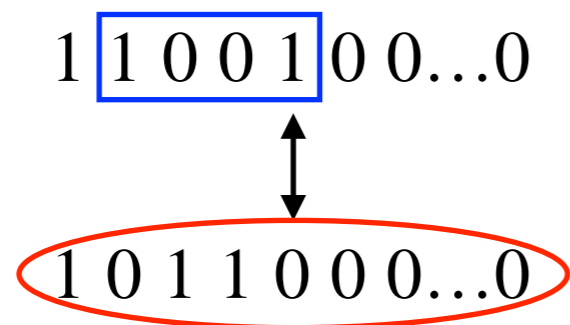
Case I A. Unfragmented + pump-loss at opposite ends

Other frozen states? $1\ 0\ 1\ 0\ 0\ 0\dots 0$ ✓
 $1\ 1\ 0\ 1\ 0\ 0\dots 0$ ✓

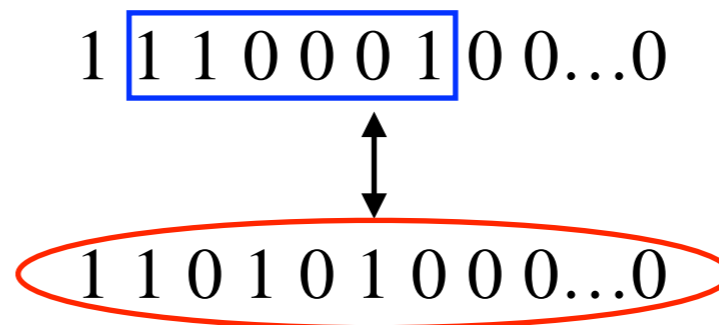
$$\Rightarrow \underbrace{1\ 1\dots 1}_{\geq 1} \boxed{0\ 1} \underbrace{0\ 0\dots 0}_{\geq 1} \quad (L - 3) \text{ frozen states}$$

How about $1\ 0\ 1\ 0\ 1\ 0\ 0\dots 0 \leftrightarrow 0\ 1\ 1\ 1\ 0\ 0\ 0\dots 0 \longrightarrow 1\ 1\ 1\ 1\ 0\ 0\ 0\dots 0$
×

× $1\ 0\ 0\ 1\ 0\ 0\dots 0 \leftrightarrow 0\ 1\ 1\ 0\ 0\ 0\dots 0 \longrightarrow 1\ 1\ 1\ 0\ 0\ 0\dots 0$



2-dimensional DFS



3-dimensional DFS

Other states w/
same (N, D)

Case I A. Unfragmented + pump-loss at opposite ends

Root configurations

$$\begin{array}{ccccccc}
 & & \text{Active block} & & & & \\
 & & \text{1 1...1} & \text{1 1...1} & \text{0 0...0} & \text{1 0 0...0} & \\
 \underbrace{\hspace{1.5cm}}_{\geq 1} & \underbrace{\hspace{1.5cm}}_{n-1} & \underbrace{\hspace{1.5cm}}_n & \underbrace{\hspace{1.5cm}}_{\geq 1} & & & n \geq 0 \\
 & \underbrace{\hspace{3cm}}_{\text{Frozen wings}} & & & & &
 \end{array}$$

Couples to other configurations with the same (N, D)

How large is the DFS?

$$\underbrace{x_1 + x_2 + \dots + x_n}_{\text{positions of 1's}} = \frac{n(n-1)}{2} + 2n \quad 1 \leq x_1 < x_2 < \dots < x_n \leq 2n$$

$$y_i := x_i - i \implies y_1 + y_2 + \dots + y_n = n \quad 0 \leq y_1 \leq y_2 \leq \dots \leq y_n \leq n$$

\implies DFS size = Number of integer partitions of $n \equiv p(n) !$

$$\text{Ramanujan: } p(n) \sim \frac{1}{4n\sqrt{3}} \exp(\pi\sqrt{2n/3}) \quad n \leq L/2 - 1$$

Case I A. Unfragmented + pump-loss at opposite ends

Root configurations

$$\begin{array}{c}
 \text{Active block} \\
 \underbrace{1 \ 1 \dots 1}_{\geq 1} \ \underbrace{1 \ 1 \dots 1}_{n-1} \ \underbrace{0 \ 0 \dots 0}_n \ \underbrace{1 \ 0 \ 0 \dots 0}_{\geq 1} \quad n \geq 0 \\
 \underbrace{\hspace{10em}}_{\text{Frozen wings}}
 \end{array}$$

DFS labeled by (N, n) or equivalently (N, D)

$$N = 1, 2, \dots, L - 1$$

$$n = 0, 1, \dots, \min(N - 1, L - N - 1)$$

Number of disjoint DFSs $\sim L^2/4$

Total number of (N, D) sectors $\sim L^3/6$

\implies Only $O(1/L)$ of the sectors become DFSs

DFS size $\Lambda_{N,n} = p(n)$

\implies Total number of decoherence-free states = $\sum_{N,n} \Lambda_{N,n} \sim \exp(\pi\sqrt{L/3})$

No net flow in steady state!

Case I B.

Unfragmented (but dipole-conserving) \hat{H}

+

Pump-loss at *both* ends

Case I B. Unfragmented + pump-loss at both ends

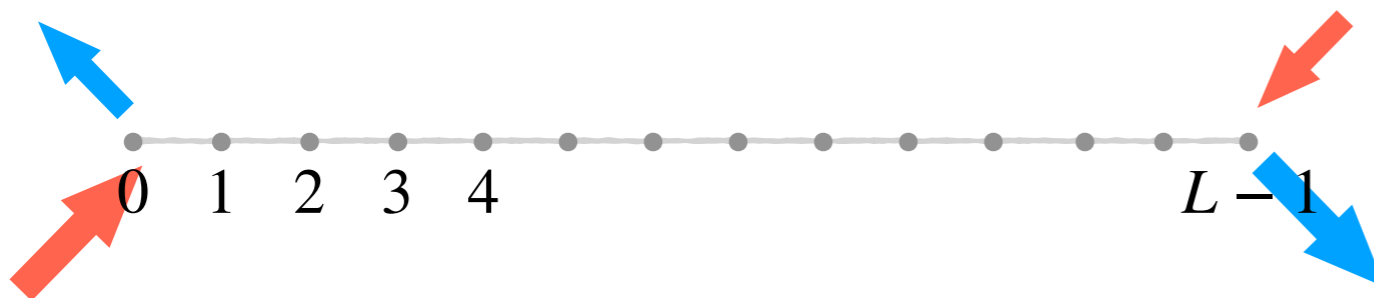
Minimal model $\hat{H} = \sum_{i,j} \hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_j \hat{c}_{j+1}^\dagger$ $\hat{L}_1 = \hat{c}_1^\dagger$ $\hat{L}_2 = \hat{c}_L$ $\hat{L}_3 = \sqrt{\gamma} \hat{c}_1$ $\hat{L}_4 = \sqrt{\gamma} \hat{c}_L^\dagger$

No frozen states

$$1\ 1\ 0\ 1\ 0\ 0\dots 0 \longrightarrow 0\ 1\ 0\ 1\ 0\ 0\dots 0 \longleftrightarrow 1\ 0\ 0\ 0\ 1\ 0\dots 0 \longrightarrow 1\ 0\ 0\ 0\ 1\ 0\dots 1 \longleftrightarrow \dots$$

In fact, No DFS.

Unique steady state? **No, because there is a strong symmetry!** Buča, Prosen, NJP 2012



$$D \rightarrow D \pm (L - 1)$$

$$\implies D \bmod (L - 1) \equiv D_{\text{mod}} \text{ is preserved!}$$

$$\implies L - 1 \text{ mixed steady states w/ } D_{\text{mod}} = 0, 1, 2, \dots, L - 2$$

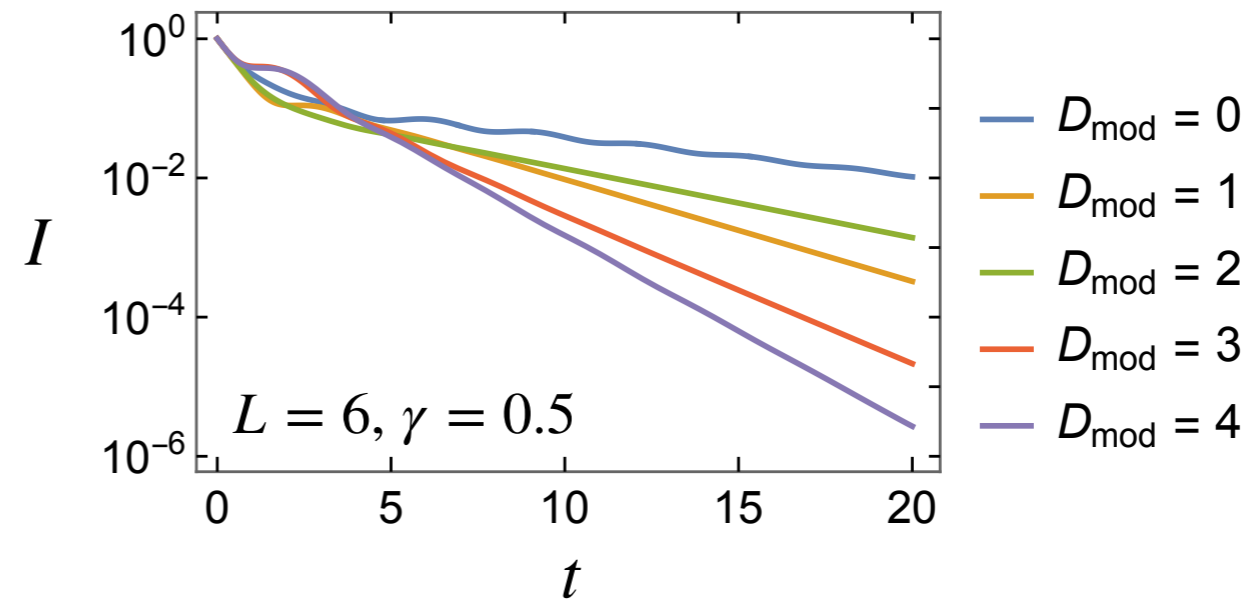
Case I B. Unfragmented + pump-loss at both ends

Current in steady states? NO

$$\hat{L}_1 = \hat{c}_1^\dagger \quad \hat{L}_2 = \hat{c}_L \quad \hat{L}_3 = \sqrt{\gamma} \hat{c}_1 \quad \hat{L}_4 = \sqrt{\gamma} \hat{c}_L^\dagger$$

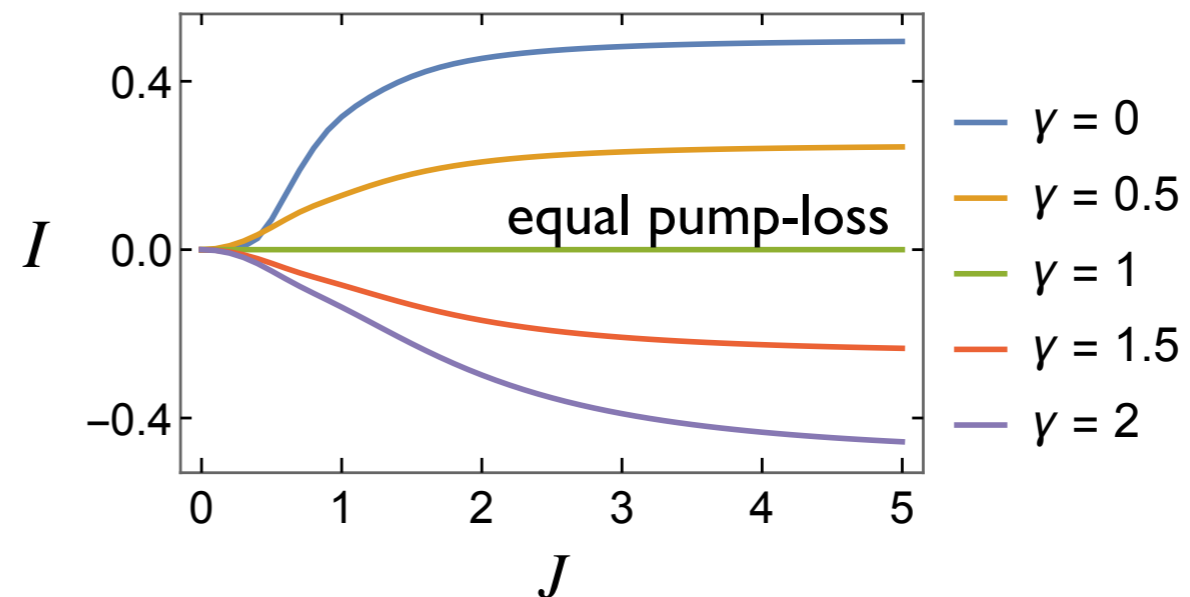
(heating within each sector)

$I \equiv$ net injection
rate at site 1



Current restored by breaking symmetry in \hat{H}

$$\hat{H} = \sum_{i,j} \hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_j \hat{c}_{j+1}^\dagger - J \sum_i \hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}$$



Case II A.

Strongly *fragmented* \hat{H}

+

Pump-loss at *opposite* ends

Case II A. *Fragmented* + pump-loss at *opposite* ends

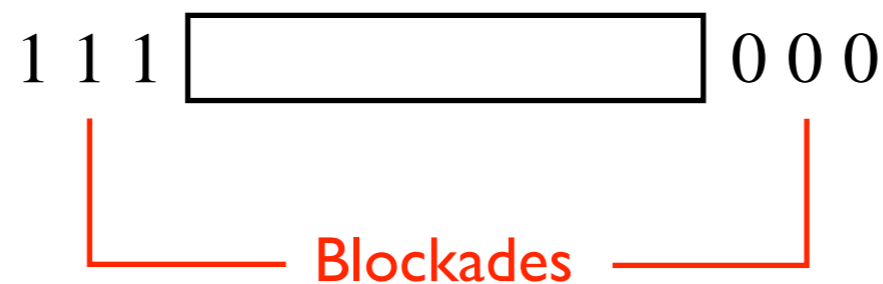
Minimal model $\hat{H} = \sum_i \hat{c}_{i-1}^\dagger \hat{c}_i \hat{c}_{i+1} \hat{c}_{i+2}^\dagger + \text{h.c.}$ $\hat{L}_1 = \hat{c}_1^\dagger$ $\hat{L}_2 = \hat{c}_L$

More constrained \implies larger number of decoherence-free states (Λ)

Previously $2L - 4$ frozen configurations — Now exponentially many:

$$\underbrace{1\ 1\ 1}_{\geq 3}\ \underbrace{0\ 0\ 0}_{\geq 3}\ 1\ 1\ 1\dots 0\ 0\ 0$$

Decoherence-free (not necessarily frozen) states:



$$\implies \Lambda \geq 2^{L-6}$$

(many disjoint subspaces)

Can we find Λ exactly? Yes!

Case II A. *Fragmented* + pump-loss at *opposite* ends

Which states start with 1, ends with 0 but NOT decoherence free?

$$1\ 0\ 0\ 1\ \boxed{1\ 0} \quad 0 \quad 2^{L-5}$$

$$1\ 0\ 1\ 0\ 0\ 1\ \boxed{} \quad 0 \quad 2^{L-7}$$

Case II A. *Fragmented* + pump-loss at *opposite* ends

Which states start with 1, ends with 0 but NOT decoherence free?

$$\begin{array}{l}
 1\ 0\ 0\ \underline{1}\ \boxed{0\ 0\ 1} \ 0 \quad 2^{L-5} \\
 1\ 0\ 1\ 0\ 0\ \underline{1}\ \boxed{} \ 0 \quad 2^{L-7} \\
 1\ 0\ 0\ 0\ 1\ \underline{1}\ 0\ \boxed{} \ 0 \quad 2^{L-8}
 \end{array}$$

Expansion rule:



⇒ 1001 family

$$\uparrow \dots \uparrow \mathbf{0} \downarrow \dots \downarrow \mathbf{1} \uparrow \dots \uparrow \mathbf{0} \downarrow \dots \dots \boxed{\downarrow \uparrow} \boxed{} \ 0$$

$$\uparrow \equiv 1\ 0 \quad \downarrow \equiv 0\ 1$$

Active edge

Case II A. *Fragmented* + pump-loss at *opposite* ends

Which states start with 1, ends with 0 but NOT decoherence free?

$$\begin{array}{l}
 1\ 0\ 0\ \underline{1}\ \boxed{0\ 0\ 1} \ 0 \quad 2^{L-5} \\
 1\ 0\ 1\ 0\ 0\ \underline{1}\ \boxed{} \ 0 \quad 2^{L-7} \\
 1\ 0\ 0\ 0\ 1\ \underline{1\ 0}\ \boxed{} \ 0 \quad 2^{L-8}
 \end{array}$$

Expansion rule:



\implies 1001 family

$$\begin{array}{l}
 \uparrow \dots \uparrow \mathbf{0} \downarrow \dots \downarrow \mathbf{1} \uparrow \dots \uparrow \mathbf{0} \downarrow \dots \dots \boxed{\uparrow \downarrow} \boxed{} \ 0 \\
 \uparrow \equiv 1\ 0 \quad \downarrow \equiv 0\ 1 \quad \text{Active edge}
 \end{array}$$

Case II A. *Fragmented* + pump-loss at *opposite* ends

Similarly, there is a 0110 family expanding from the right:

1 0 1 1 0

1 0 1 1 0 1 0

1 1 0 0 1 1 1 0

1 ↑ ↓ ↓ 1 ↑ ... ↑ 0 ↓ ... ↓ 1 ↑ ... ↑

Excluding states in these two families gives all decoherence-free states

$$\Rightarrow \Lambda \approx (2/5)^2 \times 2^L$$

Case II B.

Strongly *fragmented* \hat{H}

+

Pump-loss at *both* ends

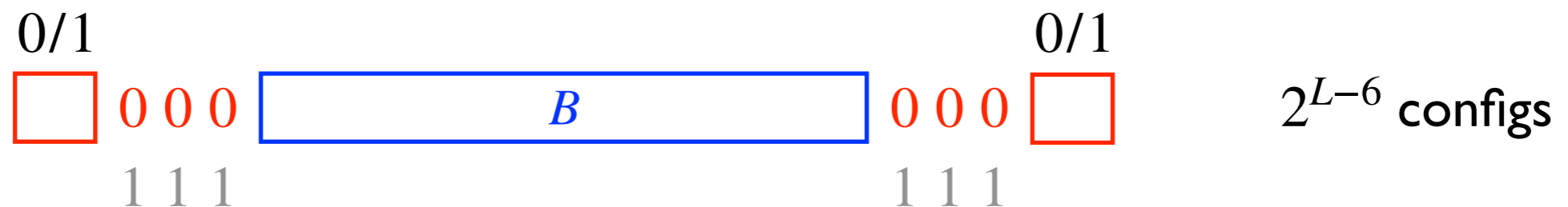
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Recall: Case I B had $(L - 1)$ mixed steady states — expect more here

But no eigenstate of \hat{H} is a null state of $\hat{L}_j \forall j \implies$ No DFS

Shield bulk from boundary by inserting blockades:

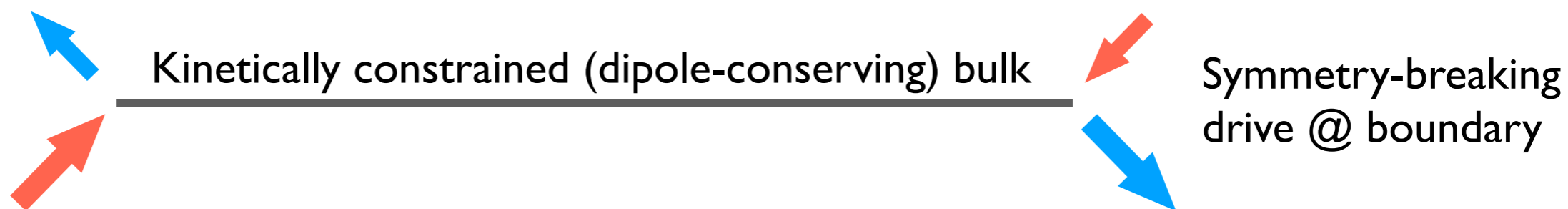


Steady states: $\hat{\rho}_A \otimes |\varphi_B^j\rangle\langle\varphi_B^k|$ $|\varphi_B^k\rangle$: eigenstates of \hat{H}_B

\implies Exponentially large (fragmented) **noiseless subsystem** — info in B preserved

Again, no net flow in steady state

Summary



- Current suppressed in steady state — restored by breaking symmetry in bulk
- Hierarchy of steady states from interplay of fragmentation & dissipation

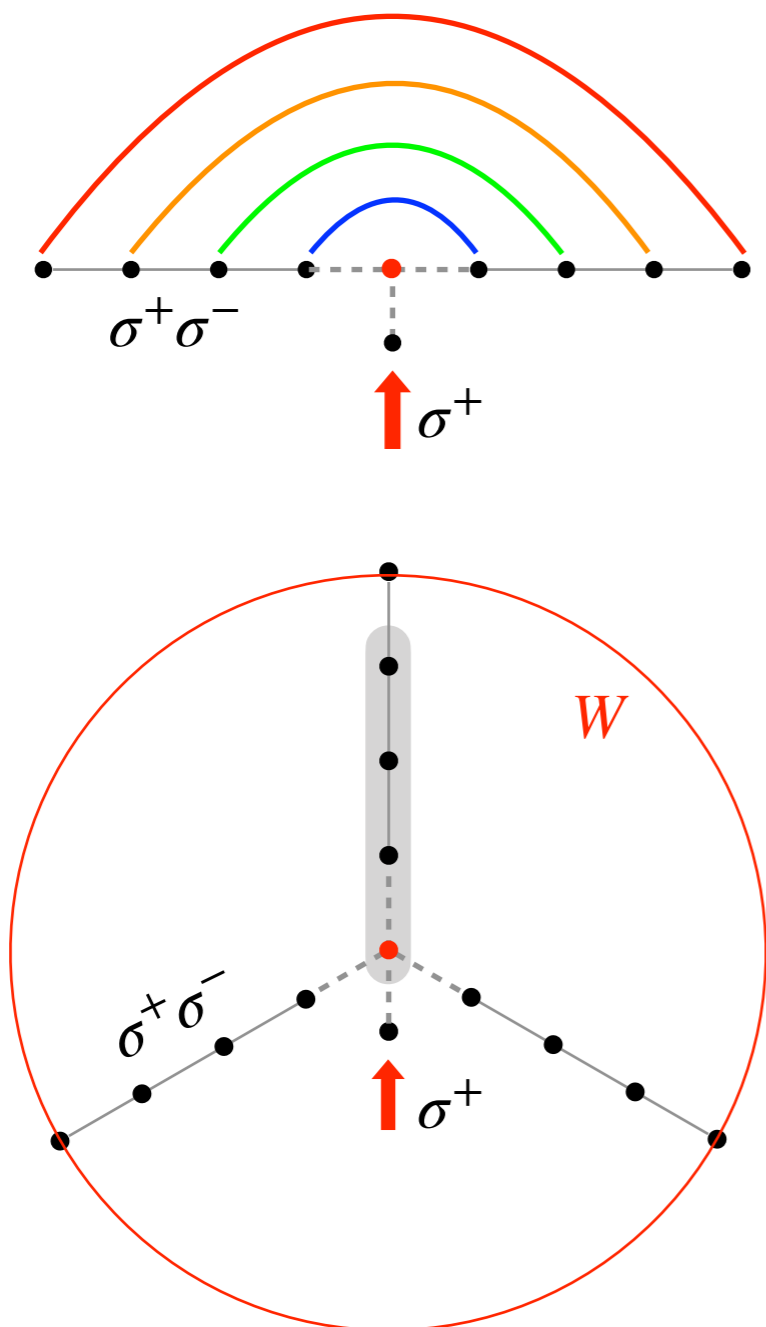
	Un-fragmented \hat{H}	Fragmented \hat{H}
Pump-loss @opposite ends	$O(L^2)$ DFSs , $\Lambda \sim e^{\pi\sqrt{L/3}}$	Fragmented DFS , $\Lambda \sim 2^L$
Pump-loss @both ends	$L - 1$ mixed steady states	Fragmented NS , $\Lambda \sim 2^L$

- Non-ergodicity in open quantum systems from fragmentation

A Srivastava and SD, in prep

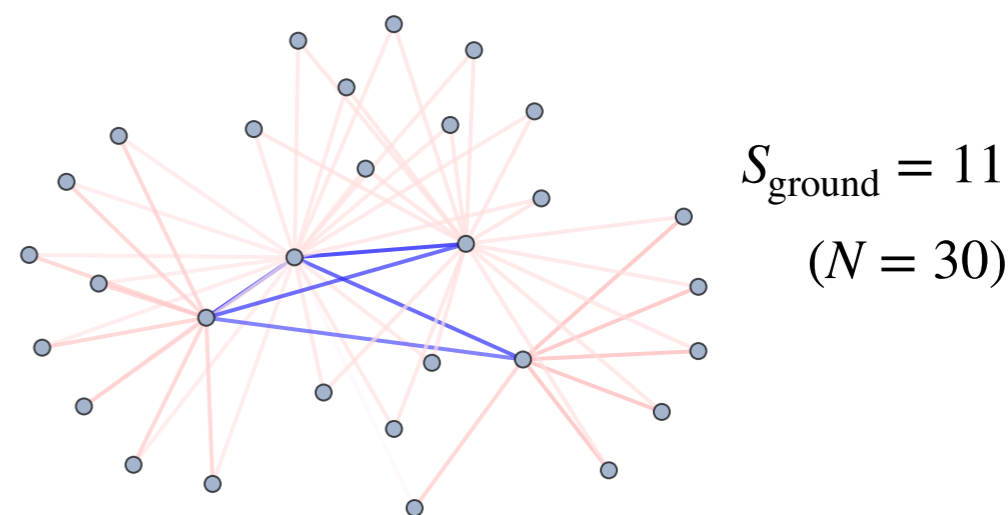
Other recent developments

Long-range multipartite entanglement from local drive & static coupling



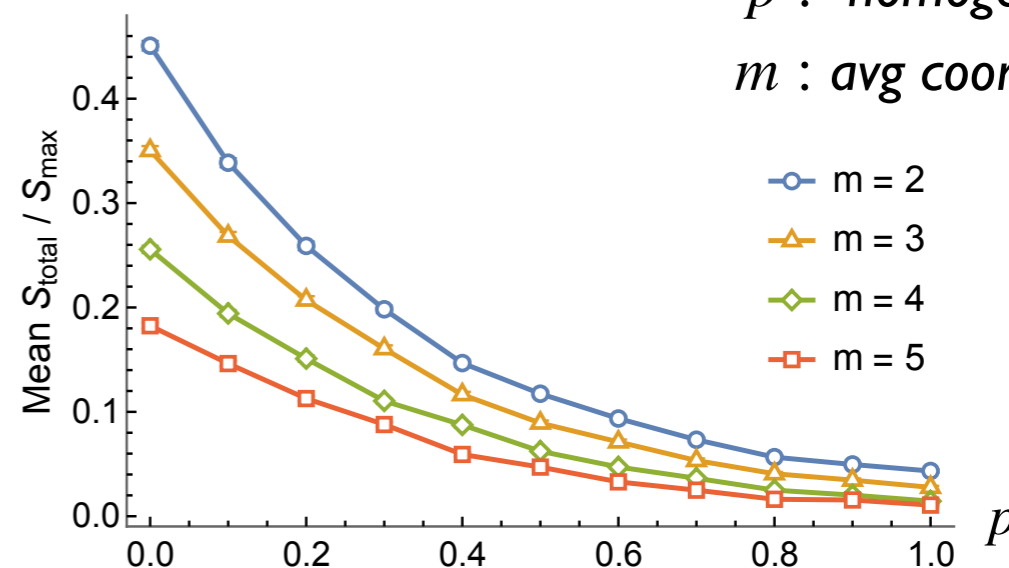
Quantum many-body physics on complex networks

qubits + $\hat{S}_i \cdot \hat{S}_j$ bonds — frustrated



$S_{\text{ground}} = 11$
($N = 30$)

p : “homogeneity”
 m : avg coord no.





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	Un-fragmented \hat{H}	Fragmented \hat{H}
Pump-loss @opposite ends	$O(L^2)$ DFSs , $\Lambda \sim e^{\pi\sqrt{L/3}}$	Fragmented DFS , $\Lambda \sim 2^L$
Pump-loss @both ends	$L - 1$ mixed steady states	Fragmented NS , $\Lambda \sim 2^L$

- Non-ergodicity in open quantum systems from fragmentation

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